Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 20

NEWTON'S LAWS OF MOTION

EQUATIONS OF MOTION & EQUATIONS OF MOTION FOR A SYSTEM OF PARTICLES

RECTANGULAR COORDINATES

Sections 10.1-10.4

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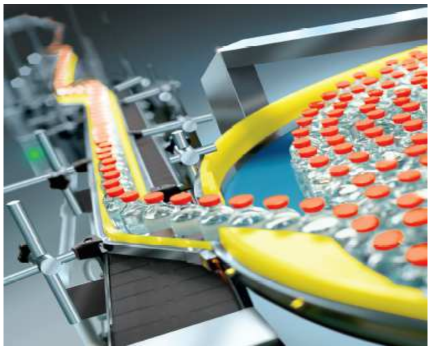
NEWTON'S LAWS OF MOTION, EQUATIONS OF MOTION, & EQUATIONS OF MOTION FOR A SYSTEM OF PARTICLES

Objectives:

Students will be able to:

1. Write the equation of motion for an accelerating body.
2. Draw the free-body and kinetic diagrams for an accelerating body.



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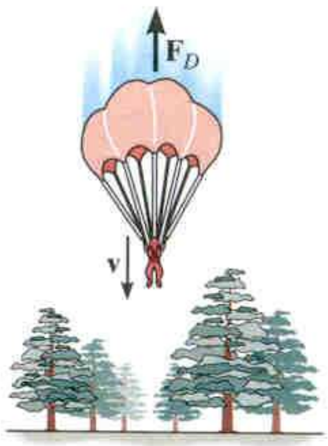
APPLICATIONS



The motion of an object depends on the forces acting on it.

A parachutist relies on the atmospheric drag resistance force to limit his velocity.

Knowing the drag force;

How can we determine the acceleration or velocity of the parachutist at any point in time?

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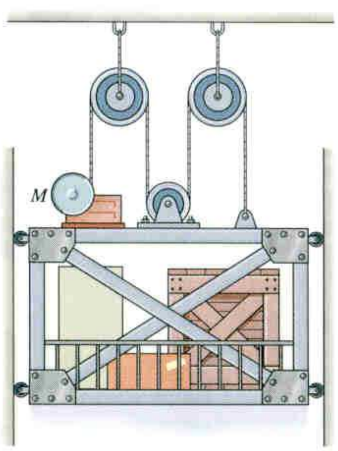
APPLICATIONS
(continued)



A freight elevator is lifted using a motor attached to a cable and pulley system as shown.

How can we determine the tension force in the cable required to lift the elevator at a given acceleration?

Is the tension force in the cable greater than the weight of the elevator and its load?

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NEWTON'S LAWS OF MOTION
(Section 13.1)

The motion of a particle is governed by Newton's three laws of motion.

First Law: A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

Second Law: If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force.
This acceleration has a magnitude proportional to the resultant force.

Third Law: Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

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NEWTON'S LAWS OF MOTION
(continued)

The first and third laws were used in developing the concepts of statics.
Newton's second law forms the basis of the study of dynamics.

Mathematically, Newton's second law of motion can be written

$$\boldsymbol{F} = m\boldsymbol{a}$$

Where:

$\boldsymbol{F}$ is the resultant unbalanced force acting on the particle, and

$\boldsymbol{a}$ is the acceleration of the particle.

The positive scalar m is called the mass of the particle.

Newton's second law cannot be used when the particle's speed approaches the speed of light, or if the size of the particle is extremely small (~ size of an atom).

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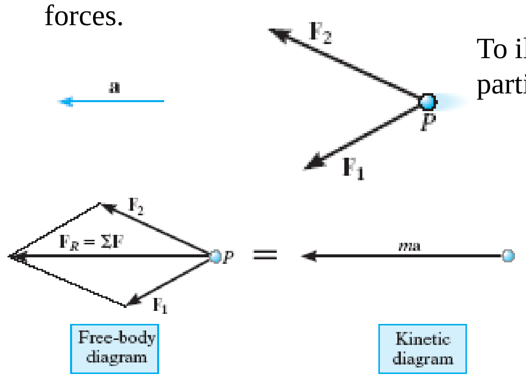
EQUATION OF MOTION

(Section 13.2-12.3)

The motion of a particle is governed by Newton's second law, relating the unbalanced forces on a particle to its acceleration. <u>If more than one force acts on the particle, the equation of motion can be written</u>

$$\sum \boldsymbol{F} = \boldsymbol{F}_R = m\boldsymbol{a}$$

where $\boldsymbol{F}_R$ is the resultant force, which is a vector summation of all the forces.



