

## Lecture 16

### INTRODUCTION RECTILINEAR KINEMATICS: CONTINUOUS MOTION OF COMPOSITES ERRATIC MOTION

Section 9.1-9.3

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#### **Today's Objectives:**

Students will be able to:

Find the kinematic quantities of a particle traveling along a straight path:

1. Position
2. Displacement
3. Velocity
4. Acceleration).



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## APPLICATIONS



The motion of large objects, such as rockets, airplanes, or cars, can often be analyzed as if they were particles.

Why?

If we measure the altitude of this rocket as a function of time, how can we determine its velocity and acceleration?

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## APPLICATIONS

(continued)



A train travels along a straight length of track.

Can we treat the train as a particle?

If the train accelerates at a constant rate, how can we determine its position and velocity at some instant?

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## An Overview of Mechanics

**Mechanics:** The study of how bodies react to forces acting on them.

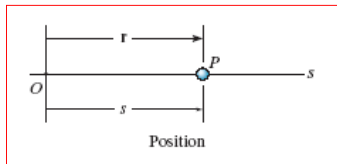
**Statics:** The study of bodies in equilibrium.

**Dynamics:**

1. **Kinematics** – concerned with the **geometric aspects** of motion
2. **Kinetics** - concerned with the **forces** causing the motion

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## RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)



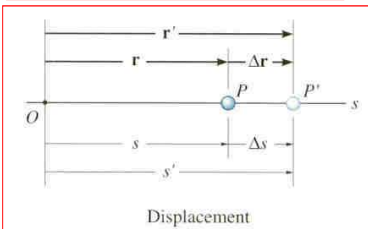
A particle travels along a straight-line path defined by the coordinate axis  $s$ .

The position of the particle at any instant, relative to the origin,  $O$ , is defined by the position vector  $\mathbf{r}$ , or the scalar  $s$ .

*Scalar  $s$  can be positive or negative.*

Typical units for  $\mathbf{r}$  and  $s$  are meters (m) or feet (ft).

The displacement of the particle is defined as its change in position.



Vector form:  $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$

Scalar form:  $\Delta s = s' - s$

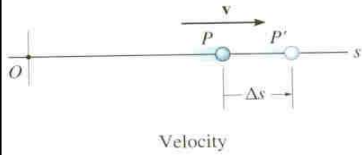
The total distance traveled by the particle,  $S_T$ , is a positive scalar that represents the total length of the path over which the particle travels.

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## VELOCITY

Velocity is a measure of the rate of change in the position of a particle. It is a vector quantity (it has both magnitude and direction).

The magnitude of the velocity is called speed, with units of m/s or ft/s.

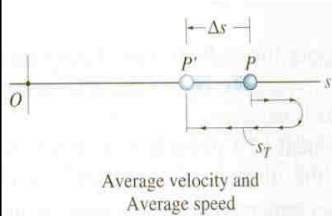


The average velocity of a particle during a time interval  $\Delta t$  is:

$$v_{avg} = \Delta r / \Delta t$$

The *instantaneous velocity* is the time-derivative of position.

$$v = dr/dt$$



Speed is the magnitude of velocity:  $v = ds/dt$

Average speed is the total distance traveled divided by elapsed time:

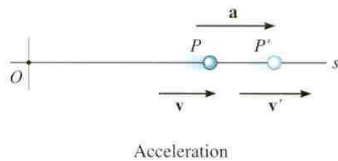
$$(v_{sp})_{avg} = S_T / \Delta t$$

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## ACCELERATION

Acceleration is the rate of change in the velocity of a particle.

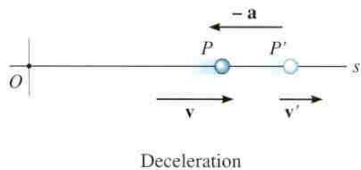
It is a vector quantity. Typical units are  $m/s^2$  or  $ft/s^2$ .



The instantaneous acceleration is the time derivative of velocity.

$$\text{Vector form: } a = dv/dt$$

$$\text{Scalar form: } a = dv/dt = d^2s/dt^2$$



Acceleration can be **positive** (speed increasing) or **negative** (speed decreasing).

The derivative equations for velocity and acceleration can be manipulated to get  $a ds = v dv$

• Important

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## SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

- Differentiate position to get velocity and acceleration.

$$v = ds/dt \quad (1) \quad \& \quad a = dv/dt \quad (2)$$

Substitute (1) into (2), and eliminate dt:

$$a \, ds = v \, dv \quad \text{or} \quad a = v \, dv/ds$$

- Integrate acceleration for velocity and position.

