Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 19

ABSOLUTE DEPENDENT MOTION ANALYSIS OF TWO PARTICLES

RELATIVE-MOTION ANALYSIS OF TWO PARTICLES USING TRANSLATING AXES

Section 9.9-9.10

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ABSOLUTE DEPENDENT MOTION ANALYSIS OF TWO PARTICLES

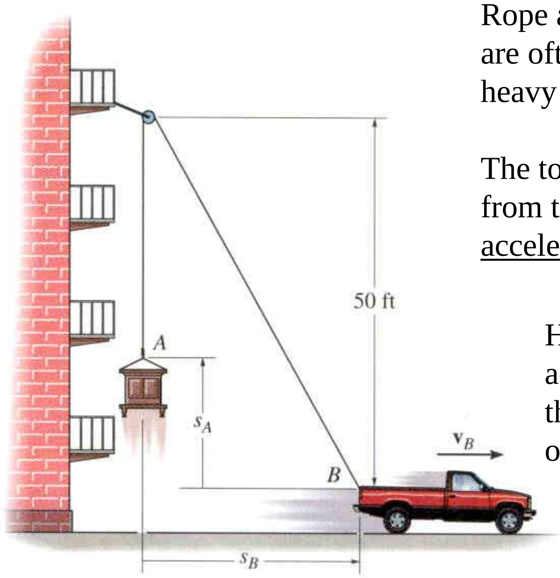
Objectives:

Students will be able to:

1. Relate the positions, velocities, and accelerations of particles undergoing dependent motion.

2

APPLICATIONS
(continued)



Rope and pulley arrangements are often used to assist in lifting heavy objects.

The total lifting force required from the truck depends on the acceleration of the cabinet.

How can we determine the acceleration and velocity of the cabinet if the acceleration of the truck is known?

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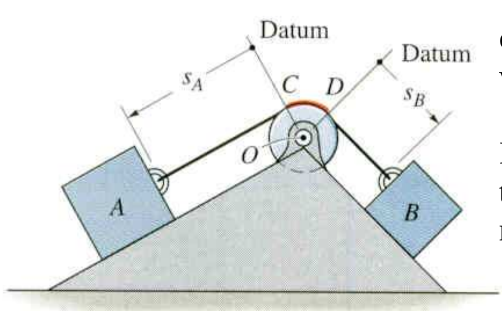
DEPENDENT MOTION
(Section 12.9)

In many kinematics problems, the motion of one object will depend on the motion of another object.



The blocks in this figure are connected by an inextensible cord wrapped around a pulley.

If block A moves downward along the inclined plane, block B will move up the other incline.

The motion of each block can be related mathematically by defining position coordinates, s_A and s_B. Each coordinate axis is defined from a fixed point or datum line, measured positive along each plane in the direction of motion of each block.

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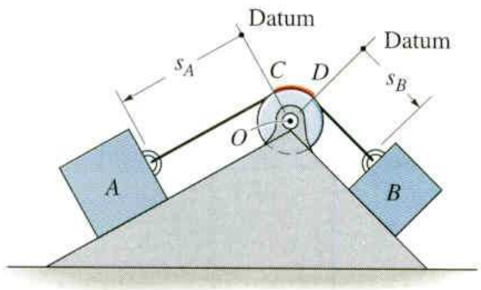
DEPENDENT MOTION
(continued)



Position coordinates s_A and s_B can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks A and B.

If the cord has a fixed length, the position coordinates s_A and s_B are related mathematically by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here l_T is the total cord length and l_{CD} is the length of cord passing over arc CD on the pulley.

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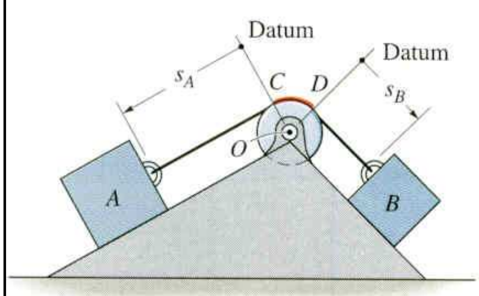
DEPENDENT MOTION
(continued)



The velocities of blocks A and B can be related by differentiating the position equation.

