Carleton University * Faculty of Engineering and Design

Mechanics I

ECOR 1101

Lecture 03

* CARTESIAN VECTORS & ADDITION & SUBTRACTION OF CARTESIAN VECTORS
* POSITION VECTORS & FORCE VECTORS
* DOT PRODUCT

Section 2.5-2.9

CARTESIAN VECTORS & ADDITION & SUBTRACTION OF CARTESIAN VECTORS

Objectives:

Students will be able to :

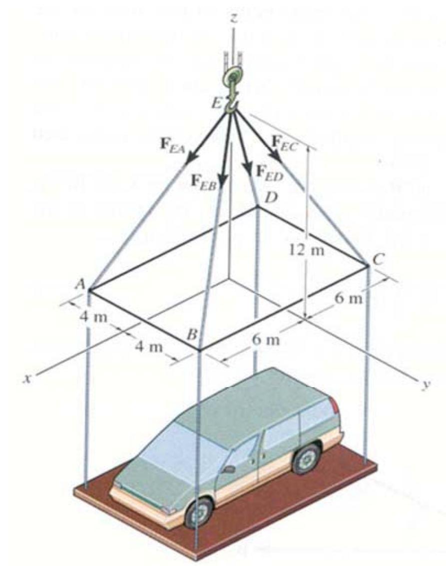
a) Represent a 3-D vector in a Cartesian coordinate system.

b) Find the magnitude and coordinate angles of a 3-D vector

c) Add vectors (forces) in 3-D space

2

APPLICATIONS



Many problems in real-life involve 3-Dimensional Space.

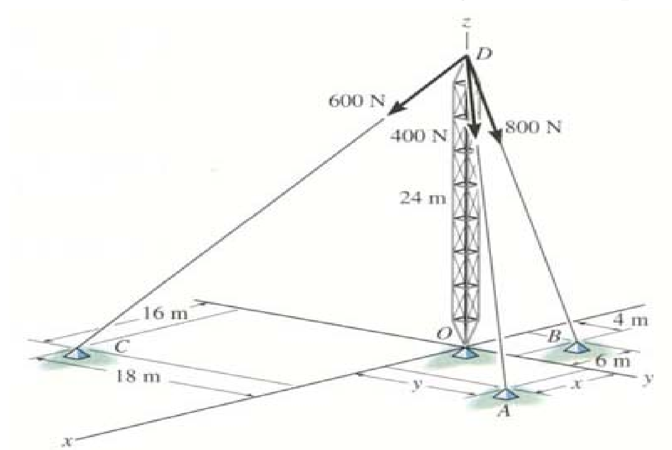
How will you represent each of the cable forces in Cartesian vector form?

3

APPLICATIONS (continued)

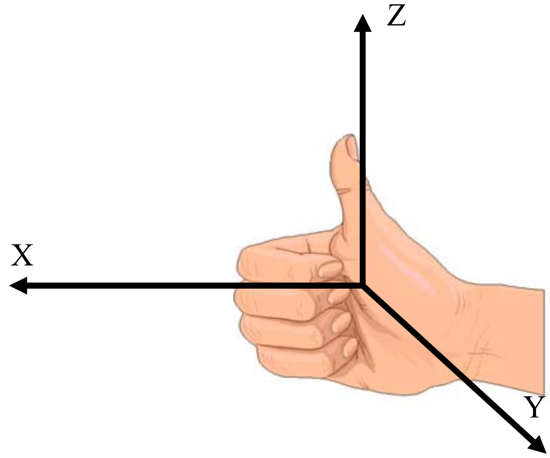
Given the forces in the cables,

How will you determine the resultant force acting at D, the top of the tower ?



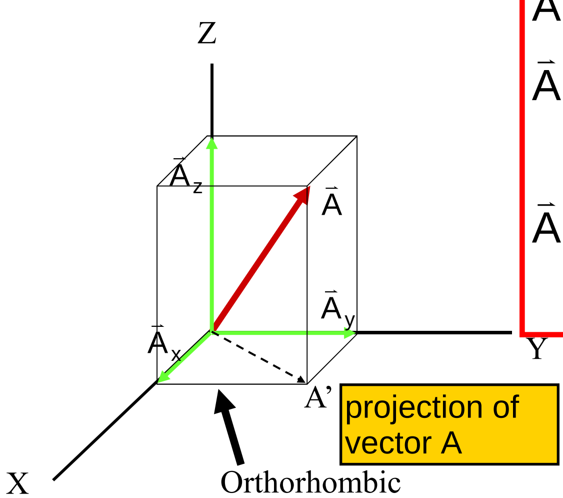
4

Right Handed Coordinate System



5

Rectangular Components



$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

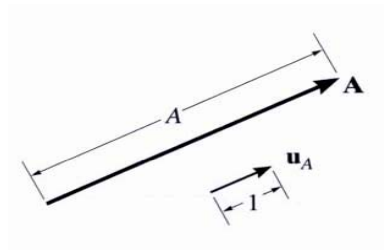
$$\vec{A} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix}$$

6

A UNIT VECTOR

For a vector ***A*** with a magnitude of A, an unit vector is defined as

$\boldsymbol{U_A}$ = ***A*** / A .