Velocity:

Position:

$$\int_{v_0}^v dv = \int_0^t a \, dt \quad \text{OR} \quad \int_{v_0}^v v \, dv = \int_{s_0}^s a \, ds \qquad \int_{s_0}^s ds = \int_0^t v \, dt$$

- Note that  $s_0$  and  $v_0$  represent the initial position and velocity of the particle at  $t = 0$ .

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## CONSTANT ACCELERATION

The three kinematic equations can be integrated for the special case when acceleration is constant ( $a = a_c$ ) to obtain very useful equations.

A common example of constant acceleration is gravity; i.e., a body freely falling toward earth.

In this case,  $a_c = g = 9.81 \, \text{m/s}^2 = 32.2 \, \text{ft/s}^2$  downward

These equations are:

$$\int_{v_0}^v dv = \int_0^t a_c \, dt \quad \text{yields} \quad v = v_0 + a_c t \qquad (v, v_0, a, t)$$

$$\int_{s_0}^s ds = \int_0^t v \, dt \quad \text{yields} \quad s = s_0 + v_0 t + (1/2) a_c t^2 \qquad (s, s_0, v_0, a, t)$$

$$\int_{v_0}^v v \, dv = \int_{s_0}^s a_c \, ds \quad \text{yields} \quad v^2 = (v_0)^2 + 2a_c(s - s_0) \qquad (s, s_0, v, v_0, a)$$

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### EXAMPLE

**Given:** A motorcyclist travels along a straight road at a speed of 27 m/s. When the brakes are applied, the motorcycle decelerates at a rate of  $-6t$  m/s<sup>2</sup>.

**Find:** The distance the motorcycle travels before it stops.

**Plan:** (1) Establish the positive coordinate  $s$  in the direction the motorcycle is traveling.

Since the acceleration is given as a function of time;

(2) Integrate it once to calculate the velocity and again to calculate the position.

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### EXAMPLE

**Solution:** (continued)

1) Integrate acceleration to determine the velocity.

$$a = dv / dt \Rightarrow dv = a dt \Rightarrow \int_{v_0}^v dv = \int_0^t (-6t) dt$$

$$\Rightarrow v - v_0 = -3t^2 \Rightarrow v = -3t^2 + v_0$$

2) We can now determine the amount of time required for the motorcycle to stop ( $v = 0$ ). Use  $v_0 = 27$  m/s.

$$0 = -3t^2 + 27 \Rightarrow t = 3 \text{ s}$$

3) Now calculate the distance traveled in 3s by integrating the velocity using  $s_0 = 0$ :

$$v = ds / dt \Rightarrow ds = v dt \Rightarrow \int_{s_0}^s ds = \int_0^t (-3t^2 + v_0) dt$$

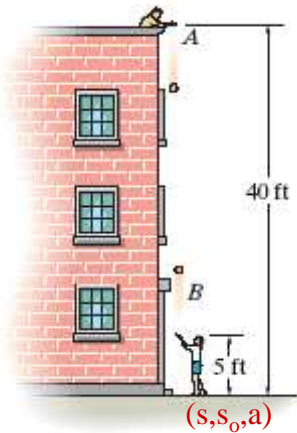
$$\Rightarrow s - s_0 = -t^3 + v_0 t$$

$$\Rightarrow s - 0 = -(3)^3 + (27)(3) \Rightarrow s = 54 \text{ m}$$

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$(s, s_o, v_o, a)$

### PROBLEM SOLVING



**Given:** Ball A is released from rest at a height of 40 ft at the same time that ball B is thrown upward, 5 ft from the ground. The balls pass one another at a height of 20 ft.

**Find:** The speed at which ball B was thrown upward.

**Plan:** Both balls experience a constant downward acceleration of  $32.2 \text{ ft/s}^2$ . Apply the formulas for constant acceleration, with  $a_c = -32.2 \text{ ft/s}^2$ .