To illustrate the equation, consider a particle acted on by two forces.

<u>First,</u> draw the particle's free-body diagram, showing all forces acting on the particle. <u>Next,</u> draw the kinetic diagram, showing the inertial force $m\boldsymbol{a}$ acting in the same direction as the resultant force $\boldsymbol{F}_R$.

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PROCEDURE FOR THE APPLICATION OF THE EQUATION OF MOTION

1) Select a convenient inertial coordinate system. Rectangular, normal/tangential, or cylindrical coordinates may be used.

2) Draw a free-body diagram showing all external forces applied to the particle. Resolve forces into their appropriate components.

3) Draw the kinetic diagram, showing the particle's inertial force, $m\boldsymbol{a}$.
   Resolve this vector into its appropriate components.

4) Apply the equations of motion in their scalar component form and solve these equations for the unknowns.

5) It may be necessary to apply the proper kinematic relations to generate additional equations.

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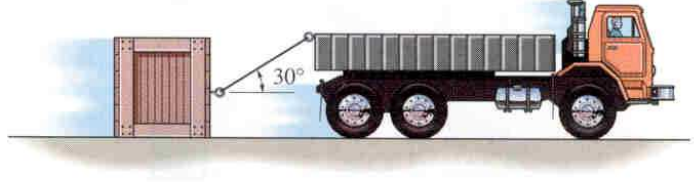
EXAMPLE



Given: A crate of mass m is pulled by a cable attached to a truck. The coefficient of kinetic friction between the crate and road is μ_k.

Find: Draw the free-body and kinetic diagrams of the crate.

Plan:
1) Define an inertial coordinate system.
2) Draw the crate's free-body diagram, showing all external forces applied to the crate in the proper directions.
3) Draw the crate's kinetic diagram, showing the inertial force vector m***a*** in the proper direction.

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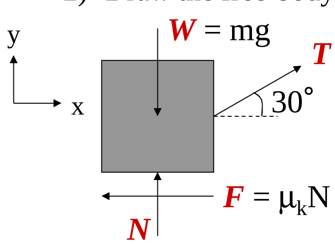
EXAMPLE
(continued)

Solution:

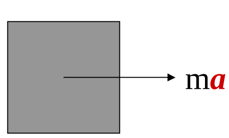
1) An inertial x-y frame can be defined as fixed to the ground.

2) Draw the free-body diagram of the crate**:**



The weight force (***W***) acts through the crate's center of mass.
T is the tension force in the cable.
The normal force (***N***) is perpendicular to the surface.
The friction force (***F*** = μ_kN) acts in a direction opposite to the motion of the crate.

3) Draw the kinetic diagram of the crate:



The crate will be pulled to the right. The acceleration vector can be directed to the right if the truck is speeding up or to the left if it is slowing down.

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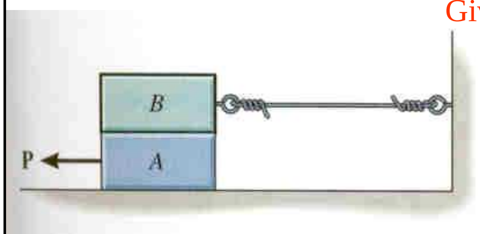
PROBLEM SOLVING



Given: Each block has a mass m. The coefficient of kinetic friction at all surfaces of contact is μ. A horizontal force ***P*** is applied to the bottom block.

Find: Draw the free-body and kinetic diagrams of each block.

Plan:
1) Define an inertial coordinate system.
2) Draw the free-body diagrams for each block, showing all external forces.
3) Draw the kinetic diagrams for each block, showing the inertial forces.

