Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 07

* MOMENT ABOUT AN AXIS
* MOMENT OF A COUPLE
* EQUIVALENT SYSTEMS

Section 4.5-4.7

Ehab Zalok

MOMENT ABOUT AN AXIS

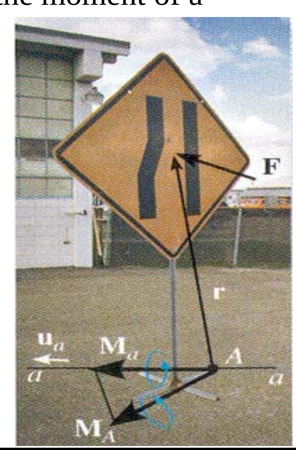
(Section 4.5)

Objectives:

Students will be able to determine the moment of a force about an axis using

a) scalar analysis, and

b) vector analysis.



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READING QUIZ

1. When determining the moment of a force about a specified axis, the axis must be along _____________.

   A) the x axis B) the y axis C) the z axis

   D) any line in 3-D space E) any line in the x-y plane

2. The triple scalar product $\boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$ results in

   A) a scalar quantity (+ or -). B) a vector quantity.

   C) zero. D) a unit vector.

   E) an imaginary number.

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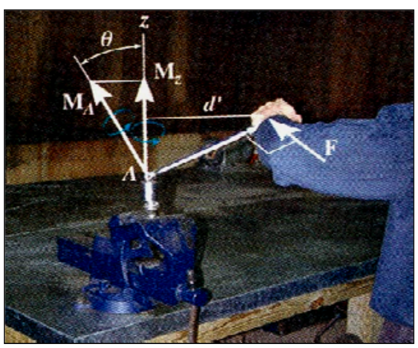
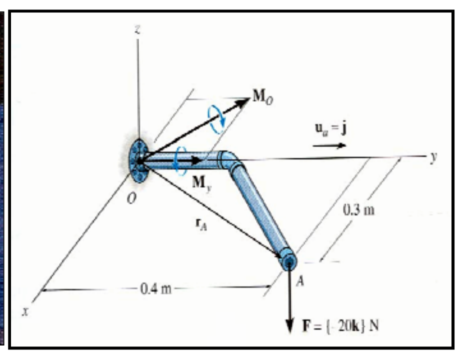
APPLICATIONS





With the force $\boldsymbol{F}$, a person is creating the moment $\boldsymbol{M_A}$. What portion of $\boldsymbol{M_A}$ is used in turning the socket?

The force $\boldsymbol{F}$ is creating the moment $\boldsymbol{M_O}$. How much of $\boldsymbol{M_O}$ acts to unscrew the pipe?

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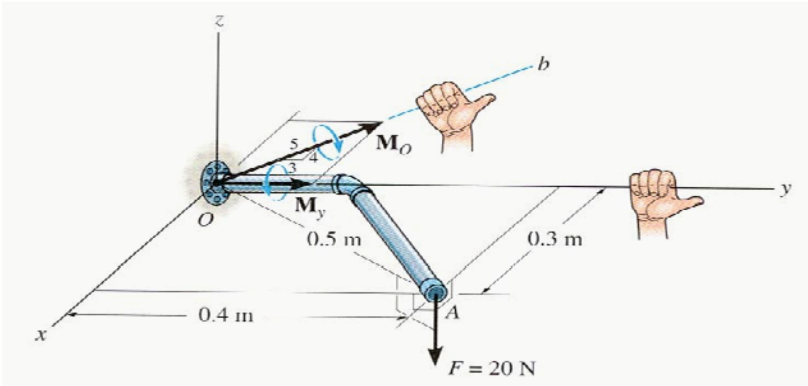
SCALAR ANALYSIS

Recall that the moment of a force about any point A is $M_A = F\, d_A$ where d_A is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.



In the figure above, the moment about the y-axis would be $M_y = 20\ (0.3) = 6$ N·m. However, this calculation is not always trivial and vector analysis may be preferable.