$(v, v_o, a, t)$

$(s, s_o, v_o, a, t)$

$(s, s_o, v, v_o, a)$

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### PROBLEM SOLVING (continued)

**Solution:**

- 1) First consider ball A. With the origin defined at the ground, ball A is released from rest ( $(v_A)_o = 0$ ) at a height of 40 ft ( $(s_A)_o = 40 \text{ ft}$ ). Calculate the time required for ball A to drop to 20 ft ( $s_A = 20 \text{ ft}$ ) using a position equation.

$$s_A = (s_A)_o + (v_A)_o t + (1/2)a_c t^2$$
$$20 \text{ ft} = 40 \text{ ft} + (0)(t) + (1/2)(-32.2)(t^2) \Rightarrow t = 1.115 \text{ s}$$

- 2) Now consider ball B. It is thrown upward from a height of 5 ft ( $(s_B)_o = 5 \text{ ft}$ ). It must reach a height of 20 ft ( $s_B = 20 \text{ ft}$ ) at the same time ball A reaches this height ( $t = 1.115 \text{ s}$ ). Apply the position equation again to ball B using  $t = 1.115 \text{ s}$ .

$$s_B = (s_B)_o + (v_B)_o t + (1/2) a_c t^2$$
$$20 \text{ ft} = 5 + (v_B)_o(1.115) + (1/2)(-32.2)(1.115)^2$$
$$\Rightarrow (v_B)_o = 31.4 \text{ ft/s}$$

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## RECTILINEAR KINEMATICS: ERRATIC MOTION

### Objectives:

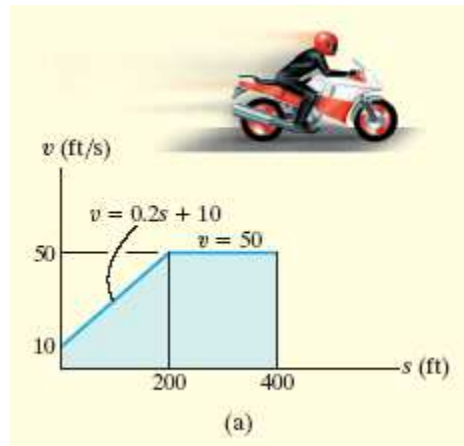
Students will be able to:

1. Determine position, velocity, and acceleration of a particle using graphs.



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## APPLICATION



In many experiments, a velocity versus position ( $v$ - $s$ ) profile is obtained.

If we have a  $v$ - $s$  graph for the motorcycle, can we determine its **acceleration at position  $s = 150$  feet?**

How?

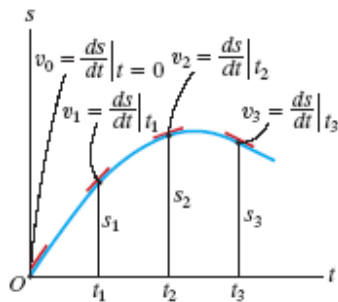
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## ERRATIC MOTION (Section 12.3)

- Graphing provides a good way to handle complex motions that would be difficult to describe with formulas.
- Graphs also provide a visual description of motion and reinforce the calculus concepts of differentiation and integration as used in dynamics.
- The approach builds on the facts that:
  - Slope and differentiation are linked
  - Integration can be thought of as finding the area under a curve.

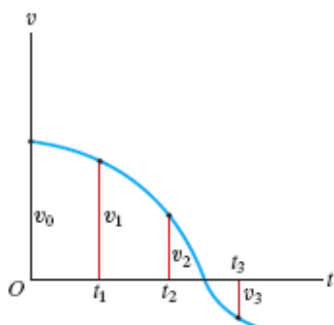
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### s-t GRAPH