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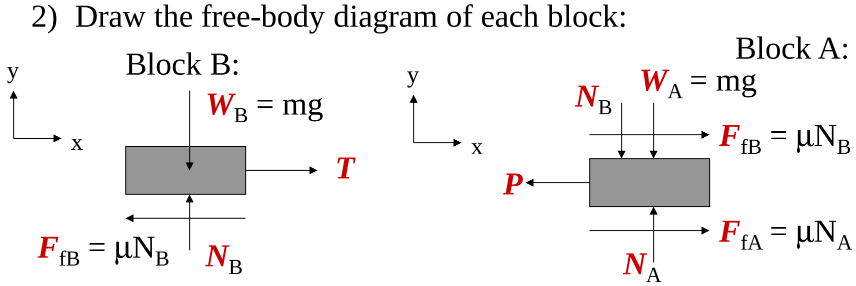
PROBLEM SOLVING (continued)

Solution:

1) An inertial x-y frame can be defined as fixed to the ground.
2) Draw the free-body diagram of each block:



The friction forces oppose the motion of each block relative to the surfaces on which they slide.

3) Draw the kinetic diagram of each block:

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EQUATIONS OF MOTION: RECTANGULAR COORDINATES

Objectives:

Students will be able to:

1. Apply Newton's second law to determine forces and accelerations for particles in rectilinear motion.

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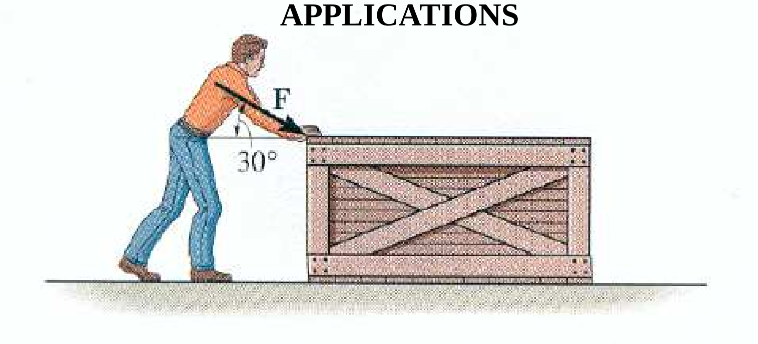
APPLICATIONS



If a man is pushing a 100 lb crate,

how large a force F must he exert to start moving the crate?

What would you have to know before you could calculate the answer?

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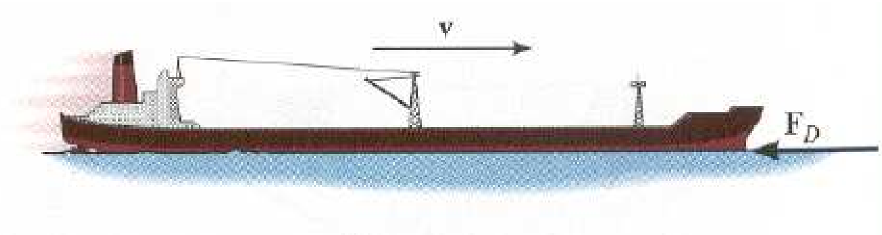
APPLICATIONS
(continued)



Objects that move in any fluid have a drag force acting on them. This drag force is a function of velocity.

If the ship has an initial velocity v_o and the magnitude of the opposing drag force at any instant is half the velocity,

how long it would take for the ship to come to a stop if its engines stop?

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RECTANGULAR COORDINATES
(Section 13.4)

The equation of motion, $\boldsymbol{F} = m\,\boldsymbol{a}$, is best used when the problem requires finding forces (especially forces perpendicular to the path), accelerations, velocities or mass.
Remember, unbalanced forces cause acceleration!

Three scalar equations can be written from this vector equation.
The equation of motion, being a vector equation, may be expressed in terms of its three components in the Cartesian (rectangular) coordinate system as

$$\sum \boldsymbol{F} = m\boldsymbol{a} \quad \text{or} \quad \sum F_x\,\boldsymbol{i} + \sum F_y\,\boldsymbol{j} + \sum F_z\,\boldsymbol{k} = m(a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k})$$

or, as scalar equations, $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$.

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PROCEDURE FOR ANALYSIS

• **Free Body Diagram**

• Establish your coordinate system and draw the particle's free body diagram showing only external forces.