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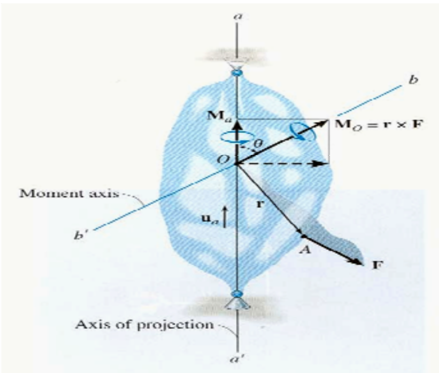
VECTOR ANALYSIS



Our goal is to find the moment of ***F*** (the tendency to rotate the body) about the axis a'-a.

First compute the moment of ***F*** about any arbitrary point O that lies on the a'a axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

Now, find the component of $\boldsymbol{M_O}$ along the axis a'-a using the dot product.

$$M_a = \boldsymbol{u_a} \cdot \boldsymbol{M_O}$$

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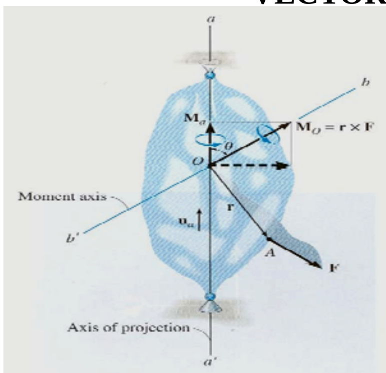
VECTOR ANALYSIS (continued)



M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the triple scalar product.

In the this equation,

u_a represents the unit vector along the axis a'-a axis,

r is the position vector from any point on the a'-a axis to any point A on the line of action of the force, and

F is the force vector.

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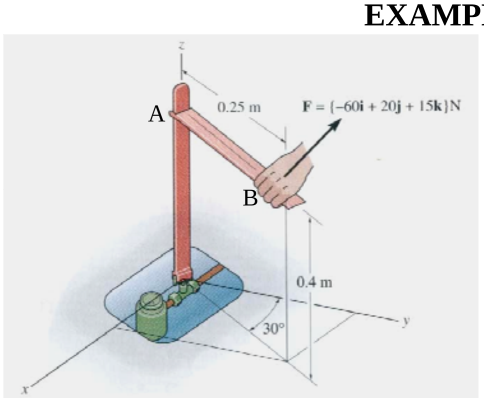
EXAMPLE



Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

Plan:

1) We need to use $M_z = u \bullet (r \times F)$.

2) Note that ***u*** = 1 ***k***.

3) The vector ***r*** is the position vector from A to B.

4) Force ***F*** is already given in Cartesian vector form.

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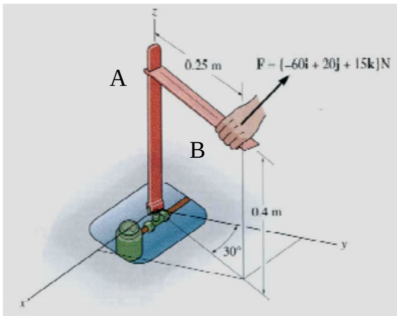
EXAMPLE (continued)



$\boldsymbol{u} = 1\ \boldsymbol{k}$

$\boldsymbol{r_{AB}} = \{0.25 \sin 30°\ \boldsymbol{i} + 0.25 \cos 30°\ \boldsymbol{j}\}$ m

$= \{0.125\ \boldsymbol{i} + 0.2165\ \boldsymbol{j}\}$ m

$\boldsymbol{F} = \{-60\ \boldsymbol{i} + 20\ \boldsymbol{j} + 15\ \boldsymbol{k}\}$ N

$M_z = \boldsymbol{u} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.2165 & 0 \\ -60 & 20 & 15 \end{vmatrix}$$

$= 1\{0.125(20) - 0.2165(-60)\}$ N·m

$= 15.5$ N·m

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CONCEPT QUIZ

1. The vector operation $(\boldsymbol{P} \times \boldsymbol{Q}) \bullet \boldsymbol{R}$ equals

A) $\boldsymbol{P} \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

B) $\boldsymbol{R} \bullet (\boldsymbol{P} \times \boldsymbol{Q})$.

C) $(\boldsymbol{P} \bullet \boldsymbol{R}) \times (\boldsymbol{Q} \bullet \boldsymbol{R})$.