Note that l_{CD} and l_T remain constant, so $dl_{CD}/dt = dl_T/dt = 0$

$$ds_A/dt + ds_B/dt = 0 \quad => \quad v_B = -v_A$$

The negative sign indicates that as <u>A moves down the incline</u> (positive s_A direction), <u>B moves up the incline</u> (negative s_B direction).

Accelerations can be found by differentiating the velocity expression. Prove to yourself that $a_B = -a_A$.

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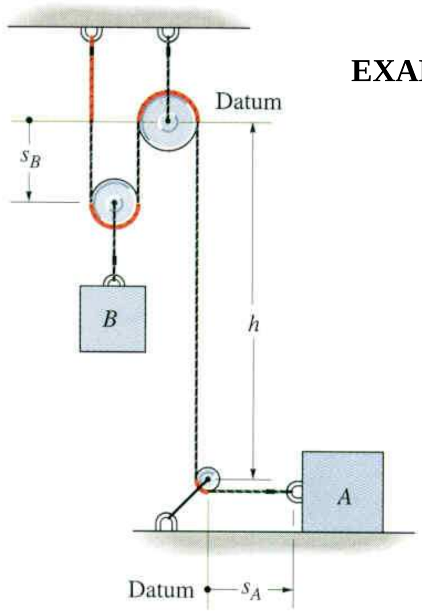

EXAMPLE

Consider a more complicated example.
Position coordinates (s_A and s_B) are defined from fixed datum lines, measured along the direction of motion of each block.

Note that s_B is only defined to the center of the pulley above block B, since this block moves with the pulley. Also, h is a constant.

The red colored segments of the cord remain constant in length during motion of the blocks.

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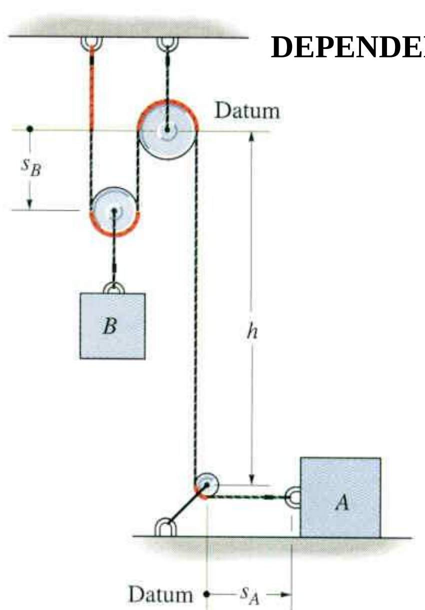

DEPENDENT MOTION EXAMPLE
(continued)

The position coordinates are related by the equation

$$2s_B + h + s_A = l$$

Where l is the total cord length minus the lengths of the red segments.

Since l and h remain constant during the motion, the velocities and accelerations can be related by two successive time derivatives:

$$2v_B = -v_A \quad \text{and} \quad 2a_B = -a_A$$

When block B moves downward ($+s_B$), block A moves to the left ($-s_A$). Remember to be consistent with the sign convention!

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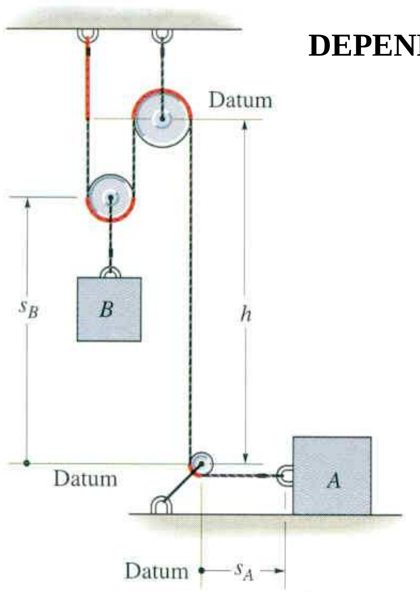
DEPENDENT MOTION EXAMPLE
(continued)



This example can also be worked by defining the position coordinate for B (s_B) from the bottom pulley instead of the top pulley.

