Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 22

- THE WORK OF A FORCE
- PRINCIPLE OF WORK AND ENERGY
- PRINCIPLE OF WORK AND ENERGY FOR A SYSTEM OF PARTICLES
- POWER AND EFFICIENCY

Section 11.1-11.4

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THE WORK OF A FORCE, PRINCIPLE OF WORK AND ENERGY, & PRINCIPLE OF WORK AND ENERGY FOR A SYSTEM OF PARTICLES

Objectives:

Students will be able to:

1. Calculate the work of a force.
2. Apply the principle of work and energy to a particle or system of particles.

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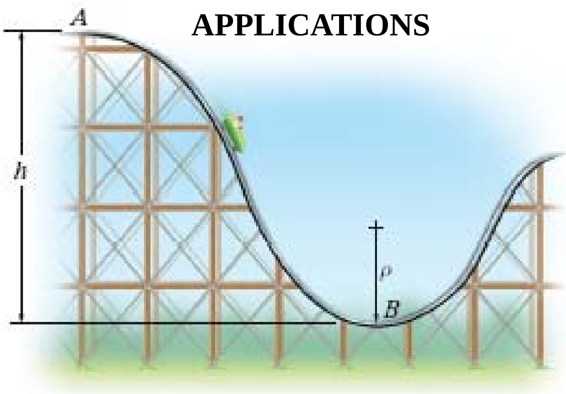
APPLICATIONS



A roller coaster makes use of gravitational forces to assist the cars in reaching <u>high speeds</u> in the “valleys” of the track.

How can we design the track to control the forces experienced by the passengers? e.g.:
• The height, h,
• The radius of curvature, ρ

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APPLICATIONS
(continued)

Crash barrels are often used along roadways for crash protection.

The barrels absorb the car’s kinetic energy by deforming.

If we know the typical <u>velocity</u> of an oncoming car and the amount of <u>energy that can be absorbed by each barrel,</u>

How can we design a crash cushion?

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WORK AND ENERGY

Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion (***F*** = m***a***) with respect to displacement.

By substituting $a_t = v\,(dv/ds)$ into $F_t = ma_t$, the result is integrated to yield an equation known as the principle of work and energy.

This principle is useful for solving problems that involve force, velocity, and displacement.
It can also be used to explore the concept of power.

To use this principle, we must first understand how to calculate the work of a force.

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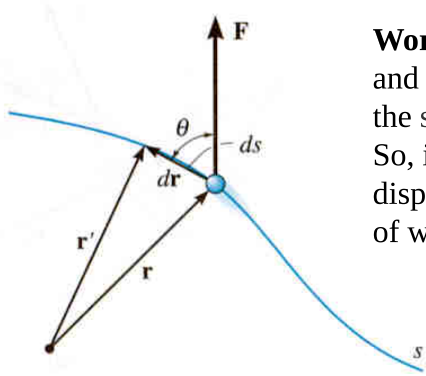
WORK OF A FORCE (Section 14.1)

A force does work on a particle when the particle undergoes a displacement along the line of action of the force.



Work is defined as the product of force and displacement components acting in the same direction.
So, if the angle between the force and displacement vector is θ, the increment of work dU done by the force is

$$dU = F \cos\theta \, ds$$

By using the definition of the dot product and integrating, the total work can be written as:

$$U_{1\text{-}2} = \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} \boldsymbol{F} \cdot d\boldsymbol{r}$$

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WORK OF A FORCE
(continued)

If ***F*** is a function of position (a common case) this becomes

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

If both F and θ are constant ($F = F_c$), this equation further simplifies to

$$U_{1\text{-}2} = F_c \cos\theta \, (s_2 - s_1)$$

- Work is positive if the force and the movement are in the same direction.
- If they are opposing, then the work is negative.
- If the force and the displacement directions are perpendicular, the work is zero. Recall cos 90 = 0

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WORK OF A WEIGHT

The work done by the gravitational force acting on a particle (or weight of an object) can be calculated by using

$$U_{1\text{-}2} = \int_{y_1}^{y_2} -W \, dy = -W (y_2 - y_1) = -W \Delta y$$

The work of a weight is the product of the magnitude of the particle's weight and its vertical displacement.