D) $(\boldsymbol{P} \times \boldsymbol{R}) \bullet (\boldsymbol{Q} \times \boldsymbol{R})$.

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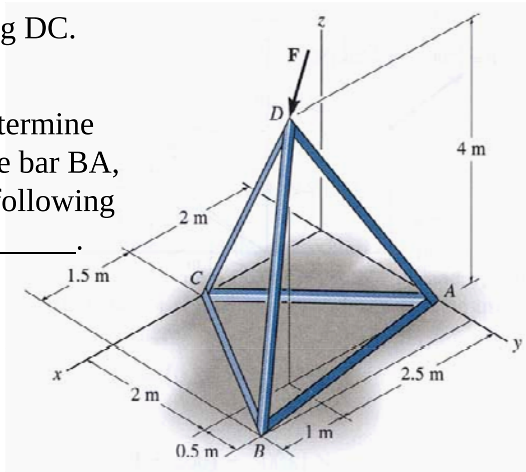
CONCEPT QUIZ

2. The force ***F*** is acting along DC.



Using the triple product to determine the moment of ***F*** about the bar BA, you could use any of the following position vectors except ______.

A) $\mathbf{r}_{BC}$ B) $\mathbf{r}_{AD}$

C) $\mathbf{r}_{AC}$ D) $\mathbf{r}_{DB}$

E) $\mathbf{r}_{BD}$

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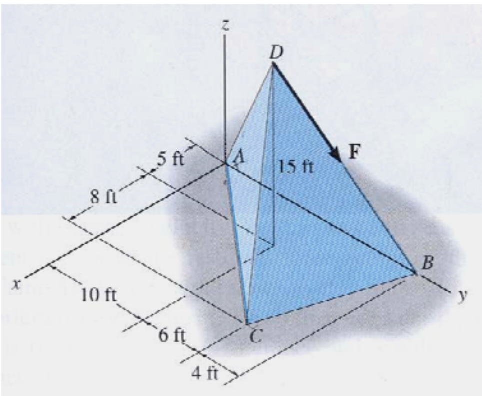
PROBLEM SOLVING



Given: A force of 80 lb acts along the edge DB.

Find: The magnitude of the moment of this force about the axis AC.

Plan:

1) We need to use $M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F}_{DB})$

2) Find $\mathbf{u}_{AC} = \mathbf{r}_{AC} / r_{AC}$

3) Find $\mathbf{F}_{DB} = 80 \text{ lb } \mathbf{u}_{DB} = 80 \text{ lb } (\mathbf{r}_{DB} / r_{DB})$

4) Complete the triple scalar product.

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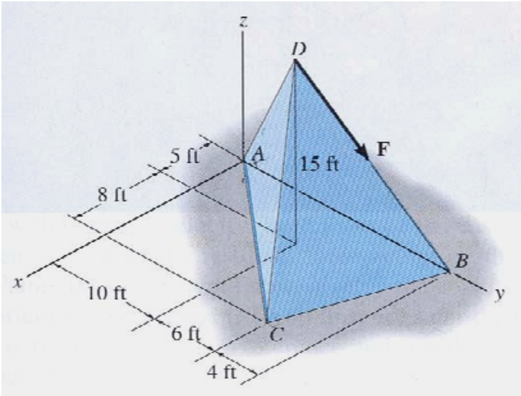
SOLUTION



$\boldsymbol{r}_{AB} = \{ 20\,\boldsymbol{j} \}$ ft

$\boldsymbol{r}_{AC} = \{ 13\,\boldsymbol{i} + 16\,\boldsymbol{j} \}$ ft

$\boldsymbol{r}_{DB} = \{ -5\,\boldsymbol{i} + 10\,\boldsymbol{j} - 15\,\boldsymbol{k} \}$ ft

$\boldsymbol{u}_{AC} = (13\,\boldsymbol{i} + 16\,\boldsymbol{j})$ ft $/ (13^2 + 16^2)^{½}$ ft

$= 0.6306\,\boldsymbol{i} + 0.7761\,\boldsymbol{j}$

$\boldsymbol{F}_{DB} = 80 \{\boldsymbol{r}_{DB} / (5^2 + 10^2 + 15^2)^{½} \}$ lb

$= \{-21.38\,\boldsymbol{i} + 42.76\,\boldsymbol{j} - 64.14\,\boldsymbol{k} \}$ lb

