Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 12

* TWO- AND THREE-FORCE MEMEBERS
* EQUILIBRIUM IN THREE DIMENSIONS
* FREE-BODY DIAGRAMS

Section 6.4-6.5

Ehab Zalok

APPLICATIONS

For a given load on the platform, how can we determine the forces at the joint A and the force in the link (cylinder) BC?

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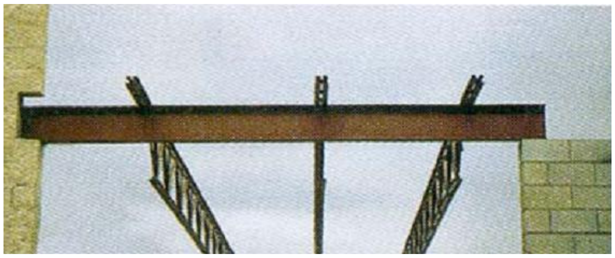
APPLICATIONS
(continued)



A steel beam is used to support roof joists. How can we determine the support reactions at each end of the beam?

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Objectives:

Students will be able to:

a) Recognize two-and three force members.

b) Apply equations of equilibrium to solve for unknowns, and,

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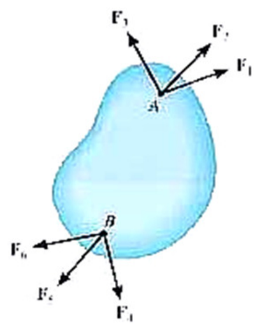
READING QUIZ

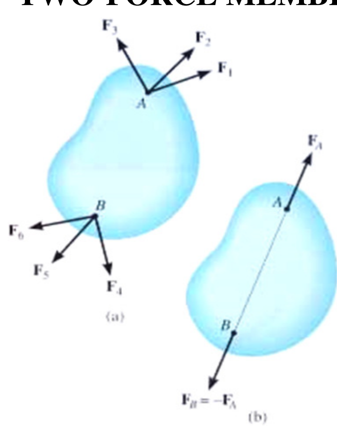
1. A rigid body is subjected to forces as shown. This body can be considered as a _______ member.

   A) single-force B) two-force

   C) three-force D) six-force



TWO-FORCE MEMBERS & THREE FORCE-MEMBERS (Section 5.4)

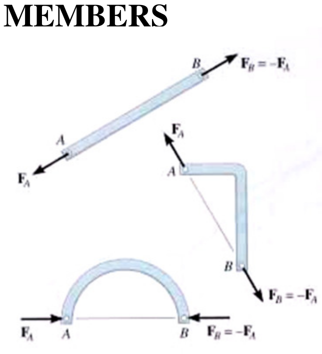


Two-force member

The solution to some equilibrium problems <u>can be simplified</u> if we recognize members that are subjected to forces at only two points (e.g., at points A and B).

If we apply the equations of equilibrium to such a member, we can quickly determine that <u>the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.</u>

EXAMPLE OF TWO-FORCE MEMBERS



Two-force members

In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.

This fact simplifies the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).

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STEPS FOR SOLVING 2-D EQUILIBRIUM PROBLEMS

1. If not given, establish a suitable x - y coordinate system.

2. Draw a free body diagram (FBD) of the object under analysis.

3. Apply the three equations of equilibrium (EofE) to solve for the unknowns.

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IMPORTANT NOTES

1. If we have more unknowns than the number of independent equations, then we have a statically indeterminate situation. We cannot solve these problems using just statics.
2. The order in which we apply equations may affect the simplicity of the solution. For example, if we have two unknown vertical forces and one unknown horizontal force, then solving $\sum F_X = O$ first allows us to find the horizontal unknown quickly.
3. If the answer for an unknown comes out as negative number, then the sense (direction) of the unknown force is opposite to that assumed when starting the problem.

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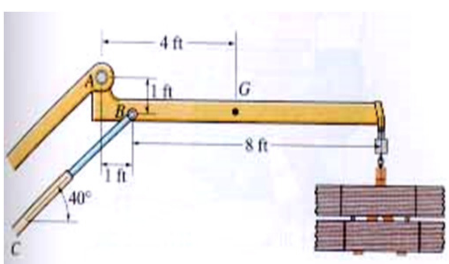
EXAMPLE



Given: Weight of the boom = 125 lb, the center of mass is at G, and the load = 600 lb.

Find: Support reactions at A and B.

Plan:

1. Put the x and y axes in the horizontal and vertical directions, respectively.
2. Determine if there are any two-force members.
3. Draw a complete FBD of the boom.
4. Apply the E-of-E to solve for the unknowns.

