Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 24

- PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM
- PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES
- ANGULAR MOMENTUM
- MOMENT OF A FORCE AND ANGULAR IMPULSE
- MOMENTUM PRINCIPLES

Section 12.1-12.2,12.5-12.7

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PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM

Sure! When a stake is struck by a sledgehammer, a large impulsive force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact,

how can we determine the magnitude of the impulsive force delivered to the stake?

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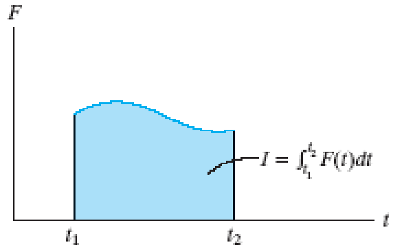
PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM
(Section 12.1)

Linear momentum: The vector m***v*** is called the linear momentum, denoted as ***L***. This vector has the same direction as ***v***.
The linear momentum vector has units of (kg·m)/s or (slug·ft)/s.

Linear impulse: The integral ∫***F*** dt is the linear impulse, denoted ***I***. *It is a vector quantity measuring the effect of a force during its time interval of action.*
I acts in the same direction as ***F*** and has units of N·s or lb·s.



Variable Force

Graphically, Impulse it can be represented by the area under the force versus time curve,
If ***F*** is constant, then

$$\boldsymbol{I} = \boldsymbol{F}\,(t_2 - t_1)\ .$$

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PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM
(continued)

The principle of linear impulse and momentum is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

$$\sum \boldsymbol{F} = m\,\boldsymbol{a} = m\,(d\boldsymbol{v}/dt)$$

Separating variables and integrating between the limits $\boldsymbol{v} = \boldsymbol{v}_1$ at $t = t_1$ and $\boldsymbol{v} = \boldsymbol{v}_2$ at $t = t_2$ results in

$$\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = m \int_{\boldsymbol{v}_1}^{\boldsymbol{v}_2} d\boldsymbol{v} = m\boldsymbol{v}_2 - m\boldsymbol{v}_1$$

This equation represents the principle of linear impulse and momentum.
It relates the particle's final velocity, $\boldsymbol{v}_2$, and initial velocity ($\boldsymbol{v}_1$) and the forces acting on the particle as a function of time.

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IMPULSE AND MOMENTUM: SCALAR EQUATIONS

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component scalar equations:

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.

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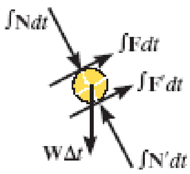
PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES

As the wheels of this pitching machine rotate, they apply frictional impulses to the ball, thereby giving it linear momentum in the direction of ***F***dt and ***F'***dt.

Does the release velocity of the ball depend on the mass of the ball?



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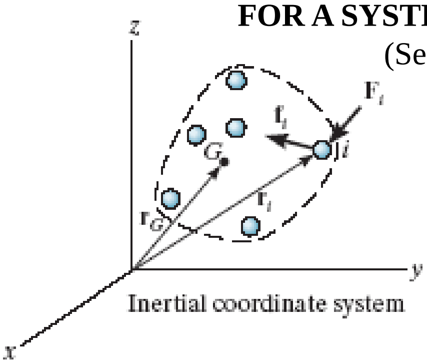
PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES

(Section 12.2)



For the system of particles shown, the internal forces f_i between particles always occur in pairs with equal magnitude and opposite directions.
Thus the internal impulses sum to zero.

The linear impulse and momentum equation for this system only includes the impulse of external forces.

$$\sum m_i(v_i)_1 + \sum \int_{t_1}^{t_2} F_i \, dt = \sum m_i(v_i)_2$$

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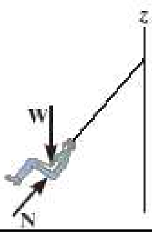
ANGULAR MOMENTUM, ANGULAR IMPULSE AND MOMENTUM PRINCIPLES

The passengers on the amusement-park ride experience conservation of angular momentum about the axis of rotation (the z-axis).

As shown on the free body diagram, the line of action of the normal force, N, passes through the z-axis and the weight's line of action is parallel to it.

Therefore, the sum of moments of these two forces about the z-axis is zero.



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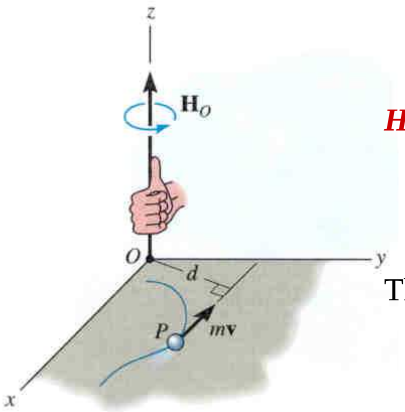
ANGULAR MOMENTUM

(Section 12.5)

The angular momentum of a particle about point O is defined as the "moment" of the particle's linear momentum about O.



$$\boldsymbol{H}_o = \boldsymbol{r} \times m\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

The magnitude of $\boldsymbol{H}_o$ is $(H_o)_z = mvd$

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