Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 06

* MOMENT OF A FORCE SCALAR FORMULATION
* CROSS PRODUCT
* MOMENT OF A FORCE VECTOR FORMULATION
* PRINCIPLE OF MOMENTS

Section 4.1-4.4

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MOMENT OF A FORCE SCALAR FORMULATION, CROSS PRODUCT, MOMENT OF A FORCE VECTOR FORMULATION, & PRINCIPLE OF MOMENTS

Objectives :

Students will be able to:

a) understand and define moment, and,

b) determine moments of a force in 2-D and 3-D cases.

http://www.dickwhitney.net/LilWillie.html

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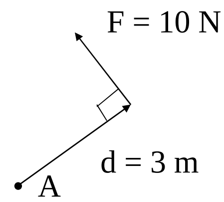
READING QUIZ

1. What is the moment of the 10 N force about point A (M_A)?

   A) 10 N·m B) 30 N·m C) 13 N·m

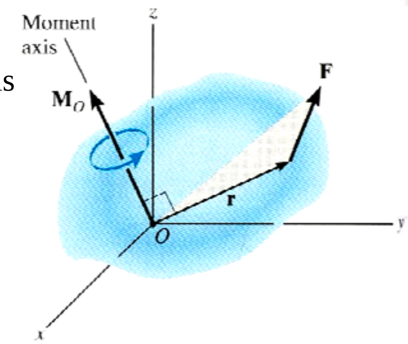
   D) (10/3) N·m E) 7 N·m



2. Moment of force ***F*** about point O is defined as $\boldsymbol{M_O}$ = ____________ .

   A) ***r* x *F***　　B) ***F* x *r***

   C) ***r* • *F***　　D) r * F



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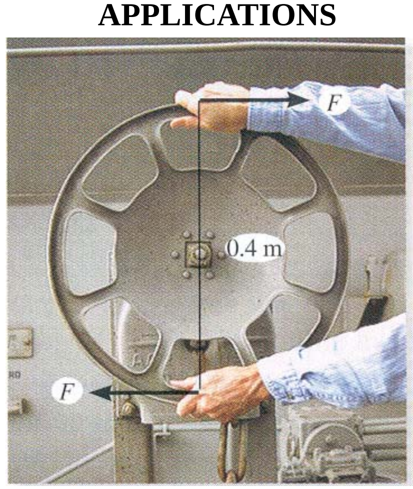
APPLICATIONS



What is the net effect of the two forces on the wheel?

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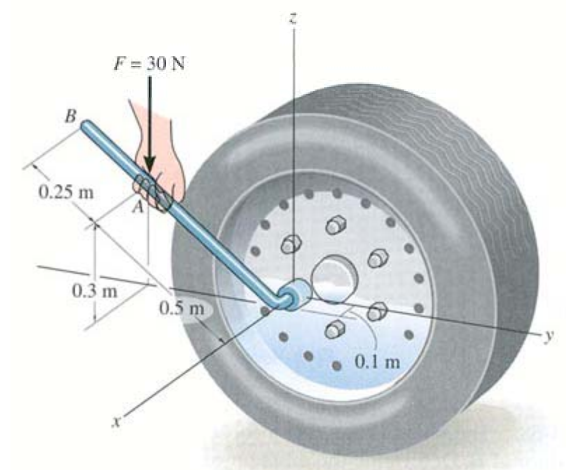
APPLICATIONS
(continued)



What is the effect of the 30 N force on the lug nut?