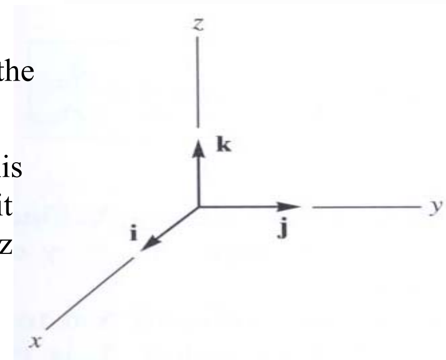
Characteristics of a unit vector**:**

a) Its magnitude is 1.

b) It is dimensionless.

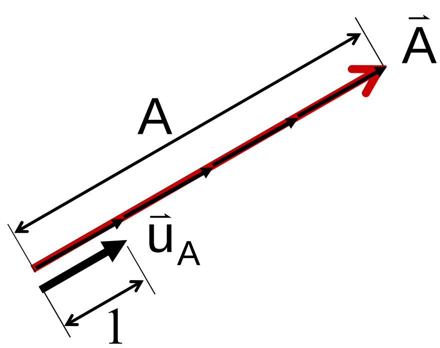
c) It points in the same direction as the original vector (***A***).

The unit vectors in the Cartesian axis system are ***i***, ***j***, and ***k***. They are unit vectors along the positive x, y, and z axes respectively.



7

Unit Direction Vector



$$\frac{\vec{A}}{A} = \vec{u}_A$$

$$\vec{A} = A\,\vec{u}_A$$

8

How to calculate a unit vector

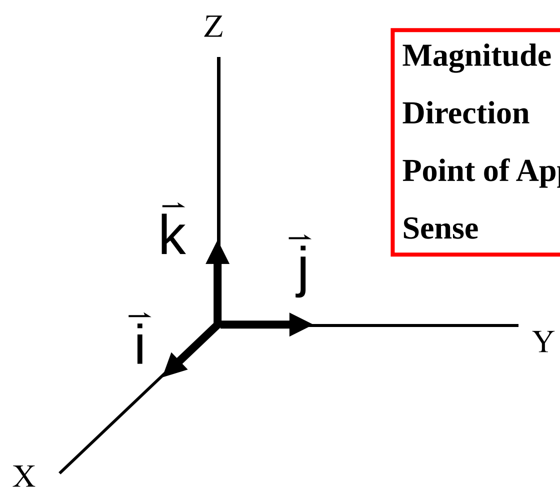
$$\vec{u}_A = \frac{\begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix}}{A} = \frac{\begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

A = magnitude

$$\vec{u}_A = \frac{\begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix}}{A} = \begin{Bmatrix} \dfrac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\ \dfrac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \\ \dfrac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \end{Bmatrix}$$

9

Cartesian Unit Vectors



Magnitude

Direction

Point of Application

Sense

10

Magnitude of a Cartesian Vector

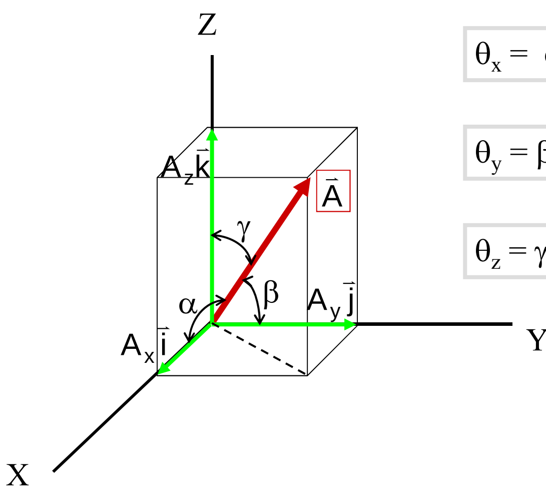
$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{A} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix}$$

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

11

Direction of a Cartesian Vector



$\theta_x = \alpha$ (alpha)

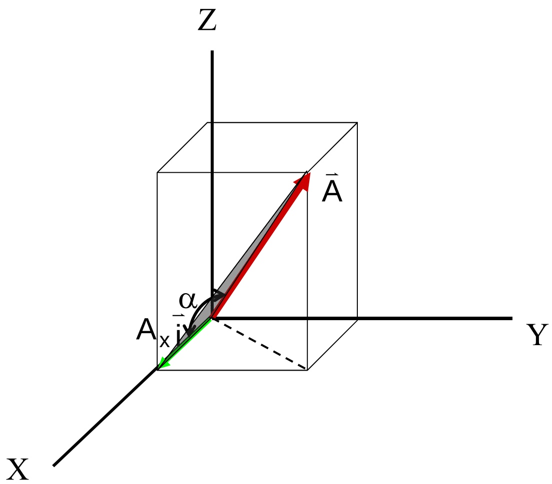
$\theta_y = \beta$ (beta)

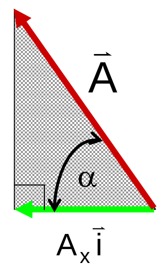
$\theta_z = \gamma$ (gamma)

12

Direction Cosines

$\theta_x = \alpha$ (alpha)



