# CONCORDIA UNIVERSITY
## FACULTY OF ENGINEERING AND COMPUTER SCIENCE
## APPLIED ORDINARY DIFFERENTIAL EQUATIONS - ENGR 213/2 sect. T, FALL 2013

**Section T**
**Date of Mid Term Test: Monday, October 7, 2013**
**Time: 1:15 – 2:45 PM**
**Instructor: Dr. Rama Bhat**

This is a closed book exam. Solve all questions. All the questions carry equal marks.
Only faculty approved calculators are allowed.

**PROBLEM No. 1.**

(a) Verify that $y(x) = e^{3x}\cos 2x$ is an explicit solution of $y'' - 6y' + 13y = 0$.

(b) Write the solution in the implicit form.

**PROBLEM No. 2.** Given the differential equation: $\frac{dy}{dx} + \left(\frac{xy + y^2}{x^2}\right) = 0$

(a) Convert the equation to the form $M(x,y)dx+N(x,y)dy$, and show that the equation is not variable separable.

(b) Show that functions $M(x,y)$ and $N(x,y)$ are homogeneous.

(c) Using an appropriate substitution, say, $y=ux$, convert the equation to variable separable form.

(d) Solve the equation to obtain $y(x)$.

**PROBLEM No. 3.** Given the following differential equation

$$\frac{dy}{dx} + \frac{2xy}{(x^2-1)} = 0$$

(a) Convert the equation to $M(x,y)dx + N(x,y)dy=0$ form and show that it is an exact equation.

(b) Solve using the method of exact differentials.

**PROBLEM No. 4.** Using integrating factor, solve the following differential equation

$$x\frac{dy}{dx} - 4y = x^6 e^x$$

**PROBLEM No. 5.**

(a) Write the complex expression in the a+ib form.

$$\frac{(4+5i)+2i^3}{(2+i)^2}$$

(b) Write the result above in (a) in the polar form.

# ENGR 213/2 — 2013 FALL

## MIDTERM TEST 1 — SOLUTIONS

**Problem 1:**

DE —: $y'' - 6y' + 13y = 0$

Verify: $y(x) = e^{3x} \cdot \cos 2x$

$y'(x) = 3e^{3x} \cdot \cos 2x - 2e^{3x} \sin 2x$

Add

$y''(x) = 9e^{3x} \cos 2x - 12e^{3x} \sin 2x - 4e^{3x} \cos 2x$

$-6y'(x) = -18e^{3x} \cos 2x + 12e^{3x} \sin 2x$

$+13y = 13e^{3x} \cos 2x$

Result of addition $= 0$. Hence verified

**Problem 2:**

(a) $(x\cancel{y} + y^2)\,dx + x^2\,dy = 0$

$M\,dx + N\,dy = 0$

$N = N(x)$, however, $M \neq M(y)$.

Hence not separable.

(b)

$M(tx, ty) = t^2 M(x, y)$

$N(tx, ty) = t^2 N(x, y)$

Hence they are homogeneous

(c) $y = ux$; $dy = x\,du + u\,dx$

Substituting in DE

$(x^2u + x^2u^2)\,dx + x^2(x\,du + u\,dx) = 0$

i.e. $(u + u^2)\,dx + x\,du + u\,dx = 0$

i.e. $(u^2+2u)\,dx = -x\,du$

$$\frac{du}{u^2+2u} = -\frac{dx}{x} \qquad \text{Separable form.}$$

(d) $$\frac{1}{u^2+2u} = \frac{a}{u} + \frac{b}{u+2}$$

i.e. $$\frac{au+2a+bu}{u(u+2)} = \frac{(a+b)u+2a}{u(u+2)}$$

Comparing numerators $2a = 1$

$a+b = 0$

Hence $a = 1/2,\quad b = -\frac{1}{2}.$

$$\int\left(\frac{1/2}{u} - \frac{1/2}{u+2}\right) du = -\int\frac{dx}{x} + c$$

$$\frac{1}{2}\left[\ln u - \ln(u+2)\right] = -\ln x + c = \ln\frac{1}{x} + c$$

$$\ln\left(\frac{u}{u+2}\right)^{1/2} = \ln\frac{1}{x} + c$$

$$\sqrt{\frac{y/x}{y/x+2}} = \frac{c_1}{x}$$

i.e. $$\frac{y}{y+2x} = \frac{c_1^2}{x^2} \quad \text{i.e. } x^2 y = (y+2x)c_1^2$$

$$y(x^2 - c_1^2) = 2c_1^2 x$$

$$y = \frac{2c_1^2 x}{x^2 - c_1^2}$$

Problem 3:

(a) $$\frac{dy}{dx} + \frac{2xy}{(x^2-1)} = 0$$

$$(x^2-1)\,dy + 2xy\,dx = 0$$

or $$\underbrace{2xy}_{M}\,dx + \underbrace{(x^2-1)}_{N}\,dy = 0$$

$$\frac{\partial M}{\partial y} = 2x \;;\; \frac{\partial N}{\partial x} = 2x .$$ Therefore homogeneous.

(b) $$f(x,y) = \int M\,dx + h(y) = x^2y + h(y)$$

$$\frac{\partial f}{\partial y} = N(x,y) = x^2 + h'(y)$$

$$\therefore \quad x^2-1 = x^2 + h'(y) \qquad \therefore \; h'(y) = -1$$

$$h(y) = -y .$$

$$\therefore f(x,y) = x^2y - y = c$$

$$y = \frac{c}{x^2-1}$$

Problem 4:

$$x\frac{dy}{dx} - 4y = x^6 e^x$$

$$\frac{dy}{dx} - \frac{4y}{x} = x^5 e^x$$

$$P(x) = -\frac{4}{x} \;;\; I.F. = e^{-\int \frac{4}{x}dx} = e^{-4\ln x} = e^{\ln \frac{1}{x^4}} = \frac{1}{x^4}.$$

$$\frac{d}{dx}\left[\frac{1}{x^4}y\right] = \frac{1}{x^4}x^5e^x = x\,e^x$$

$$\therefore \frac{1}{x^4}y = \int x\,e^x\,dx + c.$$

$$\frac{y}{x^4} = xe^x - e^x + c$$

$$y = x^4 e^x (x-1) + c\, x^4$$

Problem 5 :

a) $$z = \frac{(4+5i)+2i^3}{(2+i)^2} = \frac{4+5i-2i}{4-1+4i} = \frac{4+3i}{3+4i}$$

$$= \frac{(4+3i)(3-4i)}{9+16} = \frac{12+12+i(9-16)}{25}$$

$$= \frac{24-7i}{25} = \frac{24}{25} - \frac{7}{25}i$$

b) $$|z| = \sqrt{\left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2} = 1$$

$$\tan\theta = -\frac{7}{24} \; ; \quad \theta = -\tan^{-1}\left(\frac{7}{24}\right)$$

$z = |z| (\cos\theta + i \sin\theta)$ in polar form.

$|z| \cdot \text{Arg} \cdot \theta$