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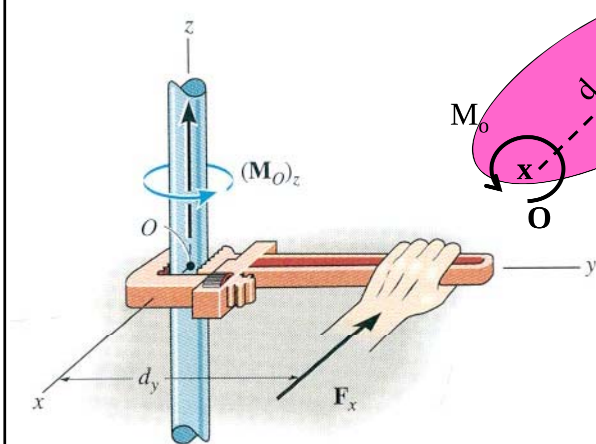
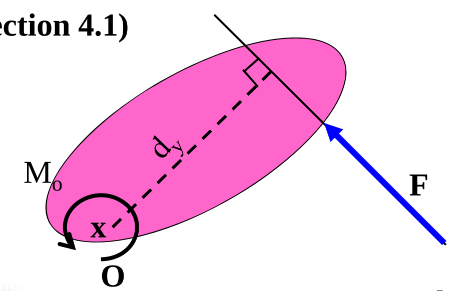
MOMENT OF A FORCE - SCALAR FORMULATION (Section 4.1)





$$M_o = F\ d_y$$

The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

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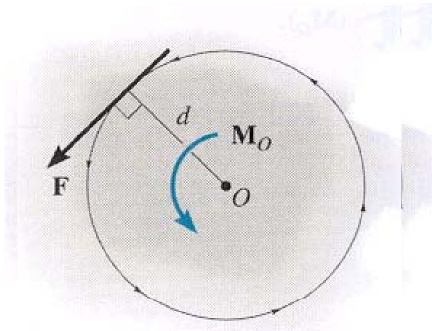
MOMENT OF A FORCE - SCALAR FORMULATION
(continued)

In the 2-D case, the magnitude of the moment is $M_o = F\,d$



As shown, d is the perpendicular distance from point O to the line of action of the force.

In 2-D, the direction of M_O is either clockwise or counter-clockwise depending on the tendency for rotation.

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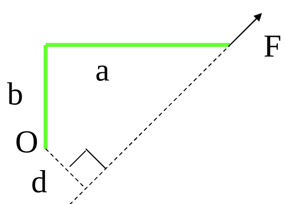
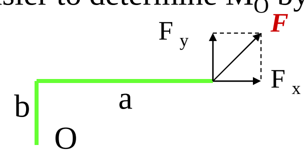
MOMENT OF A FORCE - SCALAR FORMULATION
(continued)



For example, $M_O = F\,d$ and the direction is counter-clockwise.

Often it is easier to determine M_O by using the components of ***F*** as shown.



Using this approach, $M_O = (F_Y\, a) – (F_X\, b)$. Note the different signs on the terms! The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive. We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

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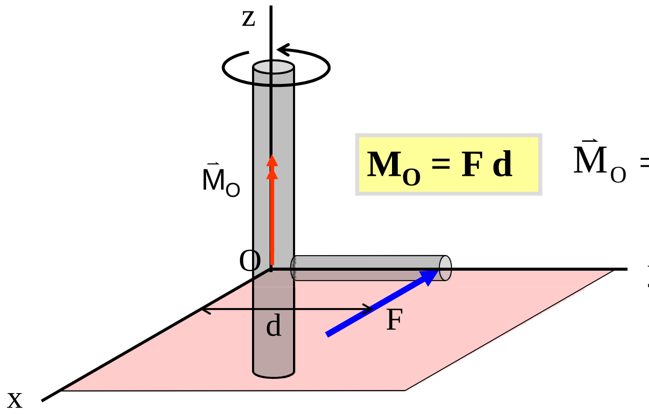
General concept of moment



$$\mathbf{M_O = F\ d}$$

$$\vec{M}_O = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F\,d \end{Bmatrix}$$

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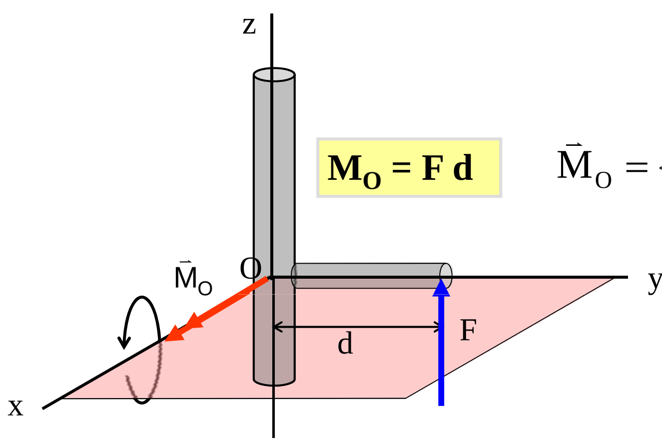
Concept of Moment



$$\mathbf{M_O = F\ d}$$

$$\vec{M}_O = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} Fd \\ 0 \\ 0 \end{Bmatrix}$$