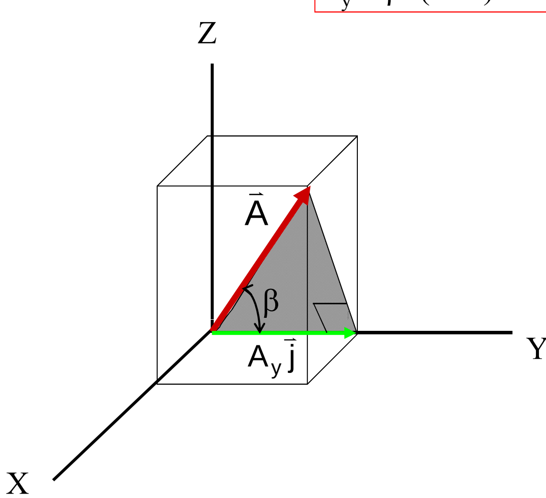


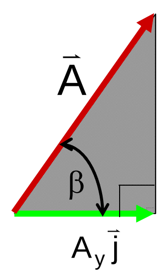
$$\cos\alpha = \frac{A_x}{A}$$

13

Direction Cosines

$\theta_y = \beta$ (beta)



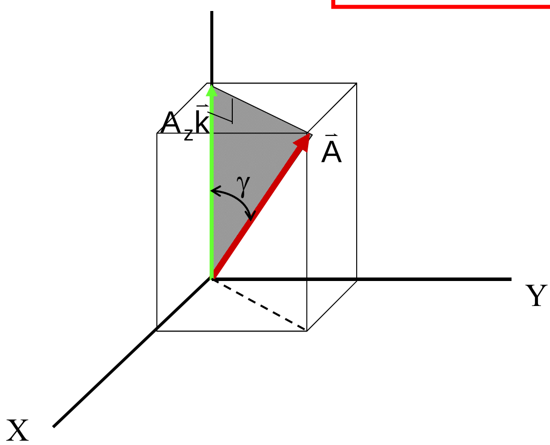


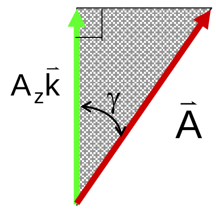
$$\cos\beta = \frac{A_y}{A}$$

14

Direction Cosines

$\theta_z = \gamma$ (gamma)

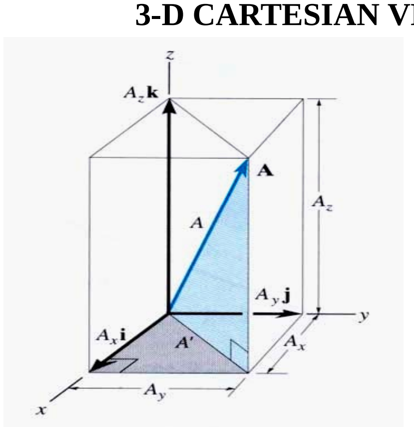




$$\cos\gamma = \frac{A_z}{A}$$

15

3-D CARTESIAN VECTOR TERMINOLOGY



Consider a box with sides A_X, A_Y, and A_Z meters long.

The vector ***A*** can be defined as
A = (A_X ***i*** + A_Y ***j*** + A_Z ***k***) m

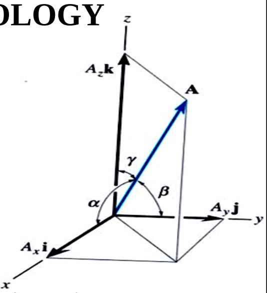
The projection of the vector ***A*** in the x-y plane is A´.

The magnitude of this projection, A´, is found by using the same approach as a 2-D vector: $A' = (A_X^2 + A_Y^2)^{1/2}$.
The magnitude of the position vector ***A*** can now be obtained as

$$A = ((A')^2 + A_Z^2)^{½} = (A_X^2 + A_Y^2 + A_Z^2)^{½}$$

16

3-D CARTESIAN VECTOR TERMINOLOGY (continued)



The direction or orientation of vector ***A*** is defined by the angles α, β, and γ.

These angles are measured between the vector and the positive X, Y and Z axes, respectively.
Their range of values are from 0° to 180°

Using trigonometry, "direction cosines" are found using the formulas

$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

These angles are not independent. They must satisfy the following equation.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\,\mathbf{i} + \frac{A_y}{A}\,\mathbf{j} + \frac{A_z}{A}\,\mathbf{k}$$

or written another way, $u_A = \cos\alpha\, \boldsymbol{i} + \cos\beta\, \boldsymbol{j} + \cos\gamma\, \boldsymbol{k}$.

17

ADDITION/SUBTRACTION OF VECTORS (Section 2.6)

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

For example, if

$\boldsymbol{A} = A_X\, \boldsymbol{i} + A_Y\, \boldsymbol{j} + A_Z\, \boldsymbol{k}$ and

$\boldsymbol{B} = B_X\, \boldsymbol{i} + B_Y\, \boldsymbol{j} + B_Z\, \boldsymbol{k}$, then

$$\boldsymbol{A} + \boldsymbol{B} = (A_X + B_X)\,\boldsymbol{i} + (A_Y + B_Y)\,\boldsymbol{j} + (A_Z + B_Z)\,\boldsymbol{k}$$

or

$$\boldsymbol{A} - \boldsymbol{B} = (A_X - B_X)\,\boldsymbol{i} + (A_Y - B_Y)\,\boldsymbol{j} + (A_Z - B_Z)\,\boldsymbol{k}\,.$$