The position, velocity, and acceleration relations then become

$$2(h - s_B) + h + s_A = l$$

and $2v_B = v_A$ $\quad$ $2a_B = a_A$

Prove to yourself that the results are the same, even if the sign conventions are different than the previous formulation.

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DEPENDENT MOTION: PROCEDURES

These procedures can be used to relate the dependent motion of particles moving along rectilinear paths (only the magnitudes of velocity and acceleration change, not their line of direction).

1. Define position coordinates from fixed datum lines, along the path of each particle. Different datum lines can be used for each particle.
2. Relate the position coordinates to the cord length. Segments of cord that do not change in length during the motion may be left out.
3. If a system contains more than one cord, relate the position of a point on one cord to a point on another cord. Separate equations are written for each cord.
4. Differentiate the position coordinate equation(s) to relate velocities and accelerations. Keep track of signs!

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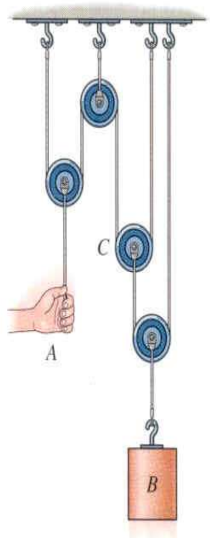
EXAMPLE II



Given: In the figure on the left, the cord at A is pulled down with a speed of 8 ft/s.

Find: The speed of block B.

Plan: There are two cords involved in the motion in this example. The position of a point on one cord must be related to the position of a point on the other cord. There will be two position equations (one for each cord).

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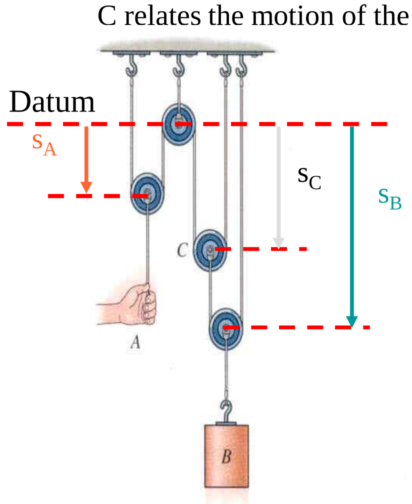
EXAMPLE (continued)

Solution:

1) Define the position coordinates from a fixed datum line. Three coordinates must be defined: one for point A (s_A), one for block B (s_B), and one relating positions on the two cords. Note that pulley C relates the motion of the two cords.



- Define the datum line through the top pulley (which has a fixed position).
- s_A can be defined to the center of the pulley above point A.
- s_B can be defined to the center of the pulley above B.
- s_C is defined to the center of pulley C.
- All coordinates are defined as positive down and along the direction of motion of each point/object.

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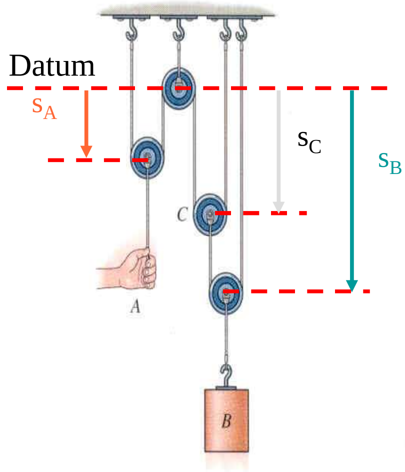

EXAMPLE
(continued)

2) Write position/length equations for each cord. Define l_1 as the length of the first cord, minus any segments of constant length. Define l_2 in a similar manner for the second cord:

Cord 1: $2s_A + 2s_C = l_1$

Cord 2: $s_B + (s_B – s_C) = l_2$

3) Eliminating s_C between the two equations, we get

$$2s_A + 4s_B = l_1 + 2l_2$$

4) Relate velocities by differentiating this expression. Note that l_1 and l_2 are constant lengths.

$$2v_A + 4v_B = 0 \quad => \quad v_B = - 0.5v_A = - 0.5(8) = - 4 \text{ ft/s}$$

The velocity of block B is 4 ft/s up (negative s_B direction).