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<u>Solution</u> (continued)

Now find the triple product, $M_{AC} = \boldsymbol{u}_{AC} \cdot (\boldsymbol{r}_{AB} \times \boldsymbol{F}_{DB})$

$$M_{AC} = \begin{vmatrix} 0.6306 & 0.7761 & 0 \\ 0 & 20 & 0 \\ -21.38 & 42.76 & -64.14 \end{vmatrix} \begin{matrix} \\ \text{ft} \\ \text{lb} \end{matrix}$$

$M_{AC} = 0.6306 \{20 (-64.14) - 0 - 0.7706 (0 - 0)\}$ lb·ft

$= -809$ lb·ft

The negative sign indicates that the sense of $\boldsymbol{M}_{AC}$ is opposite to that of $\boldsymbol{u}_{AC}$

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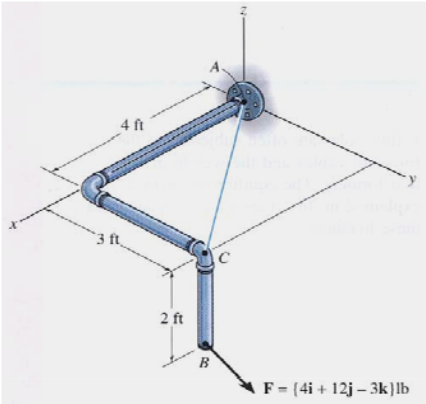
ATTENTION QUIZ

1. For finding the moment of the force ***F*** about the x-axis, the position vector in the triple scalar product should be ___ .

   A) $\mathbf{r}_{AC}$ B) $\mathbf{r}_{BA}$

   C) $\mathbf{r}_{AB}$ D) $\mathbf{r}_{BC}$



2. If $\mathbf{r}$ = {1 $\mathbf{i}$ + 2 $\mathbf{j}$} m and $\mathbf{F}$ = {10 $\mathbf{i}$ + 20 $\mathbf{j}$ + 30 $\mathbf{k}$} N, then the moment of ***F*** about the y-axis is ____ N·m.

   A) 10 B) -30

   C) -40 D) None of the above.

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MOMENT OF A COUPLE

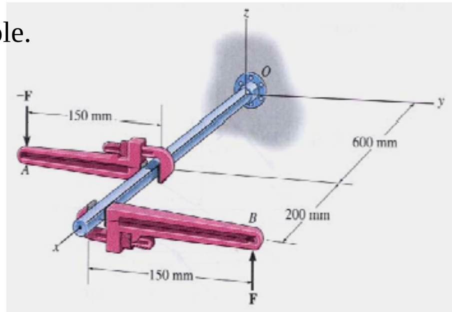
(Section 4.6)

Objectives:

Students will be able to

a) define a couple, and,

b) determine the moment of a couple.



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READING QUIZ

1. In statics, a couple is defined as __________ separated by a perpendicular distance.

   A) two forces in the same direction.

   B) two forces of equal magnitude.

   C) two forces of equal magnitude acting in the same direction.

   D) two forces of equal magnitude acting in opposite directions.

2. The moment of a couple is called a _________ vector.

   A) free   B) spin

   C) romantic   D) sliding

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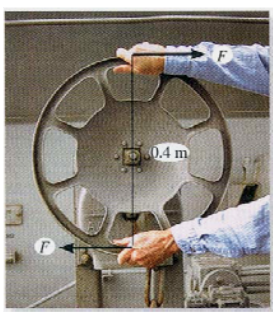
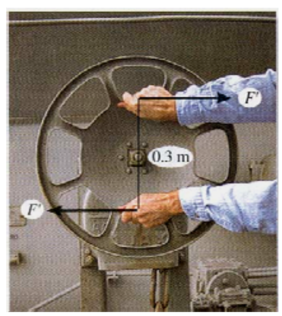
APPLICATIONS





A torque or moment of 12 N · m is required to rotate the wheel. Which one of the two grips of the wheel above will require less force to rotate the wheel?