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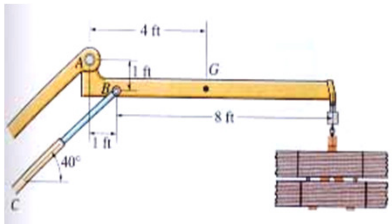
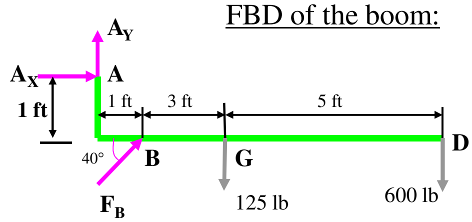
EXAMPLE
(Continued)





Note: Upon recognizing CB as a two-force member, the number of unknowns at B are reduced from two to one. Now, using Eof E, we get,

$\circlearrowleft + \sum M_A = 125 * 4 + 600 * 9 - F_B \sin 40° * 1 - F_B \cos 40° * 1 = 0$

$F_B = 4188$ lb or 4190 lb

$\rightarrow + \sum F_X = A_X + 4188 \cos 40° = 0;$ $A_X = -3210$ lb

$\uparrow + \sum F_Y = A_Y + 4188 \sin 40° - 125 - 600 = 0;$ $A_Y = -1970$ lb

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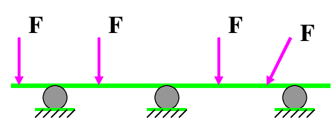
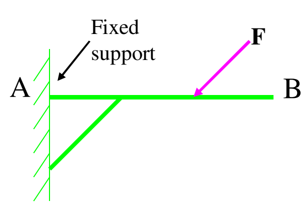
CONCEPT QUIZ

1. For this beam, how many support reactions are there and is the problem statically determinate?
   1) (2, Yes) 2) (2, No)
   3) (3, Yes) 4) (3, No)



2. The beam AB is loaded and supported as shown:
   a) how many support reactions are there on the beam,
   b) is this problem statically determinate,
   c) is the structure stable?



A) (4, Yes, No) B) (4, No, Yes)

C) (5, Yes, No) D) (5, No, Yes)

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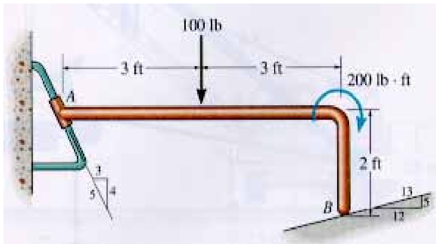
PROBLEM SOLVING



Given: The load on the bent rod is supported by a smooth inclined surface at B and a collar at A.
The collar is free to slide over the fixed inclined rod.

Find: Support reactions at A and B.

Plan:

a) Establish the x – y axes.

b) Draw a complete FBD of the bent rod.

c) Apply the E-of-E to solve for the unknowns.

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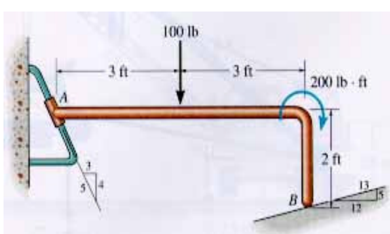
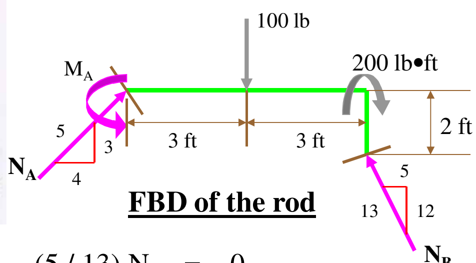
PROBLEM SOLVING (Continued)





$\rightarrow + \; \Sigma F_X = (4/5)\, N_A - (5/13)\, N_B = 0$

$\uparrow + \; \Sigma F_Y = (3/5)\, N_A + (12/13)\, N_B - 100 = 0$

Solving these two equations, we get
$N_B = 82.54$ or 82.5 lb and $N_A = 39.68$ or 39.7 lb

$\circlearrowleft + \Sigma M_A = M_A - 100 * 3 - 200 + (12/13)\, N_B * 6 - (5/13)\, N_B * 2 = 0$

$M_A = 106$ lb • ft

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QUIZ

1. If a support prevents rotation of a body about an axis, then the support exerts a ________ on the body about that axis.

   A) couple moment B) force

   C) Both A and B. D) None of the above.

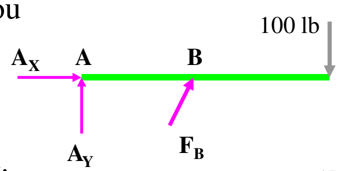
2. When doing a 3-D problem analysis, you have ________ scalar equations of equilibrium.

   2. Enter a number corresponding to your answer.

3. Which equation of equilibrium allows you to determine F_B right away?

   A) $\sum F_X = 0$ B) $\sum F_Y = 0$

   C) $\sum M_A = 0$ D) Any one of the above.