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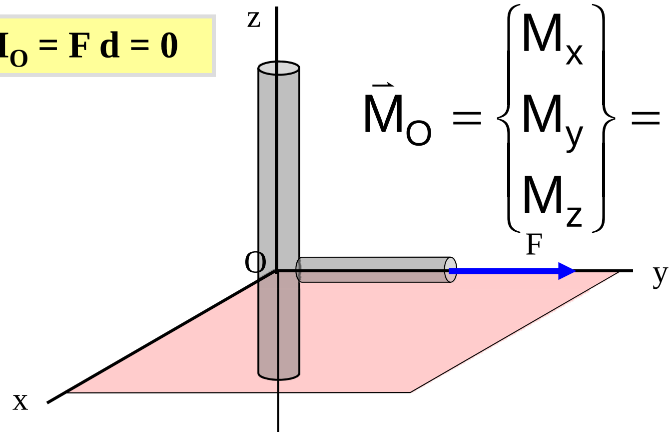
Concept of Moment

$M_O = F\,d = 0$



$$\vec{M}_O = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

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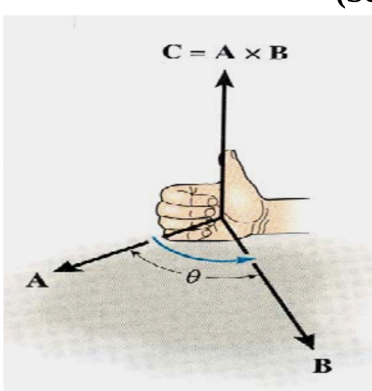
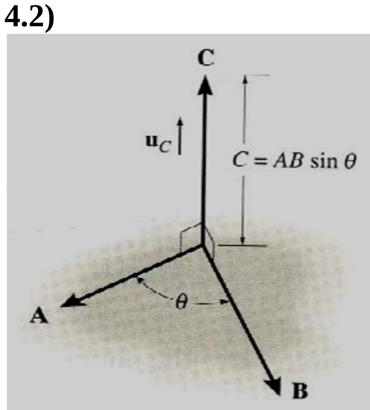
CROSS PRODUCT
(Section 4.2)





In general, the cross product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ results in another vector $\boldsymbol{C}$, i.e., $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$. The magnitude and direction of the resulting vector can be written as

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = A\,B \sin\theta\, \boldsymbol{U_C}$$

Here $\boldsymbol{U_C}$ is the unit vector perpendicular to both A and B vectors as shown (or to the plane containing the A and B vectors).

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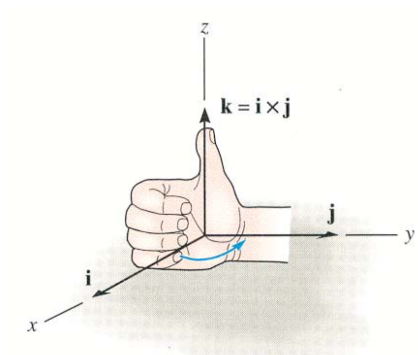
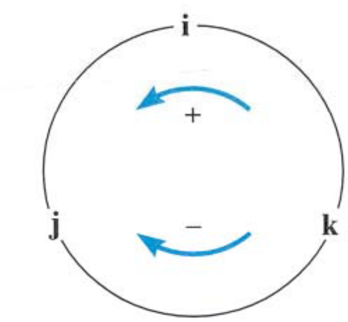
CROSS PRODUCT
(continued)