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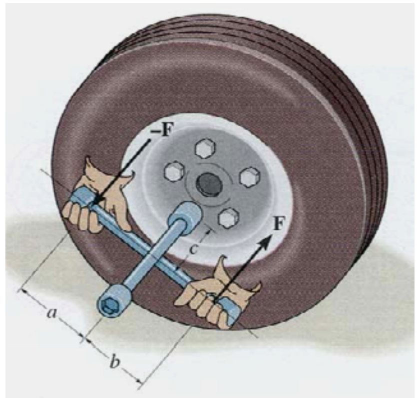
APPLICATIONS
(continued)



The crossbar lug wrench is being used to loosen a lug net. What is the effect of changing dimensions a, b, or c on the force that must be applied?

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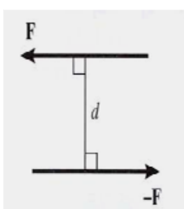
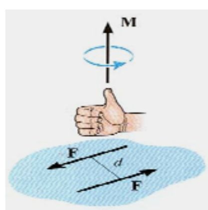
MOMENT OF A COUPLE





A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

The moment of a couple is defined as

M_O = F d (using a scalar analysis) or as

$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).

Here ***r*** is any position vector from the line of action of ***–F*** to the line of action of ***F***.

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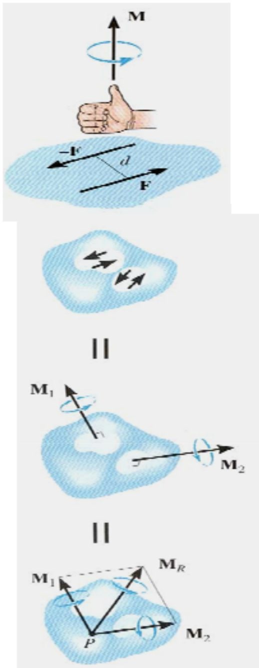

MOMENT OF A COUPLE
(continued)

The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F d

Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a free vector. It can be moved anywhere on the body and have the same external effect on the body.

Moments due to couples can be added using the same rules as adding any vectors.

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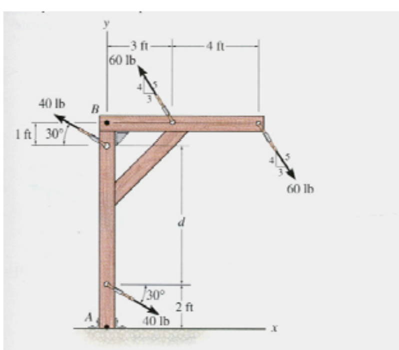
EXAMPLE - SCALAR APPROACH



Given: Two couples act on the beam and d equals 8 ft.

Find: The resultant couple

Plan:

1) Resolve the forces in x and y directions so they can be treated as couples.

2) Determine the net moment due to the two couples.

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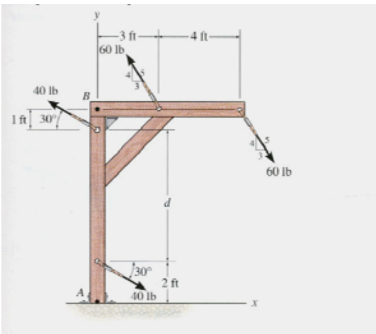
EXAMPLE - SCALAR APPROACH



The x and y components of the top 60 lb force are:

(4/5)(60 lb) = 48 lb vertically up

(3/5)(60 lb) = 36 lb to the left

Similarly for the top 40 lb force:

(40 lb) (sin 30°) up

(40 lb) (cos 30°) to the left

The net moment equals to

$+ \circlearrowleft \Sigma M =$ -(48 lb)(4 ft) + (40 lb)(cos 30º)(8ft)

= -192.0 + 277.1 = 85.1 ft·lb

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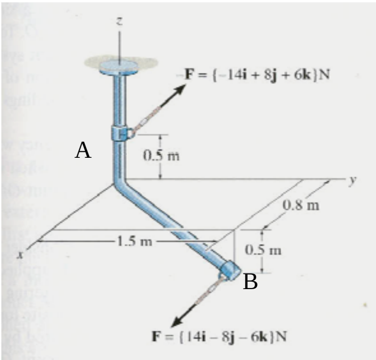
EXAMPLE – VECTOR APPROACH



Given: A force couple acting on the rod.