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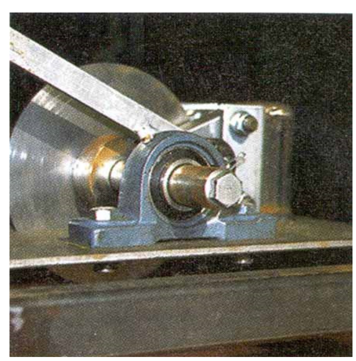
SUPPORT REACTIONS (APPLICATIONS)



Ball-and-socket joints and journal bearings are often used in mechanical systems.
How can we determine the support reactions at these joints for a given loading?

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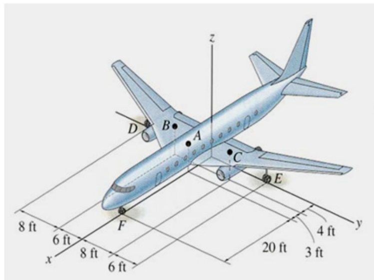
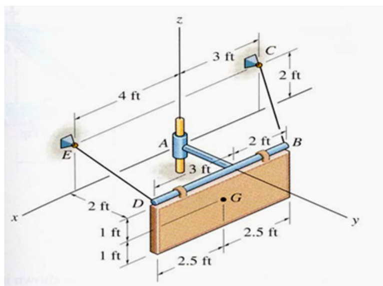
APPLICATIONS (continued)



The weights of the fuselage and fuel act through A, B, and C. How will we determine the reactions at the wheels D, E and F ?



A 50 lb sign is kept in equilibrium using two cables and a smooth collar. How can we determine the reactions at these supports?

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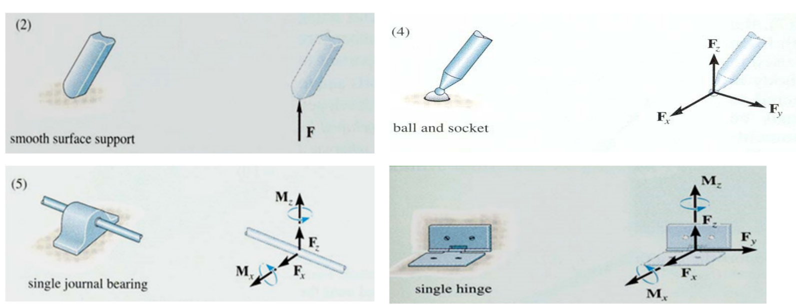
SUPPORT REACTIONS IN 3-D (Table 5-2)



As a general rule, if a support prevents translation of a body in a given direction, then a reaction force acting in the opposite direction is developed on the body. Similarly, if rotation is prevented, a couple moment is exerted on the body by the support.

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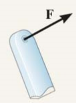
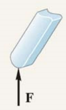
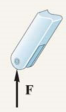
SUPPORT REACTIONS IN 3-D (Table 5-2)

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

continued

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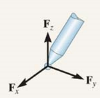
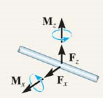
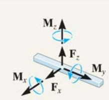
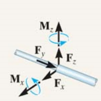
SUPPORT REACTIONS IN 3-D (Table 5-2)

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft.
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components.

continued

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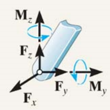
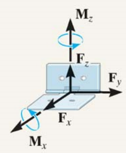
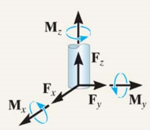
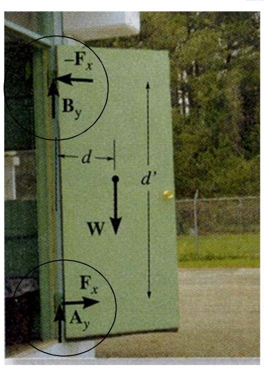
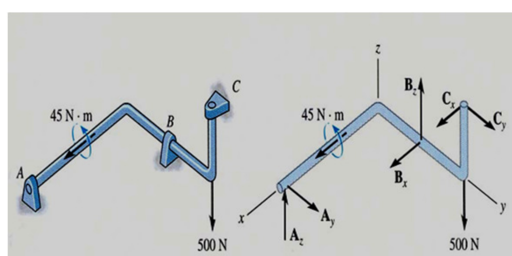
SUPPORT REACTIONS IN 3-D (Table 5-2)

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  single smooth pin		Five unknowns. The reactions are three force and two couple-moment components.
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

IMPORTANT NOTE





A single bearing or hinge can prevent rotation by providing a resistive couple moment. However, it is usually preferred to use two or more properly aligned bearings or hinges. Thus, in these cases, only force reactions are generated and there are no moment reactions created.