• These external forces usually include the weight, normal forces, friction forces, and applied forces.

• Show the 'm***a***' vector (sometimes called the inertial force) on a separate diagram.

• Make sure any friction forces act opposite to the direction of motion!

• If the particle is connected to an elastic spring, a spring force equal to *ks* should be included on the FBD.

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PROCEDURE FOR ANALYSIS
(continued)

• **Equations of Motion**

• If the forces can be resolved directly from the free-body diagram (often the case in 2-D problems), use the scalar form of the equation of motion.

• In more complex cases (usually 3-D), a Cartesian vector is written for every force and a vector analysis is often best.

A Cartesian vector formulation of the second law is

$$\sum \boldsymbol{F} = m\boldsymbol{a} \quad \text{or}$$

$$\sum F_x \boldsymbol{i} + \sum F_y \boldsymbol{j} + \sum F_z \boldsymbol{k} = m(a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k})$$

Three scalar equations can be written from this vector equation. You may only need two equations if the motion is in 2-D.

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PROCEDURE FOR ANALYSIS
(continued)

• **Kinematics**

• The second law only provides solutions for forces and accelerations.

• *If velocity or position have to be found,* kinematics equations are used once the acceleration is found from the equation of motion.

• Any of the tools learned in Chapter 12 may be needed to solve a problem.

• Make sure you use consistent positive coordinate directions as used in the equation of motion part of the problem!

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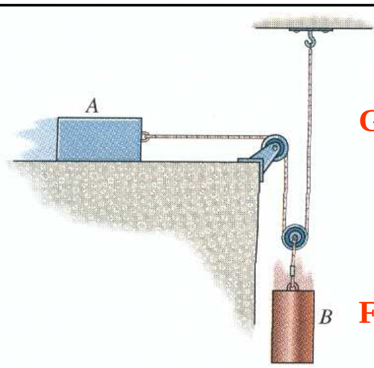

EXAMPLE

Given: $W_A = 10$ lb
$W_B = 20$ lb
$v_{oA} = 2$ ft/s →
$\mu_k = 0.2$

Find: v_A when A has moved 4 feet.

Plan: Since both forces and velocity are involved, this problem requires both the equation of motion and kinematics.

First, draw free body diagrams of A and B. Apply the equation of motion .

Using dependent motion equations, derive a relationship between $\boldsymbol{a}_A$ and $\boldsymbol{a}_B$ and use with the equation of motion formulas.

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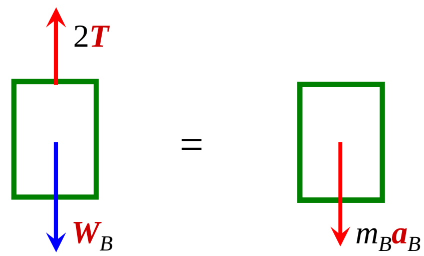
EXAMPLE
(continued)

Solution:

Free-body and kinetic diagrams of B:



Apply the equation of motion to B:

$$+\downarrow \sum F_y = ma_y$$

$$W_B - 2T = m_B a_B$$

$$20 - 2T = \frac{20}{32.2} a_B \qquad (1)$$

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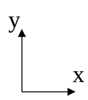
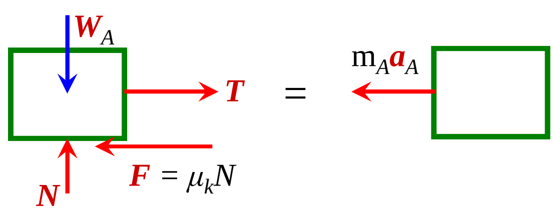
EXAMPLE
(continued)

Free-body and kinetic diagrams of A:





Apply the equations of motion to A:

$$+\uparrow \sum F_y = ma_y = 0$$

$$N = W_A = 10 \ lb$$

$$F = \mu_k N = 2 \ lb$$

$$\overset{+}{\leftarrow} \sum F_x = ma_x$$

$$F - T = m_A a_A$$

$$2 - T = \frac{10}{32.2} a_A \qquad (2)$$

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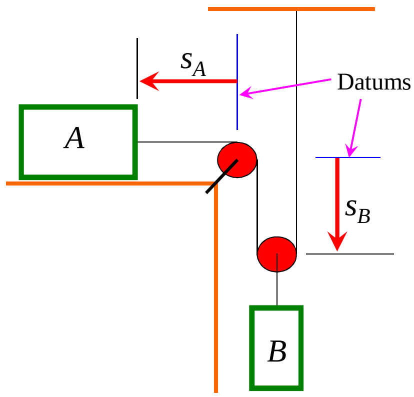
EXAMPLE
(continued)

Now consider the kinematics.