18

IMPORTANT NOTES

Sometimes 3-D vector information is given as:

a) Magnitude and the coordinate direction angles, or

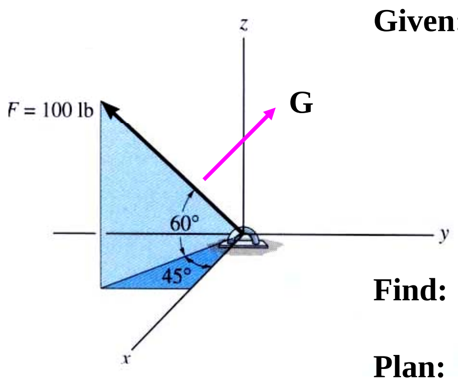
b) Magnitude and projection angles.

You should be able to use both these types of information to change the representation of the vector into the Cartesian form, i.e.,

$\boldsymbol{F} = \{10\,\boldsymbol{i} - 20\,\boldsymbol{j} + 30\,\boldsymbol{k}\}$ N .

19

EXAMPLE



Given: Two forces ***F*** and ***G*** are applied to a hook. Force ***F*** is shown in the figure and it makes 60° angle with the X-Y plane. Force ***G*** is pointing up and has a magnitude of 80 lb with α = 111° and β = 69.3°.

Find: The resultant force in the Cartesian vector form.

Plan:

1) Using geometry and trigonometry, write ***F*** and ***G*** in the Cartesian vector form.
2) Then add the two forces.

20

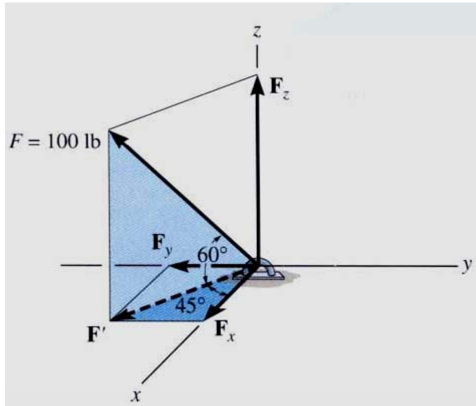
Solution : First, resolve force ***F***.

$F_z = 100 \sin 60° = 86.60$ lb

$F' = 100 \cos 60° = 50.00$ lb

$F_x = 50 \cos 45° = 35.36$ lb

$F_y = 50 \sin 45° = 35.36$ lb



Now, you can write:

F = {35.36 ***i*** – 35.36 ***j*** + 86.60 ***k***} lb

21

Now resolve force ***G***.

We are given only α and β. Hence, first we need to find the value of γ.

Recall the formula $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$.

Now substitute what we know. We have

$\cos^2(111°) + \cos^2(69.3°) + \cos^2(\gamma) = 1$.

Solving, we get γ = 30.22° or 120.2°. Since the vector is pointing up, γ = 30.22°

Now using the coordinate direction angles, we can get U_G, and determine ***G*** = 80 U_G lb.

G = {80 (cos (111°) ***i*** + cos (69.3°) ***j*** + cos (30.22°) ***k***)} lb

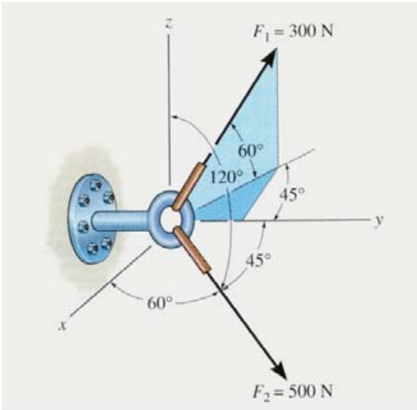
G = {- 28.67 ***i*** + 28.28 ***j*** + 69.13 ***k*** } lb

Now, *R* = *F* + *G* or

R = {6.69 ***i*** – 7.08 ***j*** + 156 ***k***} lb

22

PROBLEM SOLVING



Given: The screw eye is subjected to two forces.

Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

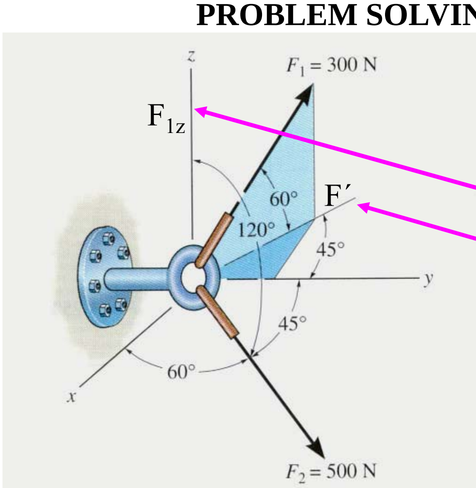
1) Using the geometry and trigonometry, write $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in the Cartesian vector form.

2) Add $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ to get $\boldsymbol{F_R}$.

3) Determine the magnitude and α, β, γ .

23

PROBLEM SOLVING (continued)



First resolve the force $\boldsymbol{F_1}$.

