Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 10

CENTROIDS OF COMPOSITES

Section 5.2

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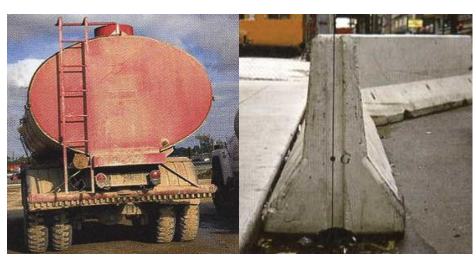
Today's Objective:

Students will be able to determine:

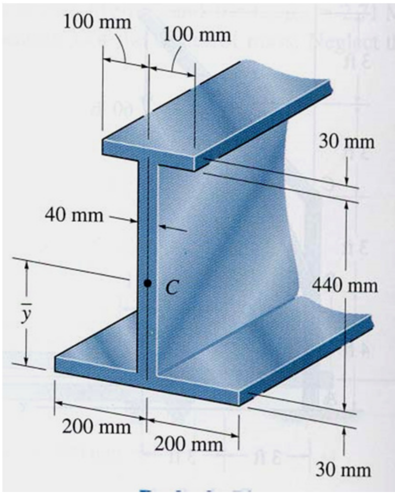
a) The location of the center of gravity.

b) The location of the center of mass.

c) The location of the centroid using the method of composite bodies.



APPLICATIONS



The I-beam is commonly used in building structures.

When doing a stress analysis on an I - beam, the location of the centroid is very important.

How can we easily determine the location of the centroid for a given beam shape?

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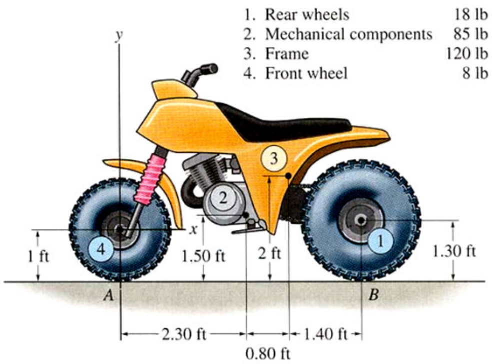
APPLICATIONS
(continued)



Cars, trucks, bikes, etc., are assembled using many individual components.

When designing for stability on the road, it is important to know the location of the bikes' center of gravity (CG).

If we know the weight and CG of individual components, how can we determine the location of the CG of the assembled unit?

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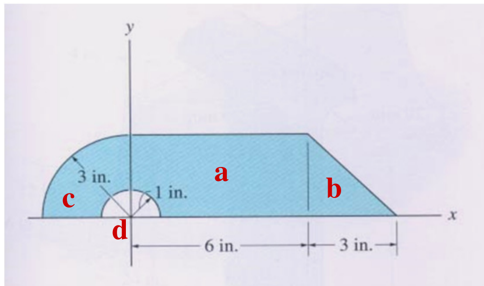
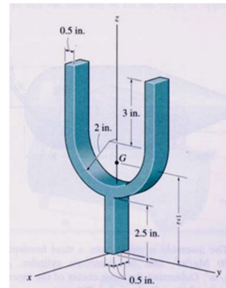
CONCEPT OF A COMPOSITE BODY





Many industrial objects can be considered as composite bodies made up of a series of connected "simpler" shaped parts or holes, like a rectangle, triangle, and semicircle.

Knowing the location of the centroid, C, or center of gravity, G, of the simpler shaped parts, we can easily determine the location of the C or G for the more complex composite body.

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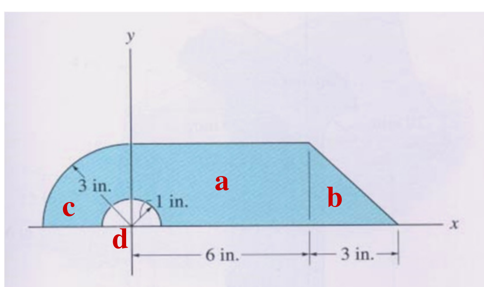
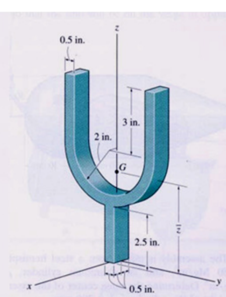
CONCEPT OF A COMPOSITE BODY
(continued)





This can be done by considering each part as a "particle" and following the procedure as described in Section 9.1.

This is a simple, effective, and practical method of determining the location of the centroid or center of gravity.

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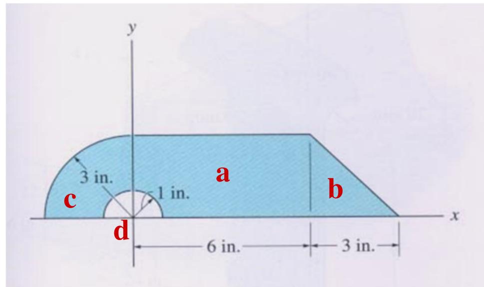
EXAMPLE

Given: The part shown.

Find: The centroid of the part.