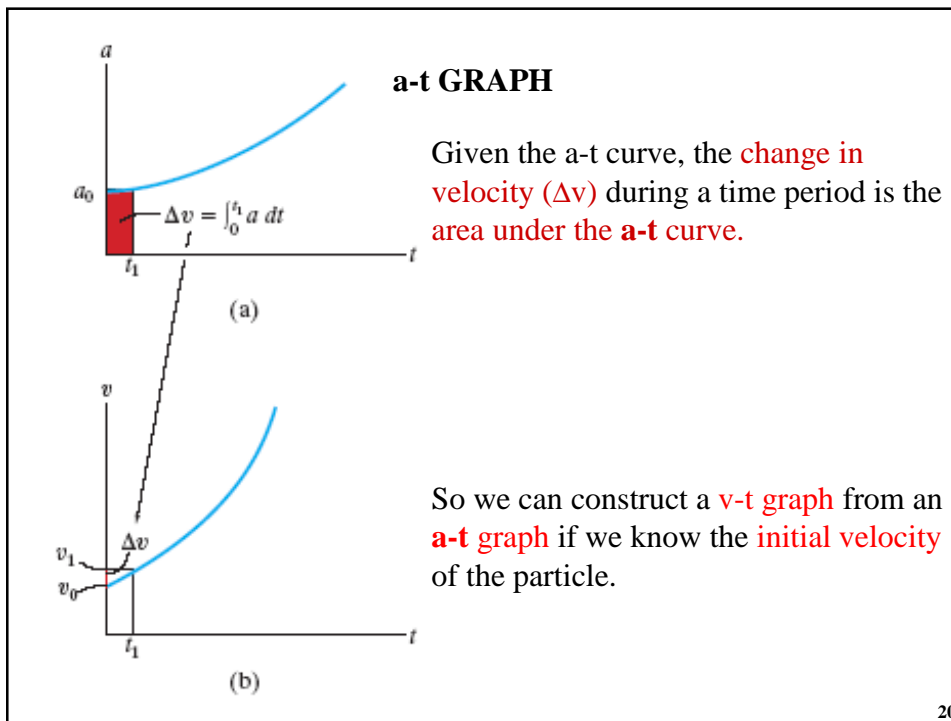
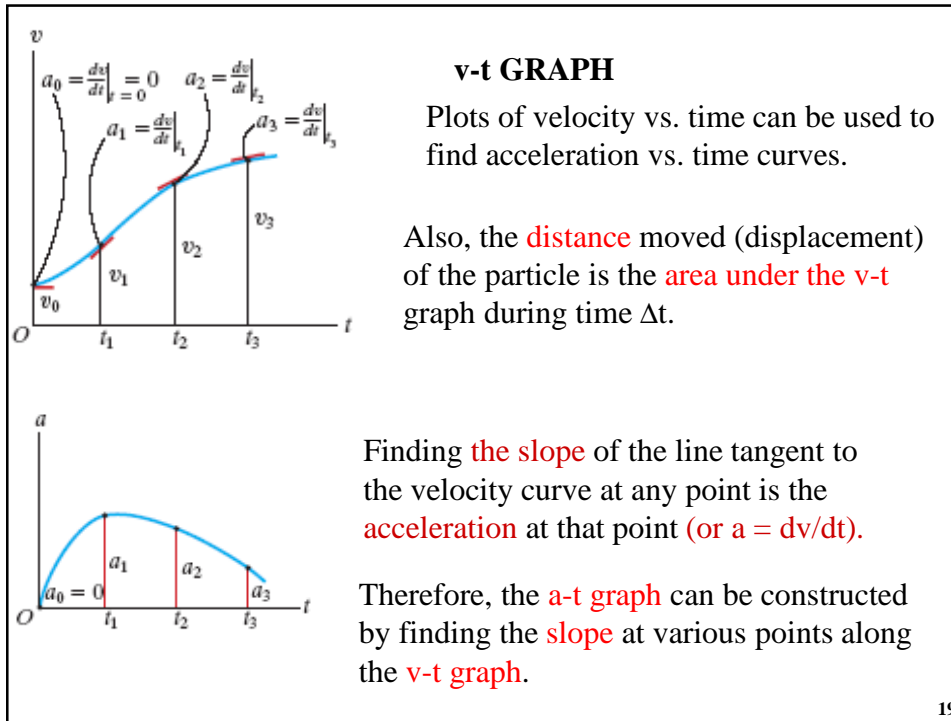
Plots of position vs. time can be used to find velocity vs. time curves.

