Carleton University * Faculty of Engineering and Design

Mechanics I

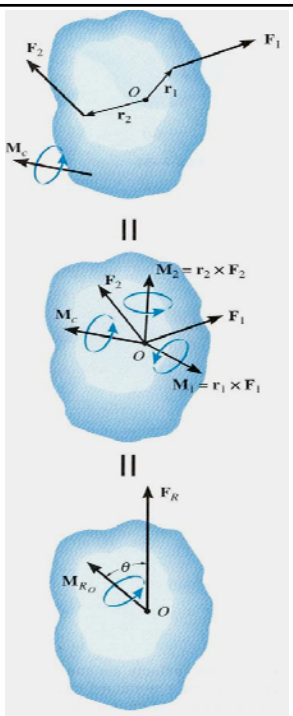
ECOR 1101

Lecture 08

* RESULTANTS OF A FORCE AND COUPLE SYSTEM
* FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM

Section 4.8-4.9

Ehab Zalok



RESULTANTS OF A FORCE AND COUPLE SYSTEM
(Section 4.8)

When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

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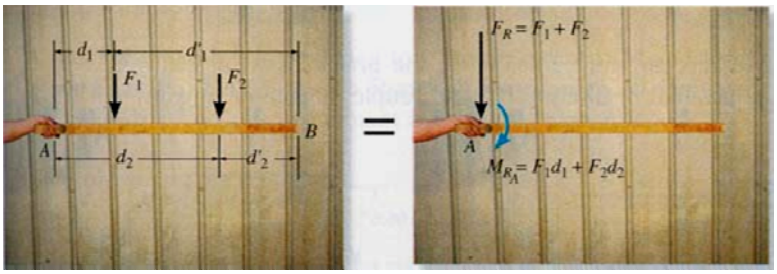
RESULTANT OF A FORCE AND COUPLE SYSTEM

(continued)



If the force system lies in the x-y plane (the 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

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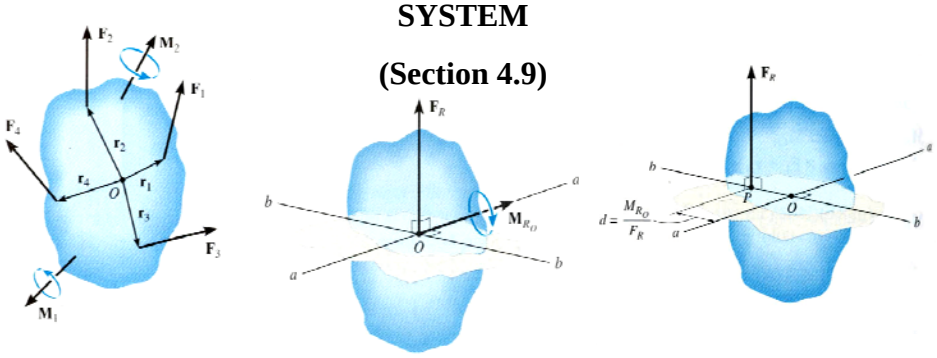
FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM

(Section 4.9)



If $\boldsymbol{F_R}$ and $\boldsymbol{M_{RO}}$ are perpendicular to each other, then the system can be further reduced to a single force, $\boldsymbol{F_R}$, by simply moving $\boldsymbol{F_R}$ from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

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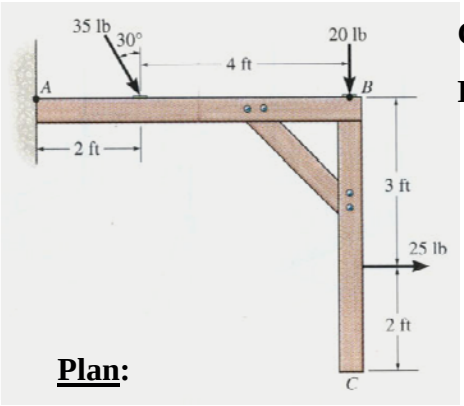
EXAMPLE #1



Given: A 2-D force as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location along the beam AB.

Plan:

1) Sum all the x and y components of the forces to find F_{RA}.
2) Find and sum all the moments resulting from moving each force to A.
3) Shift the F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

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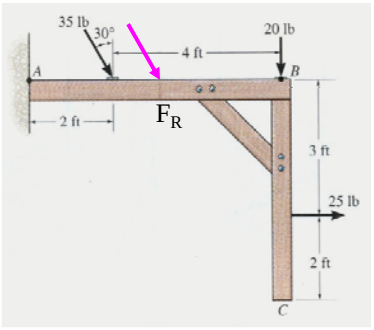
EXAMPLE #1
(continued)



$+\rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30° = 42.5$ lb

$+\downarrow \Sigma F_{Ry} = 20 + 35 \cos 30° = 50.31$ lb

$+\circlearrowleft M_{RA} = 35 \cos 30° (2) + 20(6) - 25(3)$

$= 105.6$ lb·ft

$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9$ lb

$\theta = \tan^{-1} (50.31/42.5) = 49.8°$

The equivalent single force F_R can be located on the beam AB at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 105.6/50.31 = 2.10$ ft.