Plan: Follow the steps for analysis.



Solution:

1. This body can be divided into the following pieces: rectangle (a) + triangle (b) + quarter circular (c) – semicircular area (d)

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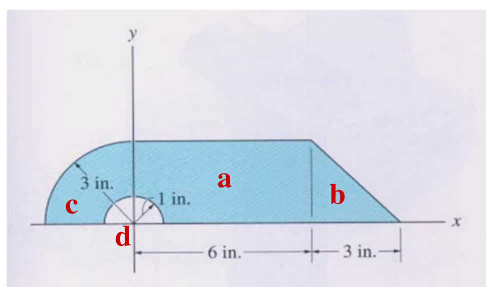
EXAMPLE (continued)

Steps 2 & 3: Make up and fill the table using parts a, b, c, and d.



Segment	Area A (in^2)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$A\tilde{x}$ (in^3)	$A\tilde{y}$ (in^3)
Rectangle a	18	3	1.5	54	27
Triangle b	4.5	7	1	31.5	4.5
Q. Circle c	$9\pi / 4$	$-4(3)/(3\pi)$	$4(3)/(3\pi)$	– 9	9
Semi-Circle d	$-\pi / 2$	0	$4(1)/(3\pi)$	0	- 2/3
Σ	28.0			76.5	39.83

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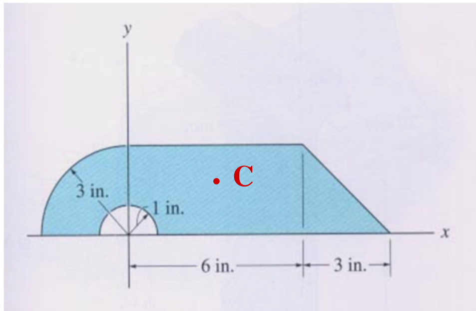
EXAMPLE
(continued)



4. Now use the table data and these formulas to find the coordinates of the centroid.

$\bar{x} = (\Sigma \tilde{x} A) / (\Sigma A) = 76.5 \text{ in}^3 / 28.0 \text{ in}^2 = 2.73 \text{ in}$

$\bar{y} = (\Sigma \tilde{y} A) / (\Sigma A) = 39.83 \text{ in}^3 / 28.0 \text{ in}^2 = 1.42 \text{ in}$

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STEPS FOR ANALYSIS

1. Divide the body into pieces that are known shapes.
   Holes are considered as pieces with negative weight or size.
2. Make a table with the first column for segment number, the second column for weight, mass, or size (depending on the problem), the next set of columns for the moment arms, and, finally, several columns for recording results of simple intermediate calculations.
3. Fix the coordinate axes, determine the coordinates of the center of gravity of centroid of each piece, and then fill-in the table.
4. Sum the columns to get $\bar{x}$, $\bar{y}$, and $\bar{z}$. Use formulas like

   $\bar{x} = (\Sigma \tilde{x}_i A_i) / (\Sigma A_i)$ or $\bar{x} = (\Sigma \tilde{x}_i W_i) / (\Sigma W_i)$

   This approach will become clear by doing examples!

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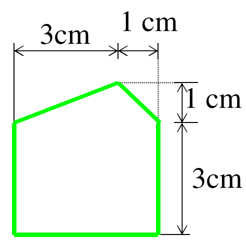
CONCEPT QUIZ

1. Based on the typical centroid information available in handbooks, what are the minimum number of segments you will have to consider for determing the centroid of the given area?

   1, 2, 3, or 4



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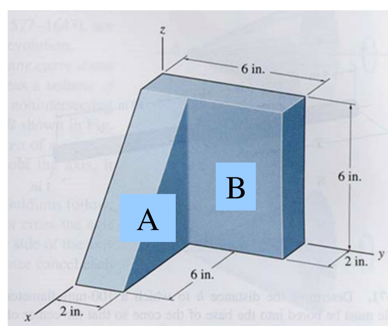
PROBLEM SOLVING

Given: Two blocks of different materials are assembled as shown.



The densities of the materials are:

ρ_A = 150 lb / ft^3 and
ρ_B = 400 lb / ft^3.

Find: The center of gravity of this assembly.

Plan: Follow the steps for analysis.

<u>Solution</u>

1. In this problem, the blocks A and B can be considered as two segments.

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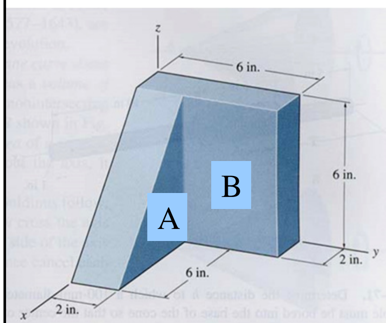
PROBLEM SOLVING (continued)



Weight = w = ρ (Volume in ft^3)

w_A = 150 (0.5) (6) (6) (2) / $(12)^3$ = 3.125 lb

w_B = 400 (6) (6) (2) / $(12)^3$ = 16.67 lb

Segment	w (lb)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$\tilde{z}$ (in)	w $\tilde{x}$ (lb·in)	w $\tilde{y}$ (lb·in)	w$\tilde{z}$ (lb·in)
A	3.125	4	1	2	12.5	3.125	6.25
B	16.67	1	3	3	16.67	50.00	50.00
Σ	19.79				29.17	53.12	56.25