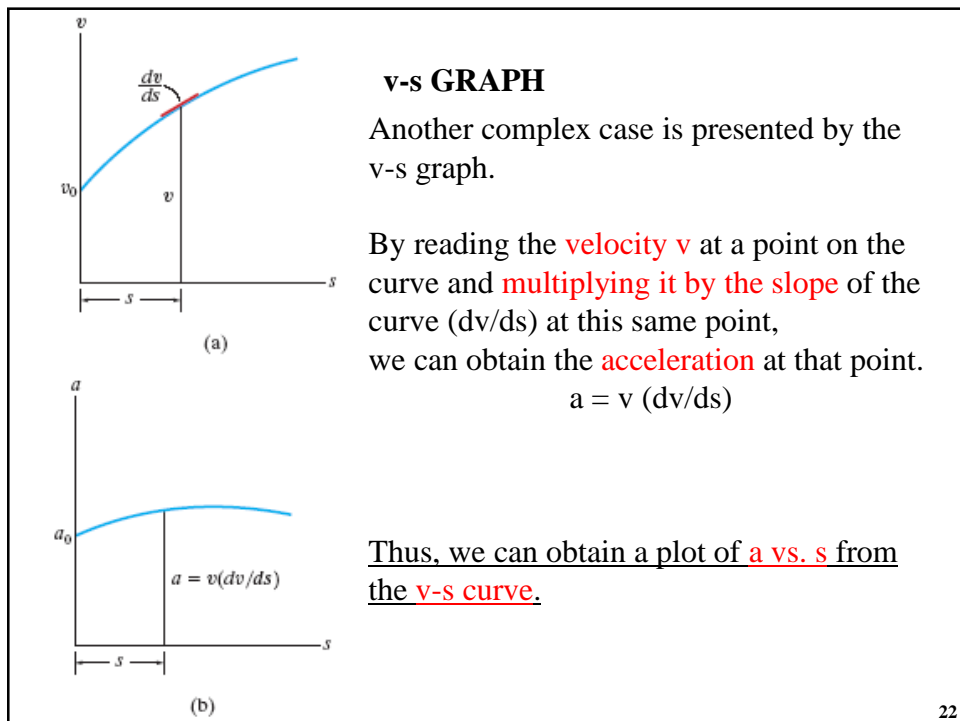
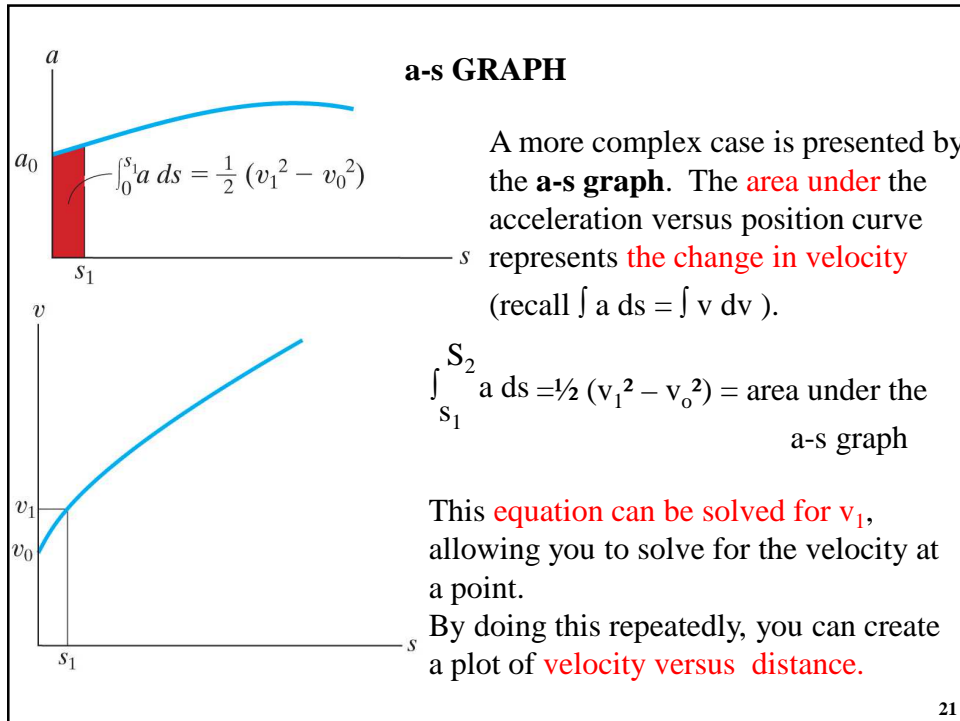
Finding the **slope of the line** tangent to the motion curve at any point **is the velocity** at that point (or  $v = ds/dt$ ).



Therefore, the **v-t graph** can be constructed by finding the **slope** at various points along the **s-t graph**.

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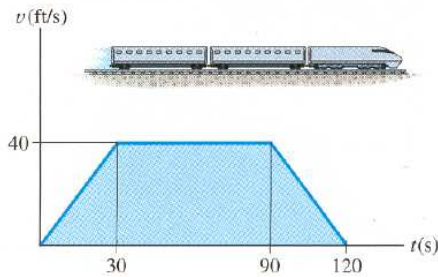




### EXAMPLE

**Given:** v-t graph for a train moving between two stations

**Find:** a-t graph and s-t graph over this time interval



Think about your plan of attack for the problem!