Find: The couple moment acting on the rod in Cartesian vector notation.

Plan:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

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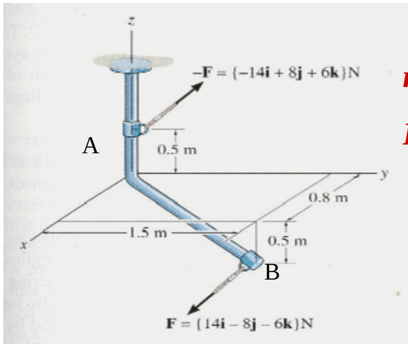
EXAMPLE – VECTOR APPROACH



$\boldsymbol{r}_{AB} = \{0.8\,\boldsymbol{i} + 1.5\,\boldsymbol{j} - 1\,\boldsymbol{k}\}$ m

$\boldsymbol{F} = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N

$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.8 & 1.5 & -1 \\ 14 & -8 & -6 \end{vmatrix} \text{ N·m}$$

$$= \{\boldsymbol{i}\,(-9 - (8)) - \boldsymbol{j}\,(-4.8 - (-14)) + \boldsymbol{k}\,(-6.4 - 21)\} \text{ N·m}$$

$$= \{-17\,\boldsymbol{i} - 9.2\,\boldsymbol{j} - 27.4\,\boldsymbol{k}\} \text{ N·m}$$

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CONCEPT QUIZ

If three couples act on a body, the overall result is that

A) the net force is not equal to 0.

B) the net force and net moment are equal to 0.

C) the net moment equals 0 but the net force is not necessarily equal to 0.

D) the net force equals 0 but the net moment is not necessarily equal to 0 .

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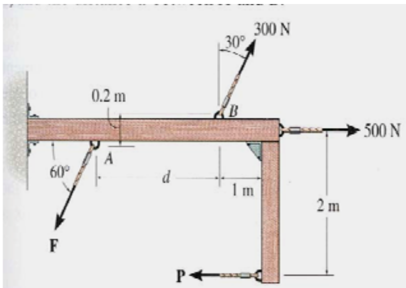
PROBLEM SOLVING – SCALAR APPROACH



Given: Two couples act on the beam. The resultant couple is zero.

Find: The magnitudes of the forces P and F and the distance d.

PLAN:

1) Use definition of a couple to find P and F.

2) Resolve the 300 N force in x and y directions.

3) Determine the net moment.

4) Equate the net moment to zero to find d.

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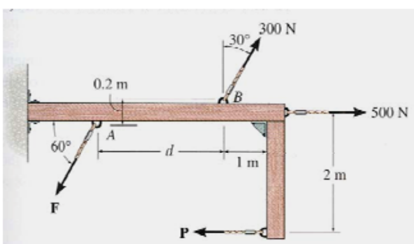
PROBLEM SOLVING – SCALAR APPROACH



From the definition of a couple:

P = 500 N and

F = 300 N.

Resolve the 300 N force into vertical and horizontal components. The vertical component is (300 cos 30º) N and the horizontal component is (300 sin 30º) N.

It was given that the net moment equals zero. So

$+\circlearrowleft \Sigma M = -(500)(2) + (300 \cos 30^\circ)(d) - (300 \sin 30^\circ)(0.2) = 0$

Now solve this equation for d.

$d = (1000 + 60 \sin 30^\circ) / (300 \cos 30^\circ) = 3.96$ m

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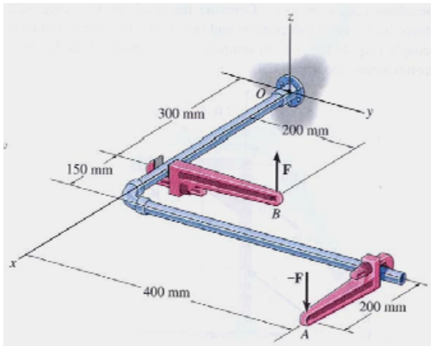
PROBLEM SOLVING – VECTOR APPROACH



Given: $\boldsymbol{F} = \{25\,\boldsymbol{k}\}$ N and
$-\boldsymbol{F} = \{-\ 25\,\boldsymbol{k}\}$ N

Find: The couple moment acting on the pipe assembly using Cartesian vector notation.