If Δy is upward, the work is negative since the weight force always acts downward (Recall cos 180 = -1).

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PRINCIPLE OF WORK AND ENERGY

(Section 14.2 & Section 14.3)

By integrating the equation of motion, $\sum F_t = ma_t = mv(dv/ds)$, the principle of work and energy can be written as

$$\sum U_{1\text{-}2} = 0.5m(v_2)^2 - 0.5m(v_1)^2 \quad \text{or} \quad T_1 + \sum U_{1\text{-}2} = T_2$$

$\sum U_{1\text{-}2}$ is the work done by all the forces acting on the particle as it moves from point 1 to point 2.
Work can be either a positive or negative scalar.

T_1 and T_2 are the kinetic energies of the particle at the initial and final position, respectively.
Thus, $T_1 = 0.5\ m\ (v_1)^2$ and $T_2 = 0.5\ m\ (v_2)^2$.
The kinetic energy is always a positive scalar (velocity is squared!).

So, the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle's final kinetic energy.

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PRINCIPLE OF WORK AND ENERGY

(continued)

Note that the principle of work and energy ($T_1 + \sum U_{1\text{-}2} = T_2$) is not a vector equation! Each term results in a scalar value.

Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a joule (J), where 1 J = 1 N·m. In the FPS system, units are ft·lb.

The principle of work and energy cannot be used, in general, to determine forces directed normal to the path, since these forces do no work (Recall cos 90 = 0).

The principle of work and energy can also be applied to a system of particles by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system.

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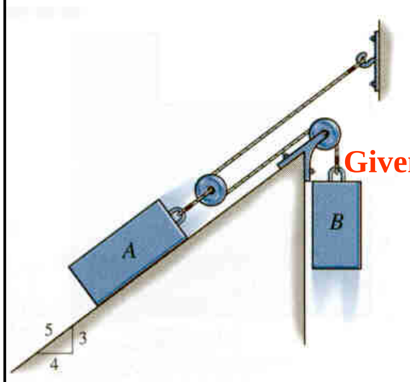
PROBLEM SOLVING



Given: Block A has a weight of 60 lb and block B has a weight of 10 lb. The coefficient of kinetic friction between block A and the incline is $\mu_k = 0.2$. Neglect the mass of the cord and pulleys

Find: The speed of block A after it moves 3 ft down the plane, starting from rest.

Plan:
1) Define the kinematic relationships between the blocks.
2) Draw the FBD of each block.
3) Apply the principle of work and energy to the system of blocks.

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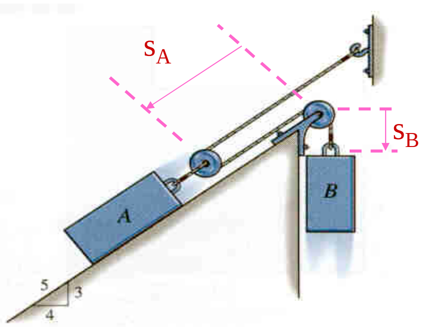
PROBLEM SOLVING
(continued)

Solution:

1) The kinematic relationships can be determined by defining position coordinates s_A and s_B, and then differentiating.



Since the cable length is constant:

$$2s_A + s_B = l$$
$$2\Delta s_A + \Delta s_B = 0$$
$$\Delta s_A = 3\text{ft} \quad \Rightarrow \quad \Delta s_B = -6 \text{ ft}$$

and $2v_A + v_B = 0$

$$\Rightarrow v_B = -2v_A$$

Note that, by this definition of s_A and s_B, positive motion for each block is defined as downwards.