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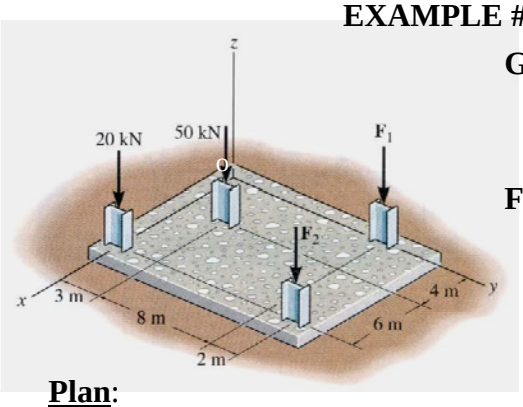
EXAMPLE #2



Given: The building slab has four columns. F_1 and $F_2 = 0$.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x,y) of the single equivalent resultant force.

Plan:

1) Find $\boldsymbol{F_{RO}} = \Sigma \boldsymbol{F_i} = F_{Rzo}\,\boldsymbol{k}$

2) Find $\boldsymbol{M_{RO}} = \Sigma\,(\boldsymbol{r_i} \times \boldsymbol{F_i}) = M_{RxO}\,\boldsymbol{i} + M_{RyO}\,\boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

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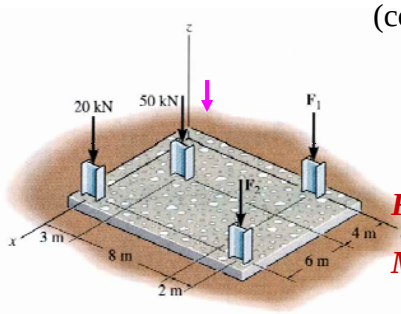
EXAMPLE #2
(continued)



$\boldsymbol{F_{RO}} = \{-50\,\boldsymbol{k} - 20\,\boldsymbol{k}\} = \{-70\,\boldsymbol{k}\}$ kN

$\boldsymbol{M_{RO}} = (10\,\boldsymbol{i}) \times (-20\,\boldsymbol{k}) + (4\,\boldsymbol{i} + 3\,\boldsymbol{j}) \times (-50\,\boldsymbol{k})$

$= \{200\,\boldsymbol{j} + 200\,\boldsymbol{j} - 150\,\boldsymbol{i}\}$ kN·m

$= \{-150\,\boldsymbol{i} + 400\,\boldsymbol{j}\}$ kN·m

The location of the single equivalent resultant force is given as,

$x = -M_{Ryo}/F_{Rzo} = -400/(-70) = 5.71$ m

$y = M_{Rxo}/F_{Rzo} = (-150)/(-70) = 2.14$ m

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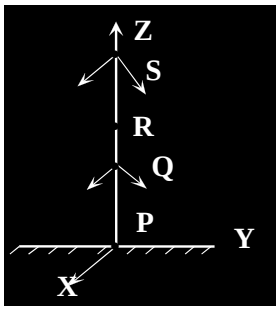
CONCEPT QUIZ



1. The forces on the pole can be reduced to a single force and a single moment at point ____ .

   1) P 2) Q 3) R

   4) S 5) Any of these points.

2. Consider two couples acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

   1) one force and one couple moment.

   2) one force.

   3) one couple moment.

   4) two couple moments.

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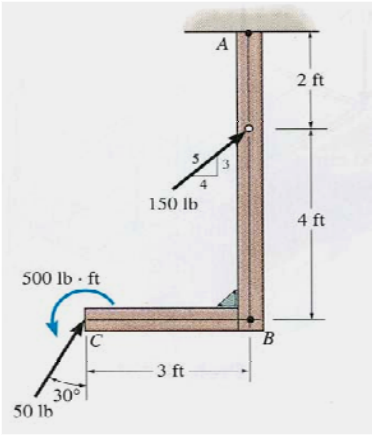
PROBLEM SOLVING



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A.

Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A and add them to the 500 lb - ft free moment to find the resultant M_{RA} .