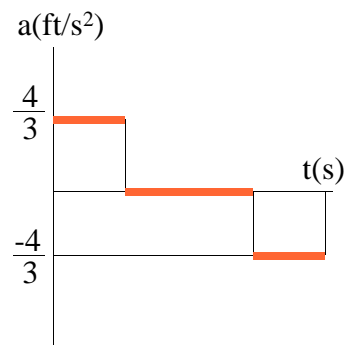
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### EXAMPLE (continued)

**Solution:** For the first 30 seconds the slope is constant and is equal to:

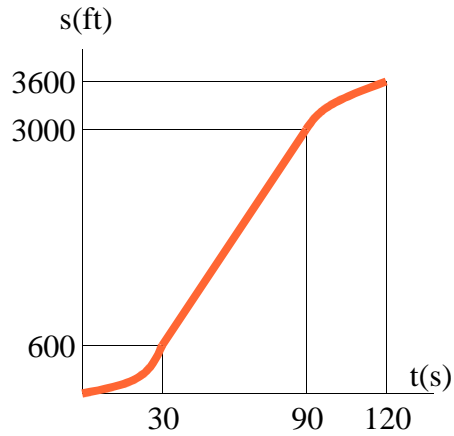
$$a_{0-30} = dv/dt = 40/30 = 4/3 \text{ ft/s}^2$$

Similarly,  $a_{30-90} = 0$  and  $a_{90-120} = -4/3 \text{ ft/s}^2$



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**EXAMPLE  
(continued)**



The area under the v-t graph represents displacement.

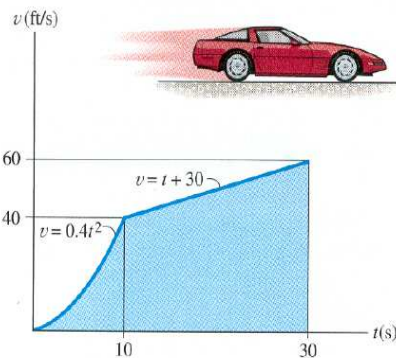
$$\Delta s_{0-30} = \frac{1}{2} (40)(30) = 600 \text{ ft}$$

$$\Delta s_{30-90} = (60)(40) = 2400 \text{ ft}$$

$$\Delta s_{90-120} = \frac{1}{2} (40)(30) = 600 \text{ ft}$$

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**PROBLEM SOLVING**



**Given:** The v-t graph shown

**Find:** The a-t graph, average speed, and distance traveled for the 30 s interval

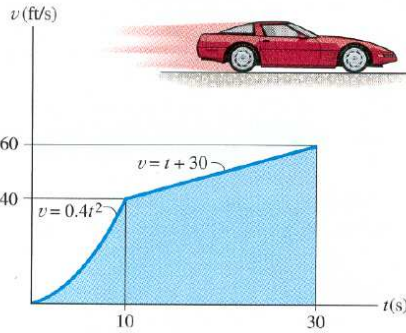
**Plan:** Find slopes of the curves and draw the a-t graph.  
Find the area under the curve--that is the distance traveled.

Finally, calculate average speed (using basic definitions!).

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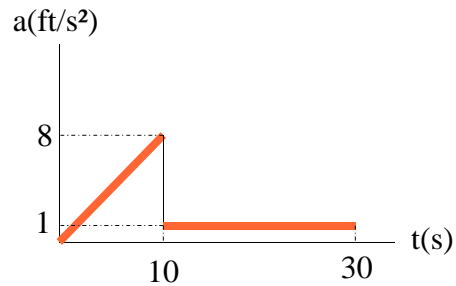
**PROBLEM SOLVING**  
(continued)

**Solution:**



For  $0 \leq t \leq 10$   $a = dv/dt = 0.8 t \text{ ft/s}^2$

For  $10 \leq t \leq 30$   $a = dv/dt = 1 \text{ ft/s}^2$



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**PROBLEM SOLVING (continued)**

$$\Delta s_{0-10} = \int v \, dt = (1/3) (.4)(10)^3 = 400/3 \text{ ft}$$

$$\begin{aligned} \Delta s_{10-30} &= \int v \, dt = (0.5)(30)^2 + 30(30) - 0.5(10)^2 - 30(10) \\ &= 1000 \text{ ft} \end{aligned}$$

$$s_{0-30} = 1000 + 400/3 = 1133.3 \text{ ft}$$

$$\begin{aligned} v_{\text{avg}(0-30)} &= \text{total distance} / \text{time} \\ &= 1133.3/30 \\ &= 37.78 \text{ ft/s} \end{aligned}$$

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