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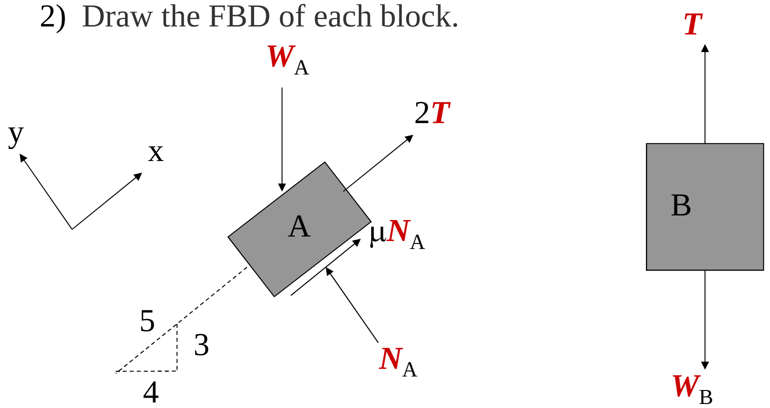
PROBLEM SOLVING
(continued)

2) Draw the FBD of each block.



Sum forces in the y-direction for block A (note that there is no motion in this direction):

$\sum F_y = 0$: $N_A - (4/5)W_A = 0$ => $N_A = (4/5)W_A$

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PROBLEM SOLVING
(continued)

3) Apply the principle of work and energy to the system (the blocks start from rest).

$$\sum T_1 + \sum U_{1\text{-}2} = \sum T_2$$

$(0.5m_A(v_{A1})^2 + 0.5m_B(v_{B1})^2) + ((3/5)W_A - 2T - \mu N_A)\Delta s_A$
$+ (W_B - T)\Delta s_B = (0.5m_A(v_{A2})^2 + 0.5m_B(v_{B2})^2)$

$v_{A1} = v_{B1} = 0$, $\Delta s_A = 3\text{ft}$, $\Delta s_B = -6\text{ ft}$, $v_B = -2v_A$, $N_A = (4/5)W_A$

=> $0 + 0 + (3/5)(60)(3) - 2T(3) - (0.2)(0.8)(60)(3) + (10)(-6)$
$- T(-6) = 0.5(60/32.2)(v_{A2})^2 + 0.5(10/32.2)(-2v_{A2})^2$

=> $v_{A2} = 3.52$ ft/s

Note that the work due to the cable tension force on each block cancels out.

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POWER AND EFFICIENCY

<u>Objectives</u>:

Students will be able to:

1. Determine the power generated by a machine, engine, or motor.
2. Calculate the mechanical efficiency of a machine.

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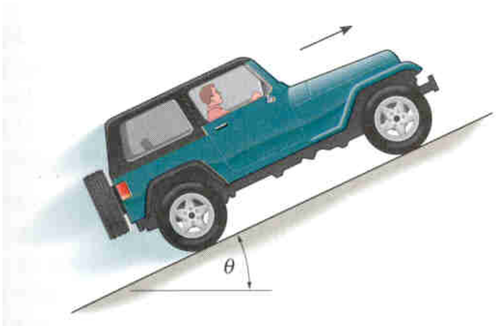
APPLICATIONS
(continued)



The speed at which a vehicle can climb a hill depends in part on the power output of the engine and the angle of inclination of the hill.

For a given angle, how can we determine the speed of this jeep, knowing the power transmitted by the engine to the wheels?

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POWER AND EFFICIENCY
(Section 14.4)

Power is defined as the amount of work performed per unit time

If a machine or engine performs a certain amount of work, dU, within a given time interval, dt, the power generated can be calculated as

$$P = dU/dt$$

Since the work can be expressed as $dU = \mathbf{F} \cdot d\mathbf{r}$, the power can be written

$$P = dU/dt = (\mathbf{F} \cdot d\mathbf{r})/dt = \mathbf{F} \cdot (d\mathbf{r}/dt) = \mathbf{F} \cdot \mathbf{v}$$

Thus, power is a scalar defined as the product of the force and velocity components acting in the same direction.