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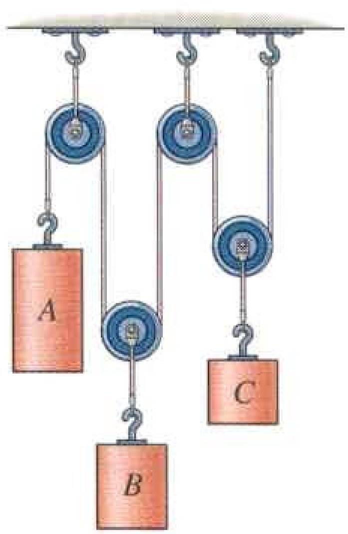

PROBLEM SOLVING

Given: In this pulley system, block A is moving downward with a speed of 4 ft/s while block C is moving up at 2 ft/s.

Find: The speed of block B.

Plan: All blocks are connected to a single cable, so only one position/length equation will be required. Define position coordinates for each block, write out the position relation, and then differentiate it to relate the velocities.

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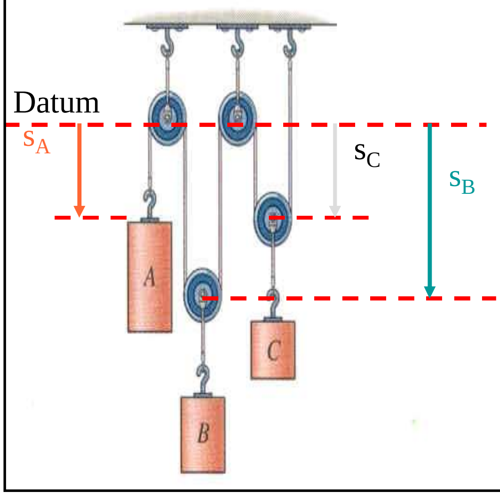
PROBLEM SOLVING (continued)

Solution:

1) A datum line can be drawn through the upper, fixed, pulleys and position coordinates defined from this line to each block (or the pulley above the block).



2) Defining s_A, s_B, and s_C as shown, the position relation can be written:

$$s_A + 2s_B + 2s_C = l$$

3) Differentiate to relate velocities:

$$v_A + 2v_B + 2v_C = 0$$

$$\Rightarrow \; 4 + 2v_B + 2(-2) = 0$$

$$\Rightarrow \; v_B = 0$$

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RELATIVE-MOTION ANALYSIS OF TWO PARTICLES USING TRANSLATING AXES

Objectives:

Students will be able to:

1. Understand translating frames of reference.
2. Use translating frames of reference to analyze relative motion.

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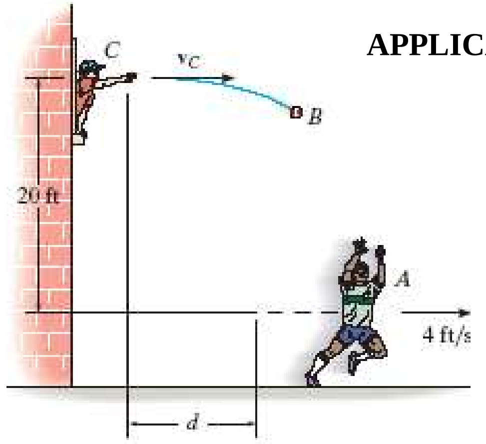
APPLICATIONS



When you try to hit a moving object, the position, velocity, and acceleration of the object must be known. Here, the boy on the ground is at d = 10 ft when the girl in the window throws the ball to him.

