Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 11

* EQUILIBRIUM OF A RIGID BODY
* FREE-BODY DIAGRAMS
* EQUATIONS OF EQUILIBRIUM

Section 6.1-6.3

Ehab Zalok

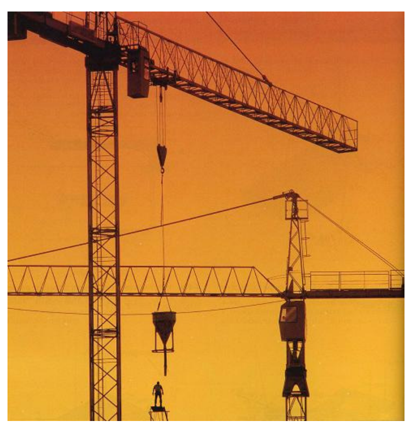
EQUILIBRIUM OF A RIGID BODY & FREE-BODY DIAGRAMS

Objectives:

Students will be able to:

a) Identify support reactions, and,

b) Draw a free-body diagram.



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READING QUIZ

1. If a support prevents translation of a body, then the support exerts a ___________ on the body.

   1) couple moment

   2) force

   3) Both 1 and 2.

   4) None of the above

2. Internal forces are _________ shown on the free body diagram of a whole body.

   A) always

   B) often

   C) rarely

   D) never

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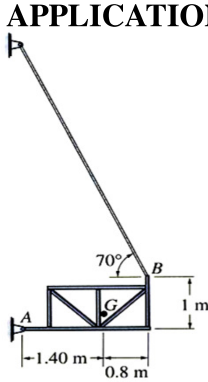
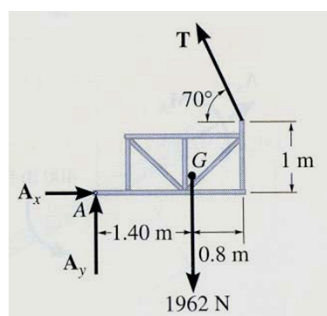
APPLICATIONS





A 200 kg platform is suspended off an oil rig.

How do we determine the force reactions at the joints and the forces in the cables?

How are the idealized model and the free body diagram used to do this?

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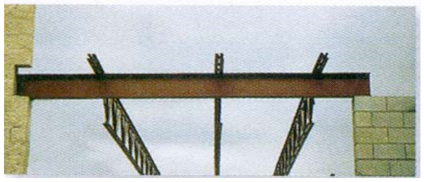
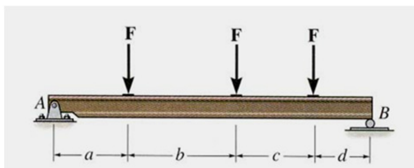
APPLICATIONS
(continued)





A steel beam is used to support roof joists.

How can we determine the support reactions at A & B?

How can we make use of an idealized model and a free body diagram to answer this question?

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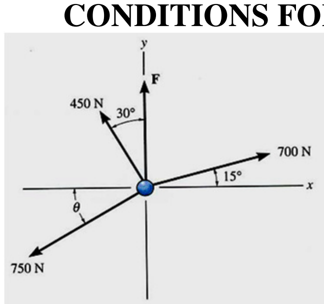
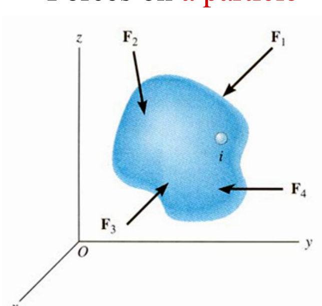
CONDITIONS FOR RIGID-BODY EQUILIBRIUM
(Section 5.1)



Forces on a particle

In contrast to the forces on a particle, the forces on a rigid-body <u>are not usually concurrent</u> and may cause rotation of the body
(due to the moments created by the forces).



Forces on a rigid body

For a rigid body to be in equilibrium, the net force as well as the net moment about any arbitrary point O must be equal to zero.