Constraint equation:

$$s_A + 2\,s_B = \text{constant}$$

or

$$v_A + 2\,v_B = 0$$

Therefore

$$a_A + 2\,a_B = 0$$

$$a_A = -2\,a_B \qquad (3)$$

(Notice a_A is considered positive to the left and a_B is positive downward.)

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EXAMPLE
(continued)

Now combine equations (1), (2), and (3).

$$T = \frac{22}{3} = 7.33\ lb$$

$$a_A = -17.16\ ft/s^2 = 17.16\ ft/s^2 \rightarrow$$

Now use the kinematic equation:

$$v_A^{\,2} = v_{oA}^{\,2} + 2\,a_A(s_A - s_{oA})$$

$$v_A^{\,2} = 2^2 + 2(17.16)(4)$$

$$v_A = 11.9\ ft/s \rightarrow$$

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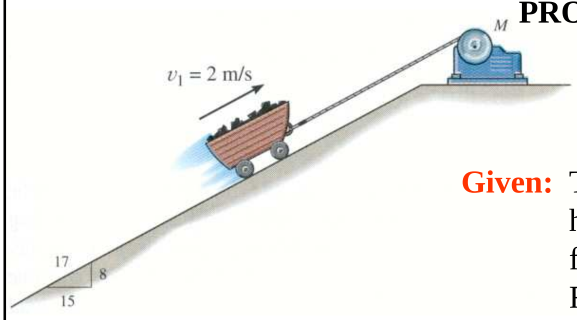
PROBLEM SOLVING



Given: The 400 kg mine car is hoisted up the incline. The force in the cable is $F = (3200t^2)$ N. The car has an initial velocity of $v_i = 2$ m/s at $t = 0$.

Find: The velocity when $t = 2$ s.

Plan: Draw the free-body diagram of the car and apply the equation of motion to determine the acceleration. Apply kinematics relations to determine the velocity.

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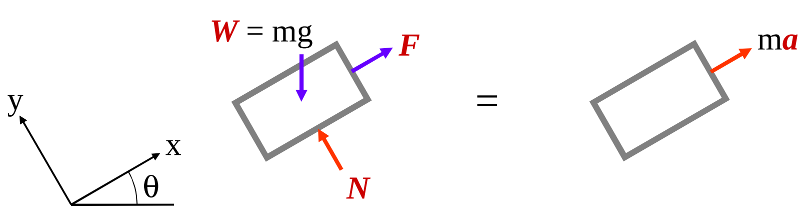
PROBLEM SOLVING (continued)

Solution:

1) Draw the free-body and kinetic diagrams of the mine car:



Since the motion is up the incline, rotate the x-y axes.

$\theta = \tan^{-1}(8/15) = 28.07°$

Motion occurs only in the x-direction.

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PROBLEM SOLVING (continued)

2) Apply the equation of motion in the x-direction:

$+\nearrow \sum F_x = ma_x \;\; \Rightarrow \;\; F - mg(\sin\theta) = ma_x$

$\Rightarrow \;\; 3200t^2 - (400)(9.81)(\sin 28.07°) = 400a$

$\Rightarrow \;\; a = (8t^2 - 4.616)\ m/s^2$

3) Use kinematics to determine the velocity:

$a = dv/dt \;\; \Rightarrow \;\; dv = a\,dt$

$\int_{v_1}^{v} dv = \int_0^t (8t^2 - 4.616)\,dt, \quad v_1 = 2\ m/s,\ t = 2\ s$

$v - 2 = (8/3t^3 - 4.616t)\Big|_0^2 = 12.10 \;\; \Rightarrow \;\; v = 14.1\ m/s$

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