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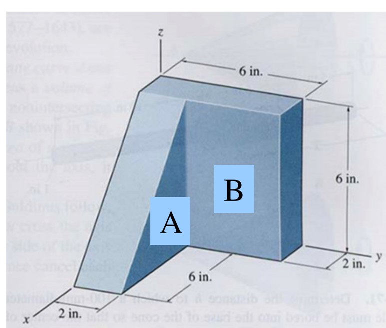
PROBLEM SOLVING (continued)



$\bar{x}$ = (Σ $\tilde{x}$ w) / (Σw) = 29.17/19.79 = 1.47 in

$\bar{y}$ = (Σ $\tilde{y}$ w) / (Σw) = 53.12/ 19.79 = 2.68 in

$\bar{z}$ = (Σ $\tilde{z}$ w) / (Σw) = 56.25 / 19.79 = 2.84 in

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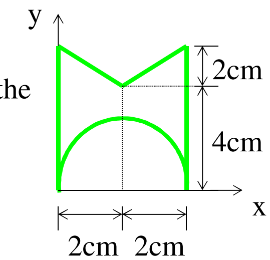
ATTENTION QUIZ

1. A rectangular area has semicircular and triangular cuts as shown. For determining the centroid, what is the minimum number of pieces that you can use?

   A) Two B) Three

   C) Four D) Five



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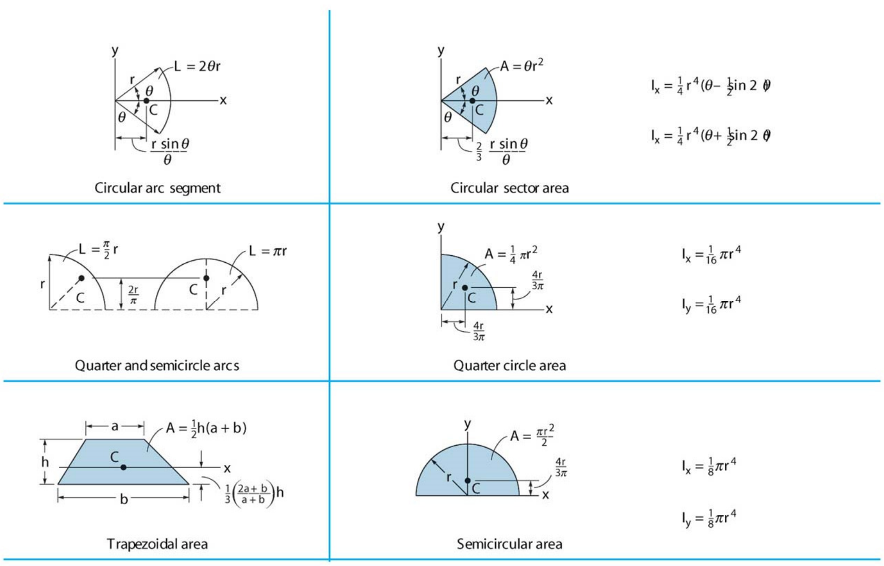
Geometric Properties of Lines and Area elements



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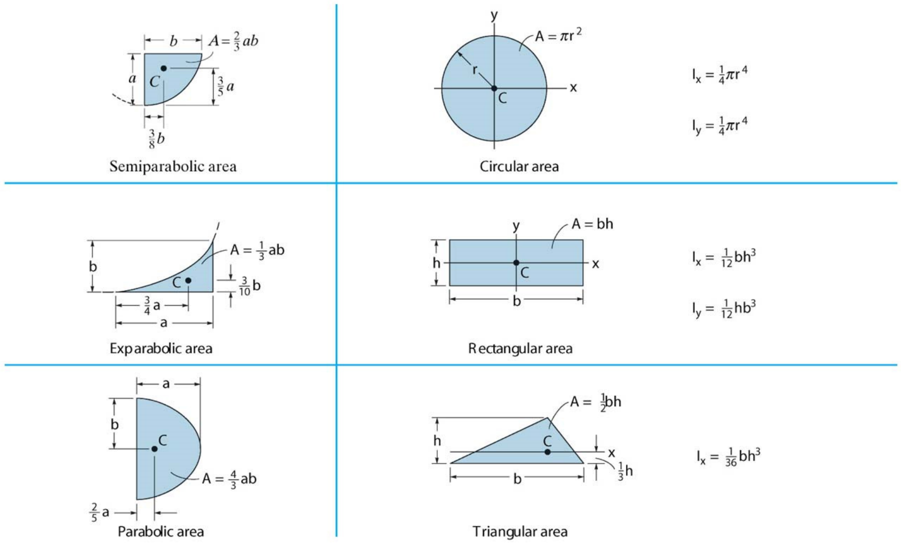
Geometric Properties of Lines and Area elements



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