$\sum \boldsymbol{F} = 0$ and $\sum \boldsymbol{M_O} = 0$

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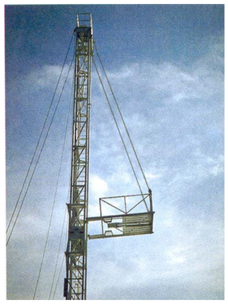
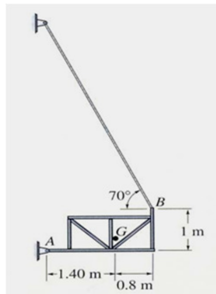
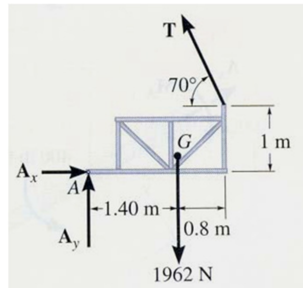
THE PROCESS OF SOLVING RIGID BODY EQUILIBRIUM PROBLEMS





Idealized model



Free-Body Diagram

(1) For analyzing an actual physical system, first we need to create an idealized model.

(2) Then we need to draw a free-body diagram showing all the external (active and reactive) forces.

(3) Finally, we need to apply the equations of equilibrium to solve for any unknowns.

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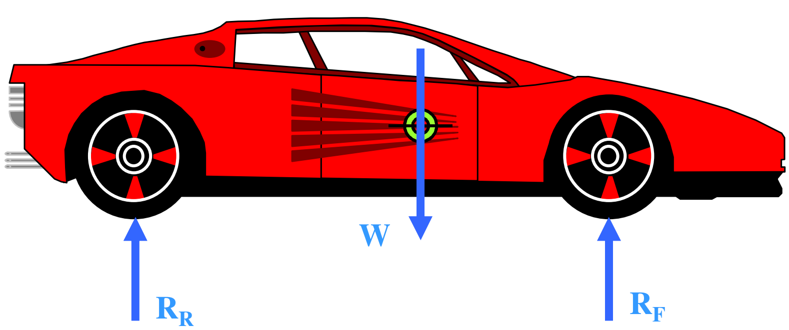
Free Body Diagrams

(Section 5.2)



External forces only

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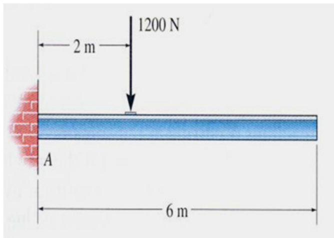
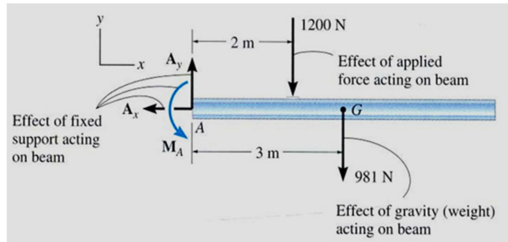
FREE-BODY DIAGRAMS
(continued)



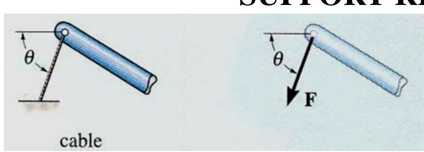
Idealized model

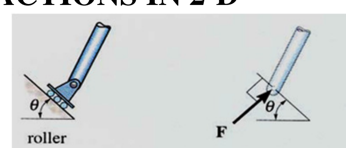


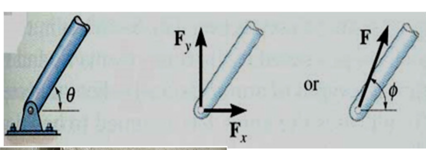
Free-body diagram

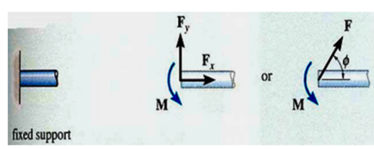
Label loads and dimensions: All known forces and couple moments should be labeled with their magnitudes and directions.
For the unknown forces and couple moments,
use letters like A_x, A_y, M_A, etc.
Indicate any necessary dimensions.