F_{1z} = 300 sin 60° = 259.8 N

F´ = 300 cos 60° = 150.0 N

F’ can be further resolved as,

F_{1x} = -150 sin 45° = -106.1 N

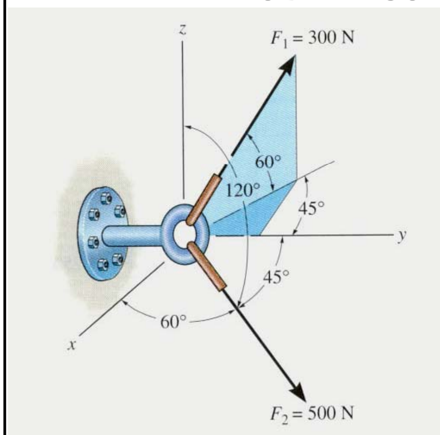
F_{1y} = 150 cos 45° = 106.1 N

Now we can write :

$\boldsymbol{F_1}$ = {-106.1 $\boldsymbol{i}$ + 106.1 $\boldsymbol{j}$ + 259.8 $\boldsymbol{k}$ } N

24

PROBLEM SOLVING (continued)



The force $\boldsymbol{F_2}$ can be represented in the Cartesian vector form as:

$\boldsymbol{F_2} = 500\{ \cos 60° \, \boldsymbol{i} + \cos 45° \, \boldsymbol{j} + \cos 120° \, \boldsymbol{k} \}$ N

$= \{ 250 \, \boldsymbol{i} + 353.6 \, \boldsymbol{j} - 250 \, \boldsymbol{k} \}$ N

$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$

$= \{ 143.9 \, \boldsymbol{i} + 459.6 \, \boldsymbol{j} + 9.81 \, \boldsymbol{k} \}$ N

$F_R = (143.9^2 + 459.6^2 + 9.81^2)^{½} = 481.7 = 482$ N

$\alpha = \cos^{-1} (F_{Rx} / F_R) = \cos^{-1} (143.9/481.7) = 72.6°$

$\beta = \cos^{-1} (F_{Ry} / F_R) = \cos^{-1} (459.6/481.7) = 17.4°$

$\gamma = \cos^{-1} (F_{Rz} / F_R) = \cos^{-1} (9.81/481.7) = 88.8°$

25

POSITION VECTORS & FORCE VECTORS

<u>Objectives:</u>

Students will be able to :

a) Represent a position vector in Cartesian coordinate form, from given geometry.

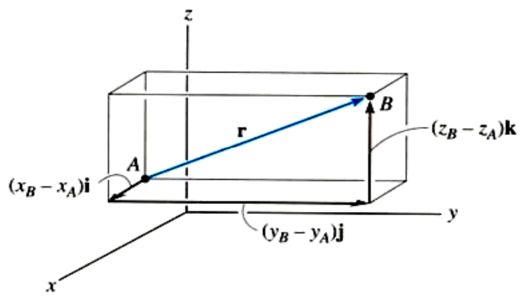
b) Represent a force vector directed along a line.

26

POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.

Consider two points, A & B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.



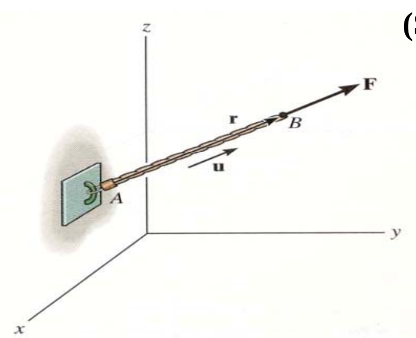
The position vector directed from A to B, $\boldsymbol{r}_{AB}$, is defined as

$\boldsymbol{r}_{AB} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}$m

Please note that B is the ending point and A is the starting point. So ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

27

FORCE VECTOR DIRECTED ALONG A LINE (Section 2.8)



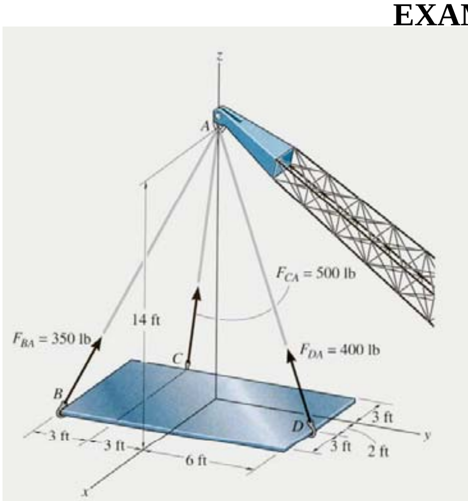
If a force is directed along a line, then we can represent the force vector in Cartesian Coordinates by using a unit vector and the force magnitude.

So we need to:

a) Find the position vector, $\boldsymbol{r}_{AB}$, along two points on that line.
b) Find the unit vector describing the line's direction, $\boldsymbol{u}_{AB} = (\boldsymbol{r}_{AB}/r_{AB})$.
c) Multiply the unit vector by the magnitude of the force, $\boldsymbol{F} = F\,\boldsymbol{u}_{AB}$.

28

EXAMPLE



Given: 400 lb force along the cable DA.