The right hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$

Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





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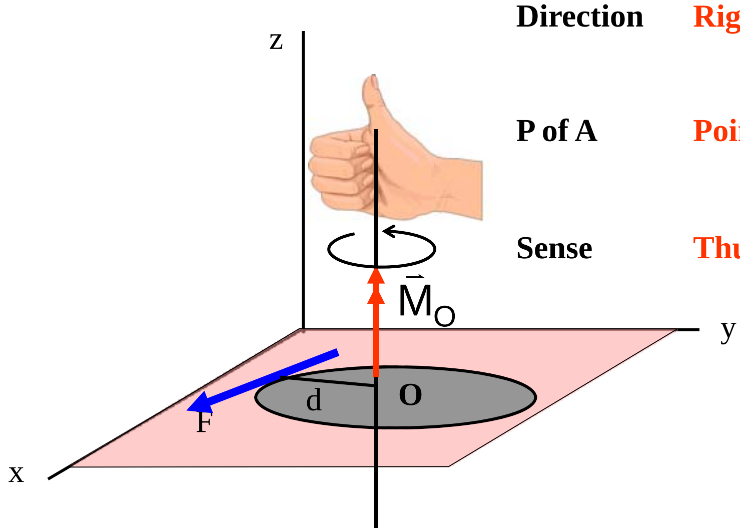
Moment Vector

Magnitude $M_O = F\,d$

Direction Right Hand Rule

P of A Point "O"

Sense Thumb



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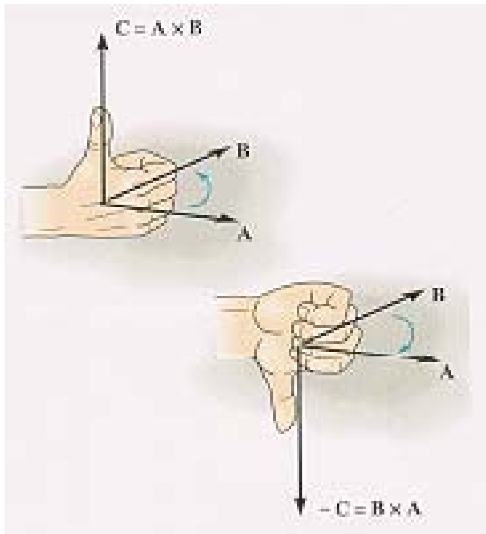
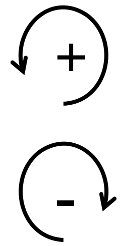
Order



CCW is (+)



CW is (-)

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CROSS PRODUCT
(continued)

Of even more utility, the cross product can be written as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y \quad A_yB_x)$

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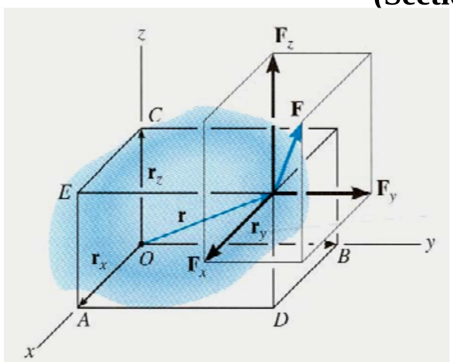
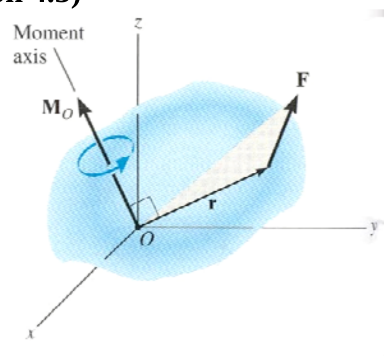
MOMENT OF A FORCE – VECTOR FORMULATION (Section 4.3)





Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$.

Here $\boldsymbol{r}$ is the position vector from point O to any point on the line of action of $\boldsymbol{F}$.

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MOMENT OF A FORCE – VECTOR FORMULATION (continued)

So, using the cross product, a moment can be expressed as

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2 × 2 determinants (see Section 4.2), we get (sample units are N - m or lb - ft)

$$\boldsymbol{M_O} = (r_y F_Z - r_Z F_y)\,\boldsymbol{i} - (r_x F_z - r_z F_x)\,\boldsymbol{j} + (r_x F_y - r_y F_x)\,\boldsymbol{k}$$

The physical meaning of the above equation becomes evident by considering the force components separately and using a 2-D formulation.