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POWER

Using scalar notation, power can be written

$$P = \mathbf{F} \cdot \mathbf{v} = F\, v \cos \theta$$

where θ is the angle between the force and velocity vectors.

So if the velocity of a body acted on by a force $\mathbf{F}$ is known, the power can be determined by calculating the dot product or by multiplying force and velocity components.

The unit of power in the SI system is the watt (W) where

$$1\text{ W} = 1\text{ J/s} = 1\text{ (N}\cdot\text{m)/s}\ .$$

In the “Foot-Pound-Second” FPS system, power is usually expressed in units of horsepower (hp) where

$$1\text{ hp} = 550\text{ (ft}\cdot\text{lb)/s} = 746\text{ W}\ .$$

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EFFICIENCY

The mechanical efficiency of a machine is the ratio of the useful power produced (output power) to the power supplied to the machine (input power):

$$\varepsilon = \text{(power output)/(power input)}$$

If energy input and removal occur at the same time, efficiency may also be expressed in terms of the ratio of output energy to input energy or

$$\varepsilon = \text{(energy output)/(energy input)}$$

Machines will always have frictional forces.
Since frictional forces dissipate energy, additional power will be required to overcome these forces.
Consequently, the efficiency of a machine is always less than 1.

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PROCEDURE FOR ANALYSIS

- Find the resultant external force acting on the body causing its motion.
  It may be necessary to draw a free-body diagram.
- Determine the velocity of the point on the body at which the force is applied.
  Energy methods or the equation of motion and appropriate kinematic relations, may be necessary.
- Multiply the force magnitude by the component of velocity acting in the direction of ***F*** to determine the power supplied to the body ($P = F\,v\cos\theta$).
- In some cases, power may be found by calculating the work done per unit of time ($P = dU/dt$).
- If the mechanical efficiency of a machine is known, either the power input or output can be determined.

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EXAMPLE

Given: A sports car has a mass of 2 Mg and an engine efficiency of $\varepsilon = 0.65$. Moving forward, the wind creates a drag resistance on the car of $F_D = 1.2v^2$ N, where v is the velocity in m/s. The car accelerates at 5 m/s^2, starting from rest.

Find: The engine's input power when t = 4 s.

Plan:
1) Draw a free body diagram of the car.
2) Apply the equation of motion and kinematic equations to find the car's velocity at t = 4 s.
3) Determine the power required for this motion.
4) Use the engine's efficiency to determine input power.

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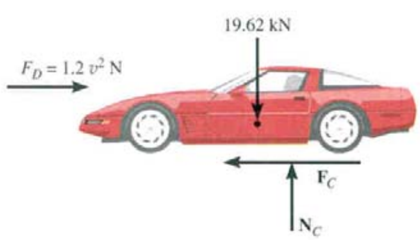
EXAMPLE
(continued)

Solution:

1) Draw the FBD of the car.



The drag force and weight are known forces. The normal force $\boldsymbol{N}_c$ and frictional force $\boldsymbol{F}_c$ represent the resultant forces of all four wheels.

The frictional force between the wheels and road pushes the car forward.

2) The equation of motion can be applied in the x-direction, with $a_x = 5$ m/s^2:

$$\overset{+}{\leftarrow}\ \Sigma F_x = ma_x \quad \Rightarrow \quad F_c - 1.2v^2 = (2000)(5)$$

$$\Rightarrow \quad F_c = (10{,}000 + 1.2v^2)\ \text{N}$$

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EXAMPLE
(continued)

3) The constant acceleration equations can be used to determine the car's velocity.

$$v_x = v_{xo} + a_x t = 0 + (5)(4) = 20 \text{ m/s}$$

4) The power output of the car is calculated by multiplying the driving (frictional) force and the car's velocity:

$$P_o = (F_c)(v_x) = [10{,}000 + (1.2)(20)^2](20) = 209.6 \text{ kW}$$

5) The power developed by the engine (prior to its frictional losses) is obtained using the efficiency equation.