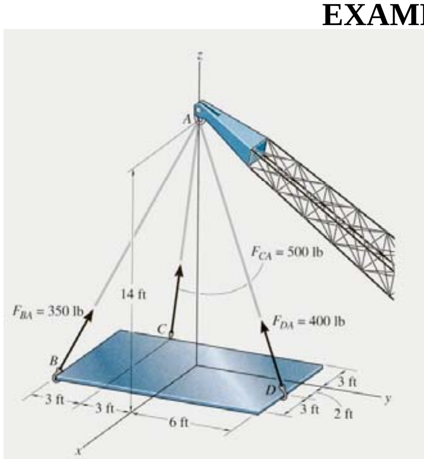
Find: The force $\boldsymbol{F_{DA}}$ in the Cartesian vector form.

Plan:

1. Find the position vector $\boldsymbol{r_{DA}}$ and the unit vector $\boldsymbol{u_{DA}}$.
2. Obtain the force vector as $\boldsymbol{F_{DA}}$ = 400 lb $\boldsymbol{u_{DA}}$.

29

EXAMPLE (continued)



The figure shows that coordinates of D and A are:

D ≡ (2,6,0) & A ≡ (0,0,14)

We can find $\boldsymbol{r_{DA}}$ by subtracting the coordinates of D from the coordinates of A. Hence,

$\boldsymbol{r_{DA}}$ = {-2 $\boldsymbol{i}$ – 6 $\boldsymbol{j}$ + 14 $\boldsymbol{k}$} ft.

$r_{DA} = (2^2 + 6^2 + 14^2)^{0.5} = 15.36$ ft

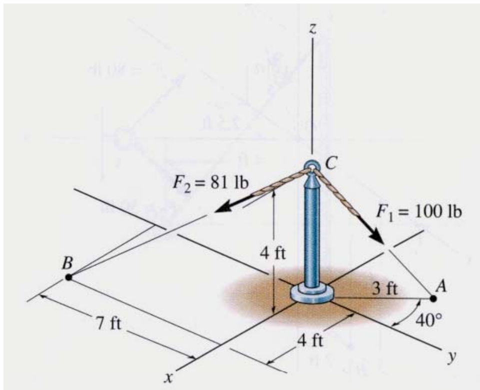
$\boldsymbol{u_{DA}} = \boldsymbol{r_{DA}}/r_{DA}$ and $\boldsymbol{F_{DA}} = 400\ \boldsymbol{u_{DA}}$ lb

$\boldsymbol{F_{DA}} = 400\{(-2\ \boldsymbol{i} - 6\ \boldsymbol{j} + 14\ \boldsymbol{k})/15.36\}$ lb

$= \{-52.1\ \boldsymbol{i} - 156\ \boldsymbol{j} + 365\ \boldsymbol{k}\}$ lb

30

PROBLEM SOLVING



Given: Two forces are acting on a pipe as shown in the figure.

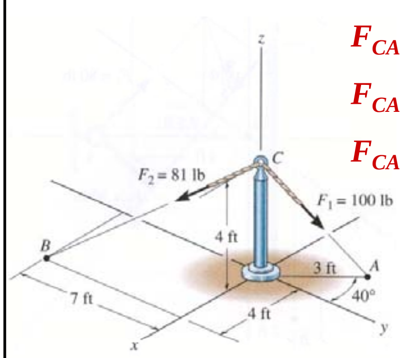
Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1. Find the forces along CA and CB in the Cartesian vector form.
2. Add the two forces to get the resultant force, $\boldsymbol{F_R}$.
3. Determine the magnitude and the coordinate angles of $\boldsymbol{F_R}$.

31

PROBLEM SOLVING (continued)



$\boldsymbol{F_{CA}}$ = 100 lb{$\boldsymbol{r_{CA}}$/r_{CA}}

$\boldsymbol{F_{CA}}$ = 100 lb(-3 sin 40° $\boldsymbol{i}$ + 3 cos 40° $\boldsymbol{j}$ – 4 $\boldsymbol{k}$)/5

$\boldsymbol{F_{CA}}$ = {-38.57 $\boldsymbol{i}$ + 45.96 $\boldsymbol{j}$ – 80 $\boldsymbol{k}$} lb

$\boldsymbol{F_{CB}}$ = 81 lb{$\boldsymbol{r_{CB}}$/r_{CB}}

$\boldsymbol{F_{CB}}$ = 81 lb(4 $\boldsymbol{i}$ – 7 $\boldsymbol{j}$ – 4 $\boldsymbol{k}$)/9

$\boldsymbol{F_{CB}}$ = {36 $\boldsymbol{i}$ – 63 $\boldsymbol{j}$ – 36 $\boldsymbol{k}$} lb

$\boldsymbol{F_R}$ = $\boldsymbol{F_{CA}}$ + $\boldsymbol{F_{CB}}$ = {-2.57 $\boldsymbol{i}$ – 17.04 $\boldsymbol{j}$ – 116 $\boldsymbol{k}$} lb

$F_R = (2.57^2 + 17.04^2 + 116^2)^{1/2} = 117.3$ lb = 117 lb

$\alpha = \cos^{-1}(-2.57/117.3) = 91.3°$, $\beta = \cos^{-1}(-17.04/117.3) = 98.4°$