If the boy on the ground is running at a constant speed of 4 ft/s, how fast should the ball be thrown?

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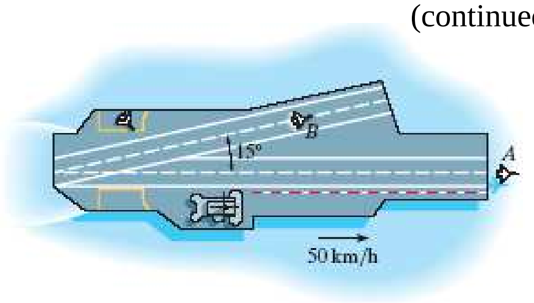
APPLICATIONS
(continued)



When fighter jets take off or land on an aircraft carrier, the velocity of the carrier becomes an issue.

If the aircraft carrier travels at a forward velocity of 50 km/hr and plane A takes off at a horizontal air speed of 200 km/hr (measured by someone on the water),
how do we find the velocity of the plane relative to the carrier?

How would you find the same thing for airplane B?

How does the wind impact this sort of situation?

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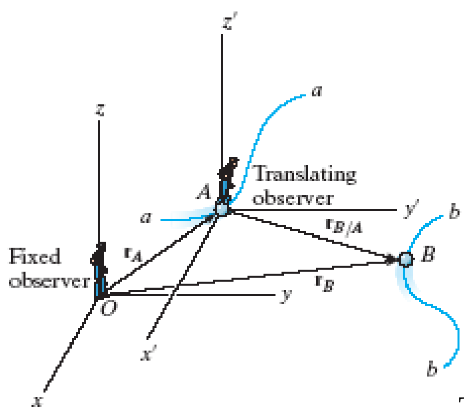
RELATIVE POSITION
(Section 12.10)



The absolute position of two particles A and B with respect to the fixed x, y, z reference frame are given by $\boldsymbol{r_A}$ and $\boldsymbol{r_B}$. The position of B relative to A is represented by

$$\boldsymbol{r_{B/A}} = \boldsymbol{r_B} - \boldsymbol{r_A}$$

Therefore, if $\boldsymbol{r_B} = (10\,\boldsymbol{i} + 2\,\boldsymbol{j})$ m

and $\boldsymbol{r_A} = (4\,\boldsymbol{i} + 5\,\boldsymbol{j})$ m,

then $\boldsymbol{r_{B/A}} = (6\,\boldsymbol{i} - 3\,\boldsymbol{j})$ m.

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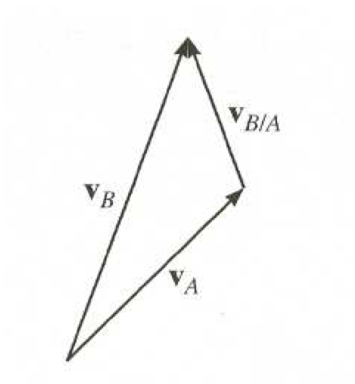
RELATIVE VELOCITY



To determine the relative velocity of B with respect to A, the time derivative of the relative position equation is taken.

$$\boldsymbol{v_{B/A}} = \boldsymbol{v_B} - \boldsymbol{v_A}$$

or

$$\boldsymbol{v_B} = \boldsymbol{v_A} + \boldsymbol{v_{B/A}}$$

$\boldsymbol{v_B}$ and $\boldsymbol{v_A}$ are called absolute velocities
$\boldsymbol{v_{B/A}}$ is the relative velocity of B with respect to A.

Note that $\boldsymbol{v_{B/A}} = -\boldsymbol{v_{A/B}}$.

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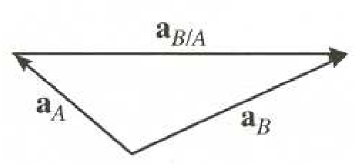
RELATIVE ACCELERATION



The time derivative of the relative velocity equation yields a similar vector relationship between the absolute and relative accelerations of particles A and B.

$$\boldsymbol{a}_{B/A} = \boldsymbol{a}_B - \boldsymbol{a}_A$$

or

$$\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A}$$

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SOLVING PROBLEMS

Since the relative motion equations are vector equations, problems involving them may be solved in one of two ways.