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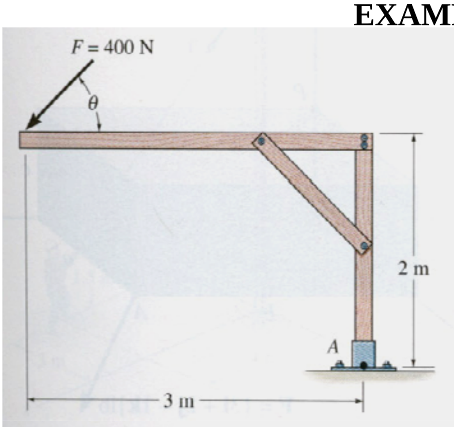
EXAMPLE #1



Given: A 400 N force is applied to the frame and θ = 20°.

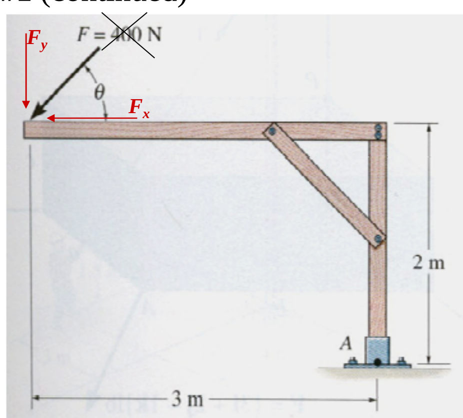
Find: The moment of the force at A.

Plan:

1) Resolve the force along x and y axes.

2) Determine M_A using scalar analysis.

EXAMPLE #1 (continued)



Solution

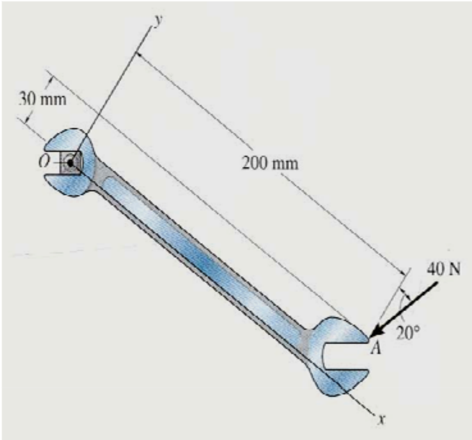
$+\uparrow\ F_y = -400 \sin 20°\ \text{N}$

$+\rightarrow\ F_x = -400 \cos 20°\ \text{N}$

$+\circlearrowleft\ M_A = \{(400 \sin 20°)(3) + (400 \cos 20°)(2)\}\ \text{N·m}$

$= 1160\ \text{N·m}$

PROBLEM SOLVING



Given: A 40 N force is applied to the wrench.

Find: The moment of the force at O.

Plan: 1) Resolve the force along x and y axes.

2) Determine M_O using scalar analysis.

Solution: $+\uparrow F_y = -40 \cos 20° \text{ N}$

$+\rightarrow F_x = -40 \sin 20° \text{ N}$

$\circlearrowleft + \quad M_O = \{-(40 \cos 20°)(200) + (40 \sin 20°)(30)\} \text{N·mm}$

$= -7107 \text{ N·mm} = -7.11 \text{ N·m}$

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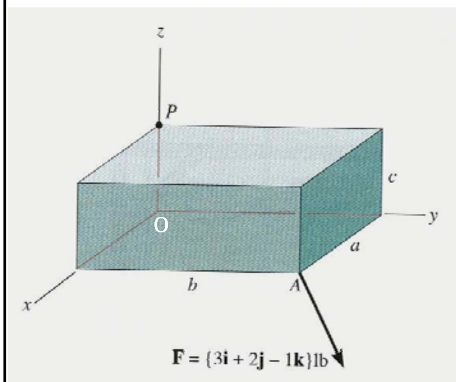
EXAMPLE # 2



Given: a = 3 in, b = 6 in and c = 2 in.

Find: Moment of ***F*** about point O.

Plan:

1) Find $\boldsymbol{r}_{OA}$.

2) Determine $\boldsymbol{M}_O = \boldsymbol{r}_{OA} \times \boldsymbol{F}$.