$\gamma = \cos^{-1}(-116/117.3) = 172°$

32

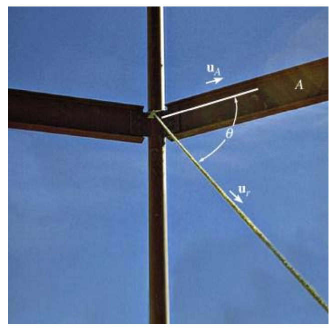
DOT PRODUCT

<u>Objective</u>:

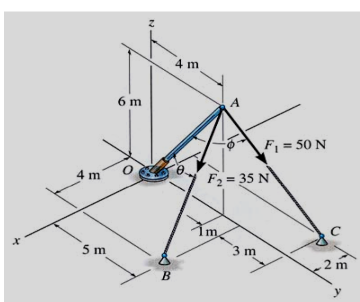
Students will be able to use the dot product to:

a) determine an angle between two vectors, and,

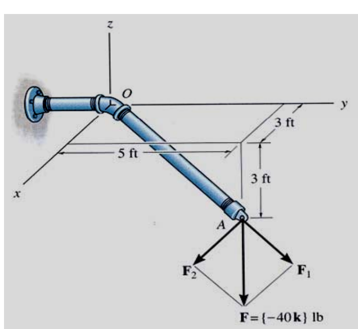
b) determine the projection of a vector along a specified line.



APPLICATIONS

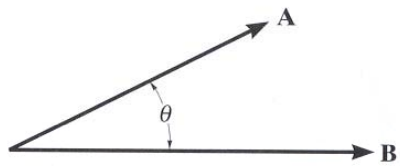


For this geometry, can you determine angles between the pole and the cables?



For force ***F*** at Point A, what component of it (F_1) acts along the pipe OA? What component (F_2) acts perpendicular to the pipe?

DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A•B*** = A B cos θ.
Angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180°.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

35

DOT PRODUCT DEFINITON (continued)

Examples: $\boldsymbol{i} \cdot \boldsymbol{j} = 0$

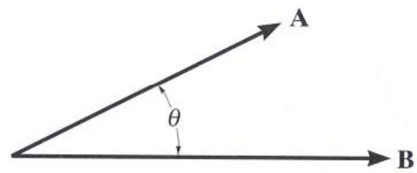
$\boldsymbol{i} \cdot \boldsymbol{i} = 1$

$$\boldsymbol{A} \cdot \boldsymbol{B} = (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \cdot (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

36

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For the given two vectors in the Cartesian form, one can find the angle by

a) Finding the dot product, $\mathbf{A} \bullet \mathbf{B} = (A_xB_x + A_yB_y + A_zB_z)$,

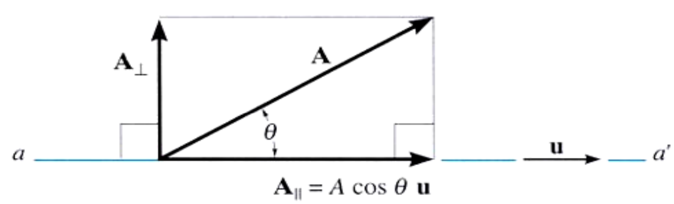
b) Finding the magnitudes (A & B) of the vectors ***A*** & ***B***, and

c) Using the definition of dot product and solving for θ, i.e.,

$\theta = \cos^{-1} [(\mathbf{A} \bullet \mathbf{B})/(A\ B)]$, where $0^\circ \leq \theta \leq 180^\circ$.

37

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\mathbf{U}_{aa'}$ along line aa´
2. Find the scalar projection of ***A*** along line aa´ by

$A_{||} = \mathbf{A} \bullet \mathbf{U} = A_xU_x + A_yU_y + A_zU_z$

38

DETERMINING THE PROJECTION OF A VECTOR (continued)

3. If needed, the projection can be written as a vector, $\boldsymbol{A}_{||}$, by using the unit vector $\boldsymbol{U}_{aa'}$ and the magnitude found in step 2.

$$\boldsymbol{A}_{||} = A_{||}\,\boldsymbol{U}_{aa'}$$

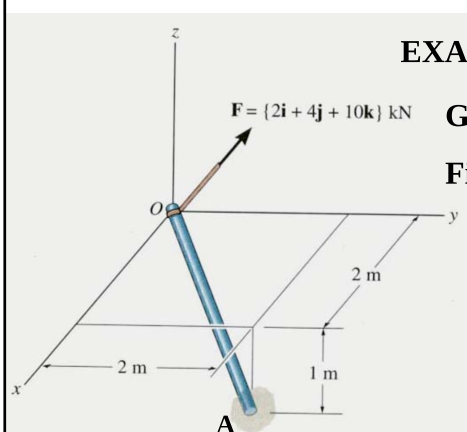
4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_{\perp} = (A^2 - A_{||}^2)^{1/2} \text{ and}$$

$$\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{||}$$

(rearranging the vector sum of $\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{||}$)

39



EXAMPLE

Given: The force acting on the pole

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole OA.