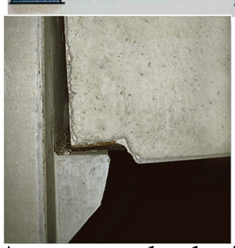
SUPPORT REACTIONS IN 2-D

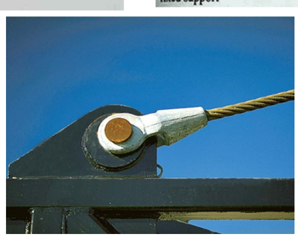


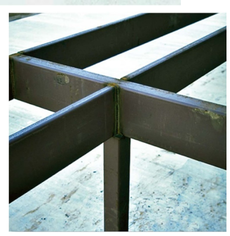










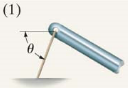
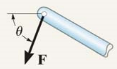
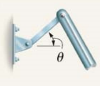
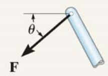
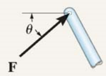
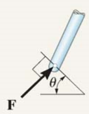
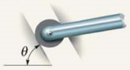
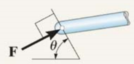
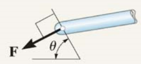


As a general rule, if a support prevents translation of a body in a given direction, then a force is developed on the body in the opposite direction.

If rotation is prevented, a couple moment is exerted on the body.

SUPPORT REACTIONS IN 2-D

TABLE 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link	 or 	One unknown. The reaction is a force which acts along the axis of the link.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  roller or pin in confined smooth slot	 or 	One unknown. The reaction is a force which acts perpendicular to the slot.

continued

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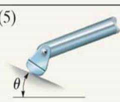
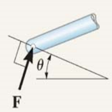
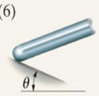
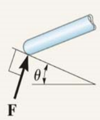
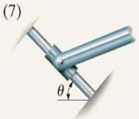
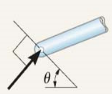
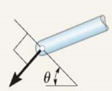
SUPPORT REACTIONS IN 2-D

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(5)  rocker		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(6)  smooth contacting surface		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(7)  member pin connected to collar on smooth rod	 or 	One unknown. The reaction is a force which acts perpendicular to the rod.

continued

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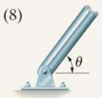
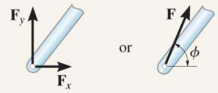
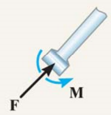
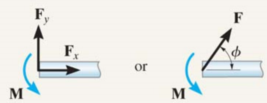
SUPPORT REACTIONS IN 2-D

TABLE 5–1 Continued

Types of Connection	Reaction	Number of Unknowns
(8)  smooth pin or hinge		Two unknowns. The reactions are two components of force, or the magnitude and direction ϕ of the resultant force. Note that ϕ and θ are not necessarily equal [usually not, unless the rod shown is a link as in (2)].
(9)  member fixed connected to collar on smooth rod		Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.
(10)  fixed support		Three unknowns. The reactions are the couple moment and the two force components, or the couple moment and the magnitude and direction ϕ of the resultant force.

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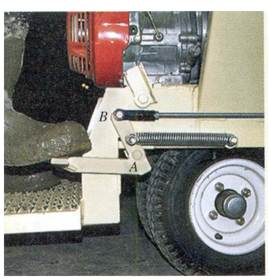
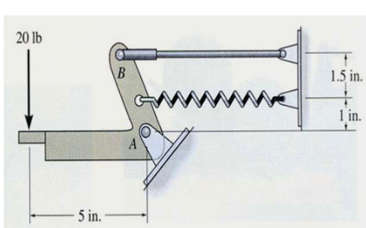
EXAMPLE

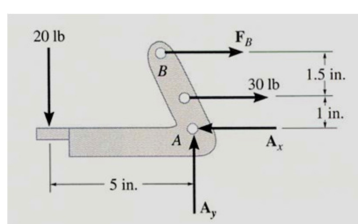


Given: An operator applies 20 lb to the foot pedal. A spring with k = 20 lb/in is stretched 1.5 in.