1. The velocity vectors in $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$ could be written as Cartesian vectors and the resulting scalar equations solved for up to two unknowns.

2. Vector problems can be solved "graphically" by use of trigonometry.
This approach usually makes use of the law of sines or the law of cosines.

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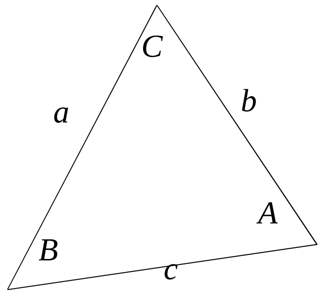
LAWS OF SINES AND COSINES



Since vector addition or subtraction forms a triangle, sine and cosine laws can be applied to solve for relative or absolute velocities and accelerations. As review, their formulations are provided below.

Law of Sines: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines:
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

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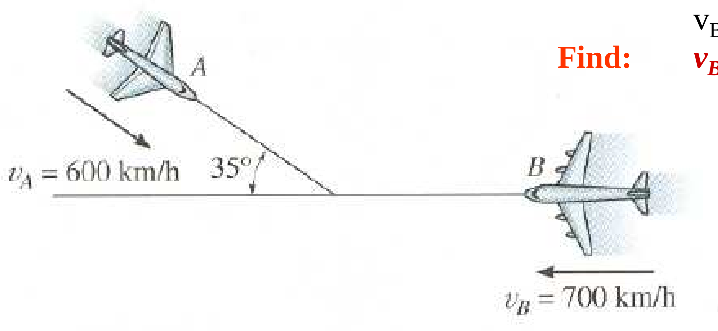
EXAMPLE



Given: $v_A = 600$ km/hr
$v_B = 700$ km/hr

Find: $\mathbf{v}_{B/A}$

Plan:

a) Vector Method: Write vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian form, then determine $\mathbf{v}_B - \mathbf{v}_A$

b) Graphical Method: Draw vectors $\mathbf{v}_A$ and $\mathbf{v}_B$ from a common point. Apply the laws of sines and cosines to determine $\mathbf{v}_{B/A}$.

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EXAMPLE
(continued)

Solution:

a) Vector Method:

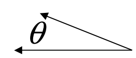
$\boldsymbol{v_A} = 600 \cos 35\, \boldsymbol{i} - 600 \sin 35\, \boldsymbol{j}$
$= (491.5\, \boldsymbol{i} - 344.1\, \boldsymbol{j})$ km/hr

$\boldsymbol{v_B} = -700\, \boldsymbol{i}$ km/hr

$\boldsymbol{v_{B/A}} = \boldsymbol{v_B} - \boldsymbol{v_A} = (-1191.5\, \boldsymbol{i} + 344.1\, \boldsymbol{j})$ km/hr

$$v_{B/A} = \sqrt{(1191.5)^2 + (344.1)^2} = 1240.2 \frac{\text{km}}{\text{hr}}$$

$$\text{where} \quad \theta = \tan^{-1}\left(\frac{344.1}{1191.5}\right) = 16.1^\circ$$



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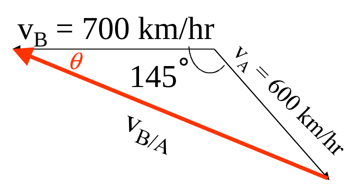
EXAMPLE
(continued)

b) Graphical Method:
Note that the vector that measures the tip of B relative to A is $\boldsymbol{v_{B/A}}$.