<u>PLAN</u>:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{25\,\boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

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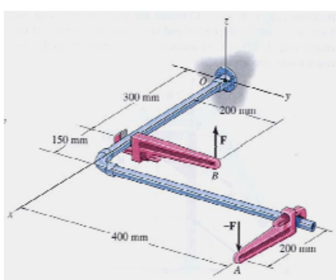
PROBLEM SOLVING – VECTOR APPROACH



$\boldsymbol{r}_{AB} = \{ -350\,\boldsymbol{i} - 200\,\boldsymbol{j} \}$ mm

$= \{ -0.35\,\boldsymbol{i} - 0.2\,\boldsymbol{j} \}$ m

$\boldsymbol{F} = \{25\,\boldsymbol{k}\}$ N

$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix} \quad \text{N} \cdot \text{m}$$

$$= \{ \boldsymbol{i}\,(-5 - 0) - \boldsymbol{j}\,(-8.75 - 0) + \boldsymbol{k}\,(0) \}\ \text{N} \cdot \text{m}$$

$$= \{ -5\,\boldsymbol{i} + 8.75\,\boldsymbol{j} \}\ \text{N} \cdot \text{m}$$

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EQUIVALENT SYSTEMS

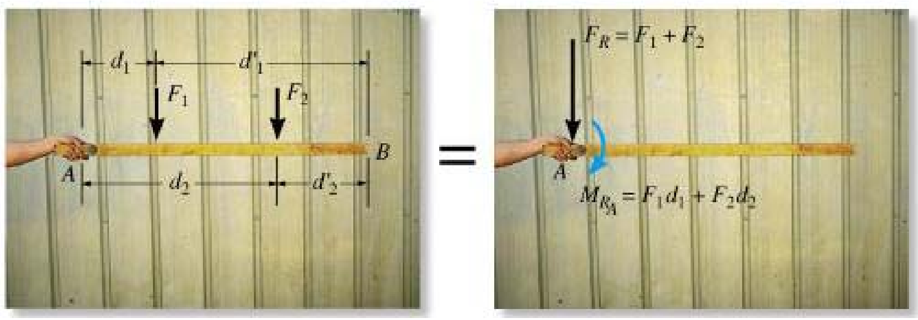
Section 4.7

Objectives:

Students will be able to:

a) Determine the effect of moving a force.

b) Find an equivalent force-couple system for a system of forces and couples.



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READING QUIZ

1. A general system of forces and couple moments acting on a rigid body can be reduced to a ___ .

   1) single force.

   2) single moment.

   3) single force and two moments.

   4) single force and a single moment.

2. The original force and couple system and an equivalent force-couple system have the same _____ effect on a body.

   1) internal 2) external

   3) internal and external 4) microscopic

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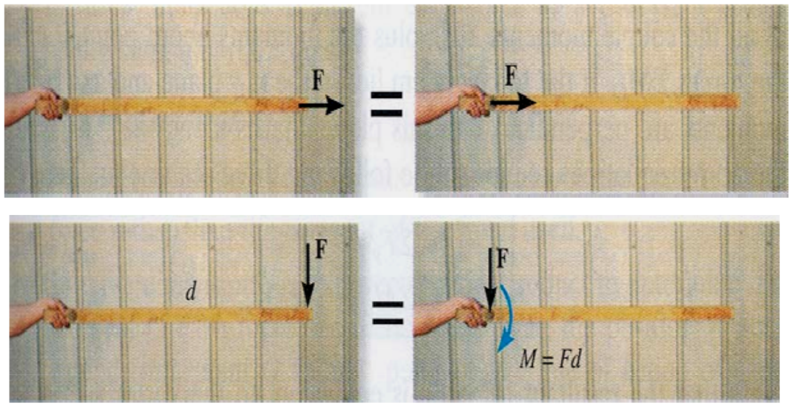
APPLICATIONS



What is the resultant effect on the person's hand when the force is applied in four different ways ?