Draw: A free-body diagram of the foot pedal.



The idealized model



The free-body diagram

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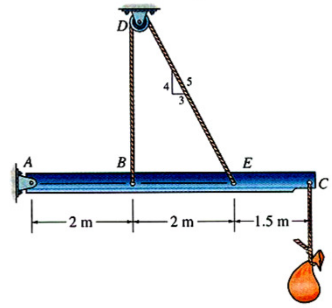
CONCEPT QUIZ



1. The beam and the cable (with a frictionless pulley at D) support an 80 kg load at C.
   In a FBD of only the beam, there are how many unknowns?
   1) 2 forces and 1 couple moment
   2) 3 forces and 1 couple moment
   3) 3 forces
   4) 4 forces

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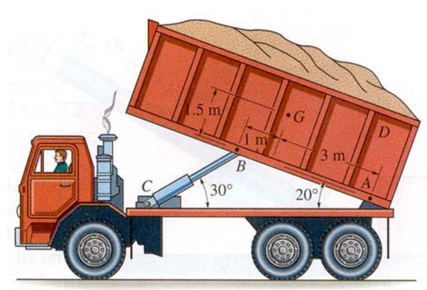
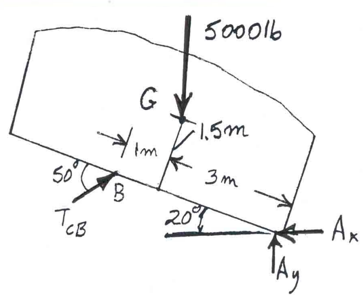
PROBLEM SOLVING



Draw a FBD of the 5000 lb dumpster (D).
It is supported by a pin at A and the hydraulic cylinder BC (treat as a short link).



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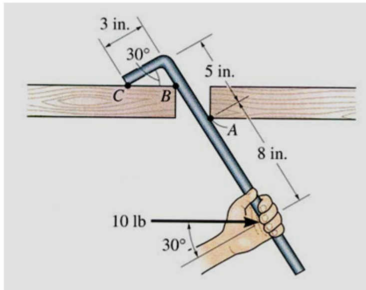
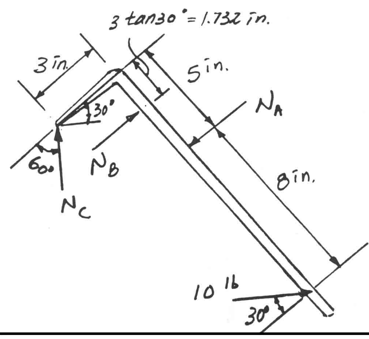
PROBLEM SOLVING



Draw a FBD of the bar, which has smooth points of contact at A, B, and C.



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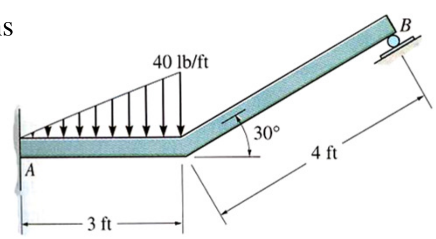
ATTENTION QUIZ

2. How many unknown support reactions are there in this problem?
   1) 2 forces and 2 couple moments
   2) 1 force and 2 couple moments
   3) 3 forces
   4) 3 forces and 1 couple moment



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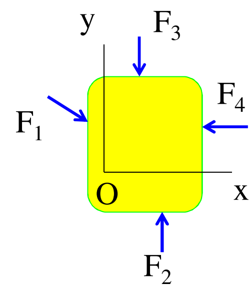
EQUATIONS OF EQUILIBRIUM
(Section 5.3)

- A body is subjected to a system of forces that lie in the x-y plane.
- When in equilibrium, the net force and net moment acting on the body are zero (as discussed earlier in Section 5.1).
- This 2-D condition can be represented by the three scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_O = 0$$

Where point O is any arbitrary point.