Law of Cosines:

$$v_{B/A}^2 = (700)^2 + (600)^2 - 2(700)(600)\cos 145^\circ$$

$$v_{B/A} = 1240.2 \frac{\text{km}}{\text{hr}}$$

Law of Sines:

$$\frac{v_{B/A}}{\sin(145^\circ)} = \frac{v_A}{\sin\theta} \quad \text{or} \quad \theta = 16.1^\circ$$

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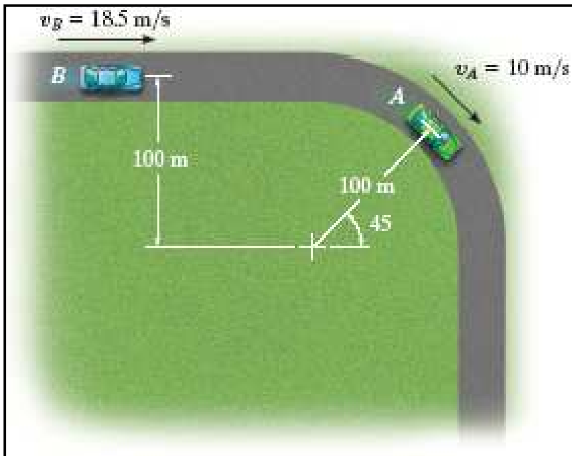

PROBLEM SOLVING

Given: $v_A = 10$ m/s
$v_B = 18.5$ m/s
$a_{tA} = 5$ m/s^2
$a_B = 2$ m/s^2

Find: $\boldsymbol{v}_{A/B}$
$\boldsymbol{a}_{A/B}$

Plan: Write the velocity and acceleration vectors for A and B and determine $\boldsymbol{v}_{A/B}$ and $\boldsymbol{a}_{A/B}$ by using vector equations.

Solution:

The velocity of A is:

$$\boldsymbol{v}_A = 10\cos(45)\boldsymbol{i} - 10\sin(45)\boldsymbol{j} = (7.07\boldsymbol{i} - 7.07\boldsymbol{j})\ \text{m/s}$$

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PROBLEM SOLVING (continued)

The velocity of B is:

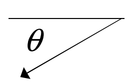
$$\boldsymbol{v}_B = 18.5\boldsymbol{i}\ \text{(m/s)}$$

The relative velocity of A with respect to B is ($\boldsymbol{v}_{A/B}$):

$$\boldsymbol{v}_{A/B} = \boldsymbol{v}_A - \boldsymbol{v}_B = (7.07\boldsymbol{i} - 7.07\boldsymbol{j}) - (18.5\boldsymbol{i}) = -11.43\boldsymbol{i} - 7.07\boldsymbol{j}$$

or $$v_{B/A} = \sqrt{(11.43)^2 + (7.07)^2} = 13.4\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{7.07}{11.43}\right) = 31.73°$$



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PROBLEM SOLVING (continued)

The acceleration of A is:

$\boldsymbol{a}_A = (\boldsymbol{a}_t)_A + (\boldsymbol{a}_n)_A = [5\cos(45)\boldsymbol{i} - 5\sin(45)\boldsymbol{j}]$

$+ [-(\frac{10^2}{100})\sin(45)\boldsymbol{i} - (\frac{10^2}{100})\cos(45)\boldsymbol{j}]$

$\boldsymbol{a}_A = 2.83\boldsymbol{i} - 4.24\boldsymbol{j}$ (m/s^2)

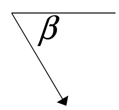
The acceleration of B is:

$\boldsymbol{a}_B = 2\boldsymbol{i}$ (m/s^2)

The relative acceleration of A with respect to B is:

$\boldsymbol{a}_{A/B} = \boldsymbol{a}_A - \boldsymbol{a}_B = (2.83\boldsymbol{i} - 4.24\boldsymbol{j}) - (2\boldsymbol{i}) = 0.83\boldsymbol{i} - 4.24\boldsymbol{j}$

$a_{A/B} = \sqrt{(0.83)^2 + (4.24)^2} = 4.32$ m/s^2



$\beta = \tan^{-1}(\frac{4.24}{0.83}) = 78.9°$

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