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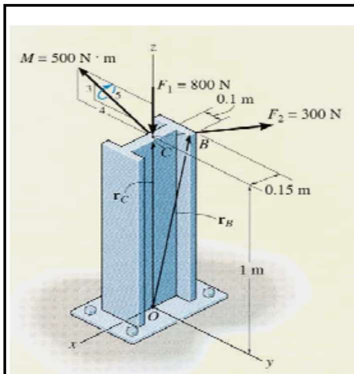

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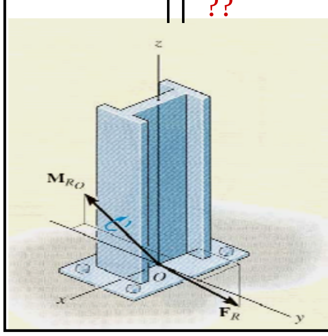

APPLICATIONS (continued)

Several forces and a couple moment are acting on this vertical section of an I-beam.

Can you replace them with just one force and one couple moment at point O that will have the same external effect? If yes, how will you do that?

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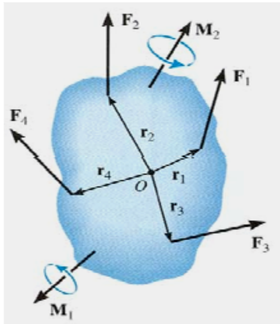
AN EQUIVALENT SYSTEM
(Section 4.7)



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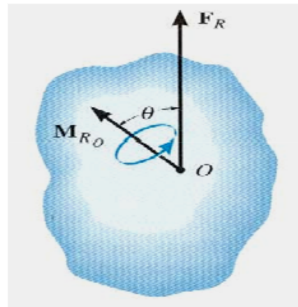

When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

The two force and couple systems are called equivalent systems since they have the same external effect on the body.

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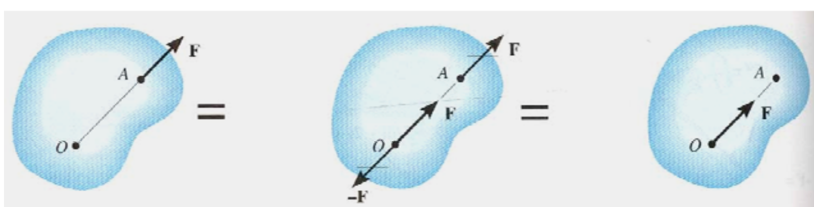
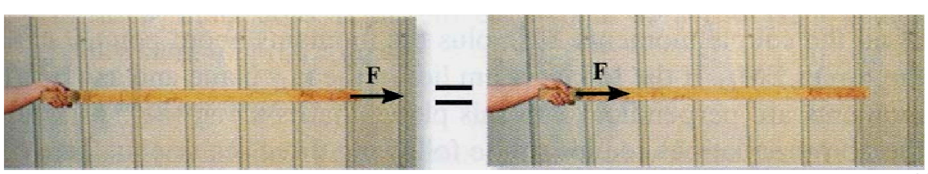
MOVING A FORCE ON ITS LINE OF ACTION



Moving a force from A to O, when both points are on the vectors' line of action, does not change the external effect. Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).



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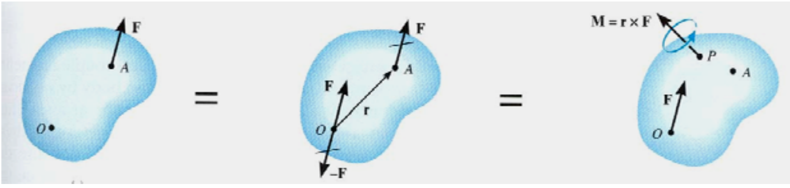
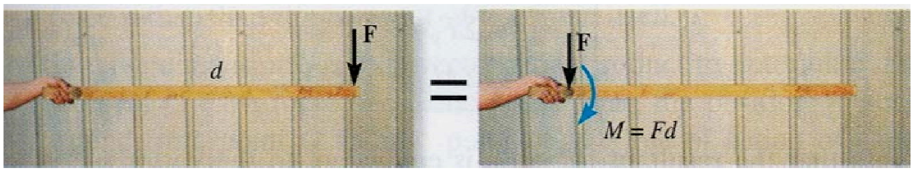
MOVING A FORCE OFF OF ITS LINE OF ACTION



Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a "free" vector, it can be applied at any point P on the body.



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