Solution $\boldsymbol{r}_{OA} = \{3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 0\,\boldsymbol{k}\}$ in

$$M_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & 0 \\ 3 & 2 & -1 \end{vmatrix} = [\{6(-1) - 0(2)\}\,\boldsymbol{i} - \{3(-1) - 0(3)\}\,\boldsymbol{j} + \{3(2) - 6(3)\}\,\boldsymbol{k}] \text{ lb·in}$$

$$= \{-6\,\boldsymbol{i} + 3\,\boldsymbol{j} - 12\,\boldsymbol{k}\} \text{ lb·in}$$

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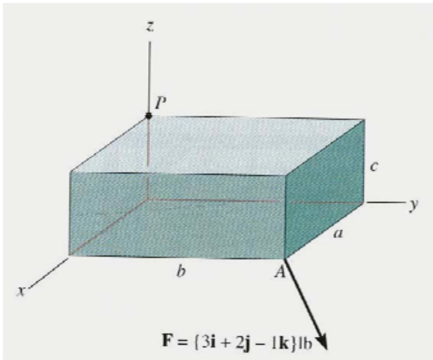
PROBLEM SOLVING



Given: a = 3 in , b = 6 in and c = 2 in

Find: Moment of F about point P

Plan: 1) Find $\boldsymbol{r}_{PA}$.

2) Determine $\boldsymbol{M}_P = \boldsymbol{r}_{PA} \times \boldsymbol{F}$

.

Solution: $\boldsymbol{r}_{PA} = \{ 3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 2\,\boldsymbol{k} \}$ in

$$\boldsymbol{M}_P = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & -2 \\ 3 & 2 & -1 \end{vmatrix} = \{ -2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 12\,\boldsymbol{k} \} \text{ lb} \cdot \text{in}$$

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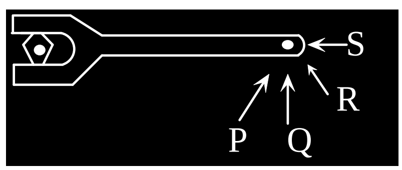
CONCEPT QUIZ

1. If a force of magnitude F can be applied in four different 2-D configurations (P,Q,R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).

A) (Q, P) B) (R, S)

C) (P, R) D) (Q, S)



2. If $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, then what will be the value of $\boldsymbol{M} \bullet \boldsymbol{r}$?

A) 0 B) 1

C) r^2 F D) None of the above.

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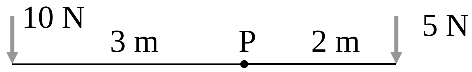
ATTENTION QUIZ



1. Using the CCW direction as positive, the net moment of the two forces about point P is

   A) 10 N ·m B) 20 N ·m C) - 20 N ·m

   D) 40 N ·m E) - 40 N ·m

2. If ***r*** = { 5 ***j*** } m and ***F*** = { 10 ***k*** } N, the moment

   r x ***F*** equals { _______ } N·m.

   A) 50 ***i*** B) 50 ***j*** C) –50 ***i***

   D) – 50 ***j*** E) 0

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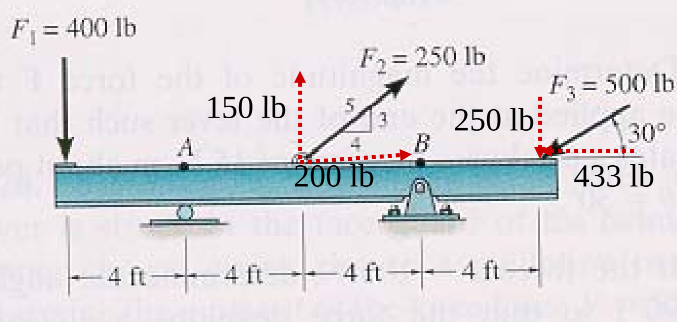
4-9. Determine the moment of each of the three forces about point *B* on the beam.



Probs. 4–8/9

M_B = +400(12) - 150(4) - 250(4) = 4800 - 600 - 1000

M_B = 3 200 lb ft

M_B = 3.2 Kip-ft

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