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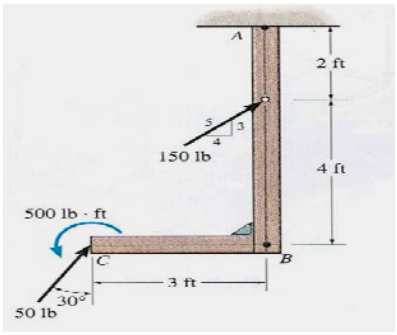
PROBLEM SOLVING (continued)



Summing the force components:

$+\rightarrow \Sigma F_x = (4/5)\ 150\ \text{lb} + 50\ \text{lb} \sin 30° = 145\ \text{lb}$

$+\uparrow \Sigma F_y = (3/5)\ 150\ \text{lb} + 50\ \text{lb} \cos 30° = 133.3\ \text{lb}$

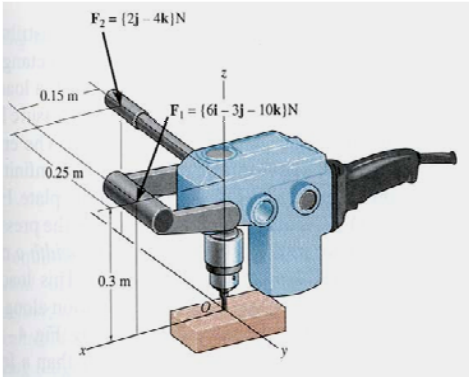
Now find the magnitude and direction of the resultant.

$F_{RA} = (145^2 + 133.3^2)^{1/2} = 197\ \text{lb}$ and $\theta = \tan^{-1}(133.3/145) = 42.6°$

$+\circlearrowleft M_{RA} = \{(4/5)(150)(2) - 50\cos 30°\,(3) + 50 \sin 30°\,(6) + 500\}$

$= 760\ \text{lb·ft}$

PROBLEM SOLVING



Given: Handle forces F_1 and F_2 are applied to the electric drill.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i}$

b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\,(\boldsymbol{r_i} \times \boldsymbol{F_i})$

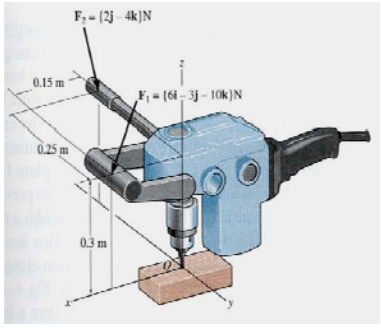
Where,

$\boldsymbol{F_i}$ are the individual forces in Cartesian Vector Notation (CVN).

$\boldsymbol{M_C}$ are any free couple moments in CVN (none in this example).

$\boldsymbol{R_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

SOLUTION



$\boldsymbol{F_1} = \{6\,\boldsymbol{i} - 3\,\boldsymbol{j} - 10\,\boldsymbol{k}\}$ N

$\boldsymbol{F_2} = \{0\,\boldsymbol{i} + 2\,\boldsymbol{j} - 4\,\boldsymbol{k}\}$ N

$\boldsymbol{F_{RO}} = \{6\,\boldsymbol{i} - 1\,\boldsymbol{j} - 14\,\boldsymbol{k}\}$ N

$\boldsymbol{r_1} = \{0.15\,\boldsymbol{i} + 0.3\,\boldsymbol{k}\}$ m

$\boldsymbol{r_2} = \{-0.25\,\boldsymbol{j} + 0.3\,\boldsymbol{k}\}$ m

$\boldsymbol{M_{RO}} = \boldsymbol{r_1} \times \boldsymbol{F_1} + \boldsymbol{r_2} \times \boldsymbol{F_2}$

$$\boldsymbol{M_{RO}} = \{ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix} \} \text{ N·m}$$

$= \{0.9\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k} + 0.4\,\boldsymbol{i} + 0\,\boldsymbol{j} + 0\,\boldsymbol{k}\}$ N·m

$= \{1.3\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k}\}$ N·m

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ATTENTION QUIZ

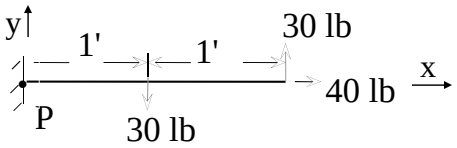
1. For this force system, the equivalent system at P is _____ .

A) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +60 ft ·lb

B) F_{RP} = 0 lb and M_{RP} = +30 ft · lb

C) F_{RP} = 30 lb (along +y-dir.) and M_{RP} = -30 ft ·lb

D) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +30 ft ·lb



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ATTENTION QUIZ

2. Consider three couples acting on a body. Equivalent systems will be _____ at different points on the body.

A) different when located

B) the same even when located

C) zero when located

D) None of the above.

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