$$P_i = P_o/\varepsilon = 209.6/0.65 = 322 \text{ kW}$$

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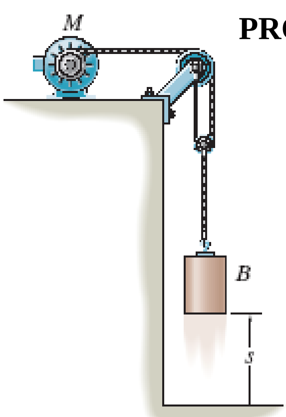

PROBLEM SOLVING

Given: A 50-lb load (B) is hoisted by the pulley system and motor M.
The motor has an efficiency of 0.76 and exerts a constant force of 30 lb on the cable.
Neglect the mass of the pulleys and cable.

Find: The power supplied to the motor when the load has been hoisted 10 ft. The block started from rest.

Plan:
1) Relate the cable and block velocities by defining position coordinates. Draw a FBD of the block.
2) Use the equation of motion or energy methods to determine the block's velocity at 10 feet.
3) Calculate the power supplied by the motor and to the motor.

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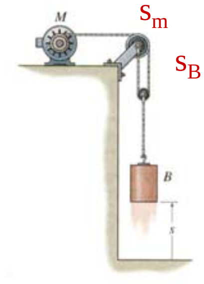
PROBLEM SOLVING (continued)

Solution:

1) Define position coordinates to relate velocities.



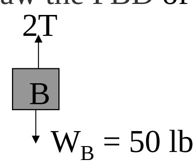
Here s_m is defined to a point on the cable. Also s_B is defined only to the lower pulley, since the block moves with the pulley. From kinematics,

$$s_m + 2s_B = l$$

$$\Rightarrow \quad v_m + 2v_B = 0$$

$$\Rightarrow \quad v_m = -2v_B$$

Draw the FBD of the block:



Since the pulley has no mass, a force balance requires that the tension in the lower cable is twice the tension in the upper cable.

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PROBLEM SOLVING (continued)

2) The velocity of the block can be obtained by applying the principle of work and energy to the block (recall that the block starts from rest).

$$+\uparrow \quad T_1 + \sum U_{1-2} = T_2$$

$$0.5m(v_1)^2 + [2T(s) - w_B(s)] = 0.5m\,(v_2)^2$$

$$0 + [2(30)(10) - (50)(10)] = 0.5(50/32.2)(v_2)^2$$

$$\Rightarrow \quad v_2 = v_B = 11.35 \text{ ft/s} \uparrow$$

Since this velocity is upwards, it is a negative velocity in terms of the kinematic equation coordinates.

The velocity of the cable coming into the motor (v_m) is calculated from the kinematic equation.

$$v_m = -2v_B = -(2)(-11.35) = 22.70 \text{ ft/s} \leftarrow$$

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PROBLEM SOLVING (continued)

3) The power supplied by the motor is the product of the force applied to the cable and the velocity of the cable:

$$P_o = \mathbf{F} \cdot \mathbf{v} = (30)(22.70) = 681 \text{ (ft}\cdot\text{lb)/s}$$

The power supplied to the motor is determined using the motor's efficiency and the basic efficiency equation.

$$P_i = P_o/\varepsilon = 681/0.76 = 896 \text{ (ft}\cdot\text{lb)/s}$$

Converting to horsepower

$$P_i = 896/550 = 1.63 \text{ hp}$$

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☺

A guy walks into work, and both of his ears are all bandaged up. The boss says, "What happened to your ears?"

He says, "Yesterday I was ironing a shirt when the phone rang and (hold iron to ear) shhh! I accidentally answered the iron."

The boss says, "Well, that explains one ear, but what happened to your other ear?"

He says, "Well, jeez, I had to call the doctor!"

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