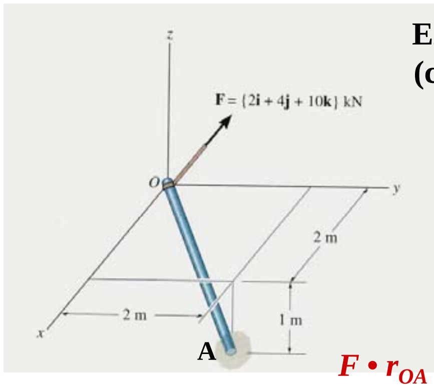
Plan:

1. Get $\boldsymbol{r}_{OA}$

2. $\theta = \cos^{-1}\{(\boldsymbol{F} \cdot \boldsymbol{r}_{OA})/(F\ r_{OA})\}$

3. $F_{OA} = \boldsymbol{F} \cdot \boldsymbol{u}_{OA}$ or $F \cos\theta$

40



EXAMPLE (continued)

$\boldsymbol{r_{OA}} = \{2\,\boldsymbol{i} + 2\,\boldsymbol{j} - 1\,\boldsymbol{k}\}$ m

$r_{OA} = (2^2 + 2^2 + 1^2)^{1/2} = 3$ m

$\boldsymbol{F} = \{2\,\boldsymbol{i} + 4\,\boldsymbol{j} + 10\,\boldsymbol{k}\}$kN

$F = (2^2 + 4^2 + 10^2)^{1/2} = 10.95$ kN

$\boldsymbol{F} \bullet \boldsymbol{r_{OA}} = (2)(2) + (4)(2) + (10)(-1) = 2$ kN·m

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{OA}})/(F\, r_{OA})\}$

$\theta = \cos^{-1}\{2/(10.95 * 3)\} = 86.5°$

$\boldsymbol{u_{OA}} = \boldsymbol{r_{OA}}/r_{OA} = \{(2/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} - (1/3)\,\boldsymbol{k}\}$

$F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u_{OA}} = (2)(2/3) + (4)(2/3) + (10)(-1/3) = 0.667$ kN

Or $F_{OA} = F \cos\theta = 10.95 \cos(86.51°) = 0.667$ kN

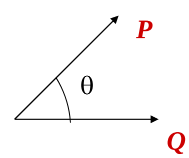
41

READING QUIZ

1. The dot product of two vectors ***P*** and ***Q*** is defined as

A) P Q cos θ   B) P Q sin θ

C) P Q tan θ   D) P Q sec θ



2. The dot product of two vectors results in a _________ quantity.

A) scalar   B) vector

C) complex   D) zero

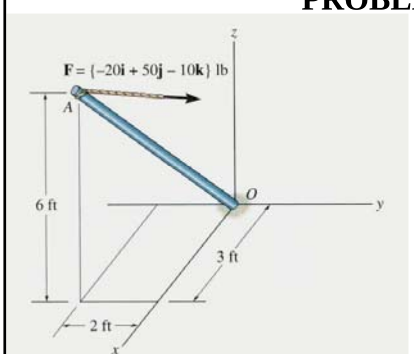
42

CONCEPT QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be ______________ to each other.
   A) parallel (pointing in the same direction)
   B) parallel (pointing in the opposite direction)
   C) perpendicular
   D) cannot be determined.

43

PROBLEM SOLVING



Given: The force acting on the pole.

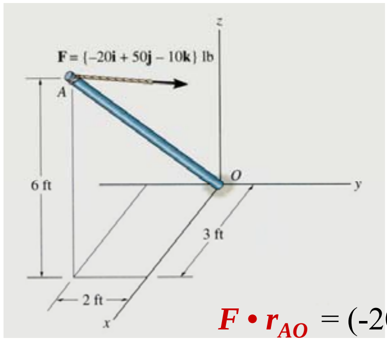
Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole AO.

Plan:

1. Get $\boldsymbol{r}_{AO}$
2. $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{AO})/(F \ r_{AO})\}$
3. $F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u}_{AO}$ or $F \cos \theta$

44

PROBLEM SOLVING (continued)



$\boldsymbol{r_{AO}} = \{-3\ \boldsymbol{i}\ +\ 2\ \boldsymbol{j}\ -\ 6\ \boldsymbol{k}\}$ ft.

$r_{AO} = (3^2\ +\ 2^2\ +\ 6^2)^{1/2} = 7$ ft.

$\boldsymbol{F}\ = \{-20\ \boldsymbol{i}\ +\ 50\ \boldsymbol{j}\ -\ 10\ \boldsymbol{k}\}$lb

$F\ = (20^2\ +\ 50^2\ +\ 10^2)^{1/2}\ =\ 54.77$ lb

$\boldsymbol{F} \bullet \boldsymbol{r_{AO}} = (-20)(-3) + (50)(2) + (-10)(-6) = 220$ lb·ft

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F\ r_{AO})\}$

$\theta = \cos^{-1}\{220/(54.77 \times 7)\} = 55.0°$

$\boldsymbol{u_{AO}} = \boldsymbol{r_{AO}}/r_{AO}\ = \{(-3/7)\ \boldsymbol{i}\ +\ (2/7)\ \boldsymbol{j}\ -\ (6/7)\ \boldsymbol{k}\}$

$F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u_{AO}} = (-20)(-3/7)\ +\ (50)(2/7)\ +\ (-10)(-6/7)\ \ =\ 31.4$ lb

Or $F_{AO}\ =\ F \cos\theta\ =\ 54.77 \cos(55.0°)\ =\ 31.4$ lb

45