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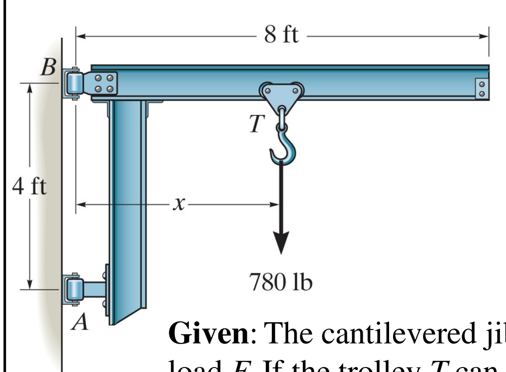

EXAMPLE

Given: The cantilevered jib crane is used to support the load *F.* If the trolley *T* can be placed anywhere in the range 1.5ft $\leq x \leq$7.5

Determine the maximum magnitude of reaction at the supports *A* and *B*.

Note that (1) the supports are collars that allow the crane to rotate freely about the vertical axis.
(2)The collar at B supports a force in the vertical direction, whereas the one at *A* does not.

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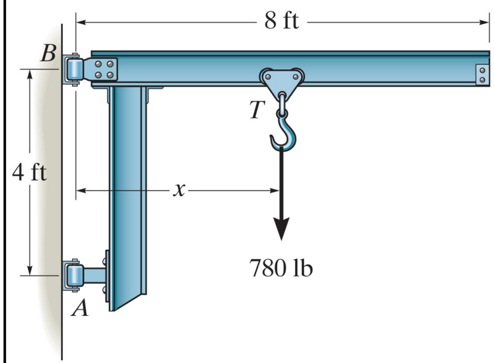
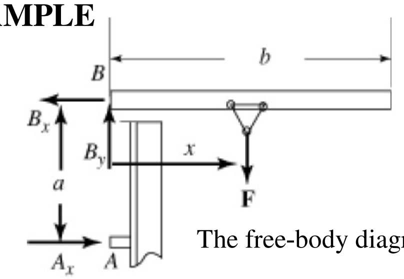
EXAMPLE





The free-body diagram

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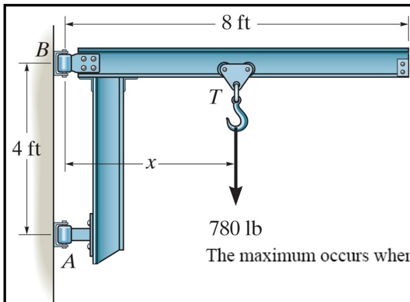
EXAMPLE

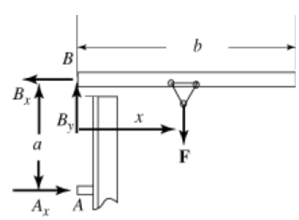


The maximum occurs when $x = x_2$

$\circlearrowleft + \ \Sigma M_A = 0; \quad -F x_2 + B_x a = 0$

$$B_x = F \frac{x_2}{a}$$

$B_x = 1462.50 \text{ lb}$



The free-body diagram

$\overset{+}{\rightarrow} \ \Sigma F_x = 0; \quad A_x - B_x = 0 \qquad A_x = B_x \qquad A_x = 1462.50 \text{ lb}$

$+\uparrow \ \Sigma F_y = 0; \quad B_y - F = 0 \qquad B_y = F \qquad B_y = 780.00 \text{ lb}$

$F_B = \sqrt{B_x^2 + B_y^2} \qquad F_B = 1.66 \text{ kip}$

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