Carleton University * Faculty of Engineering and Design

# Mechanics I

ECOR 1101

## Lecture 02

* Force vectors,
* Vector operations & addition coplanar forces

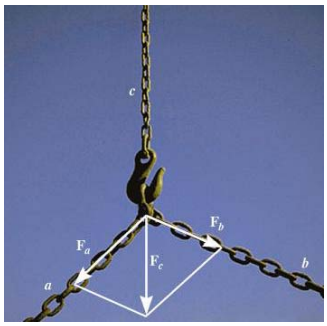
Section 2.1-2.4

## Today's Objectives

Students will be able to :

a) Resolve a 2-D vector into components.

b) Add 2-D vectors using Cartesian vector notations.

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## Reading Quiz

1. Which one of the following is a scalar quantity?

   A) Force  B) Position  C) Mass  D) Velocity

2. For vector addition you have to use _____ law.

   A) Newton's Second

   B) the arithmetic

   C) Pascal's

   D) the parallelogram

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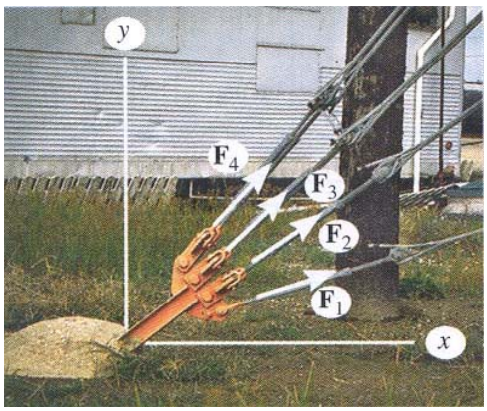
## Application Of Vector Addition




There are four concurrent cable forces acting on the bracket.

How do you determine the resultant force acting on the bracket ?

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## SCALARS AND VECTORS (Section 2.1)

| | Scalars | ***Vectors*** |
|---|---|---|
| Examples: | mass, volume, length | force, velocity |
| Characteristics: | It has a magnitude (positive or negative) | It has a magnitude and direction |
| Addition rule: | Simple arithmetic | Parallelogram law |
| Special Notation: | None | **Bold font**, a line, an arrow or a "carrot" |

In the PowerPoint presentation vector quantity is represented ***Like this*** (in **bold**, and *italics*).

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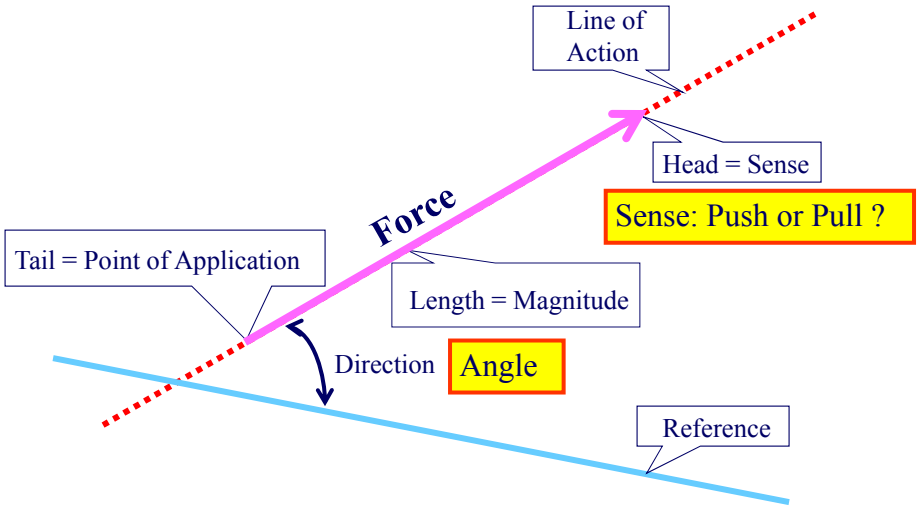
## Concept of Force as Vector

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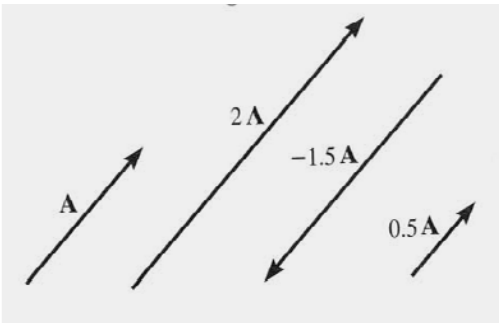
## VECTOR OPERATIONS
## (Section 2.2)




Scalar Multiplication and Division

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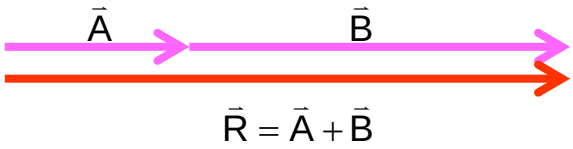
## Addition of Collinear Forces

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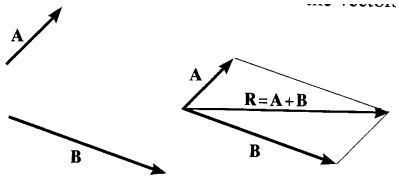
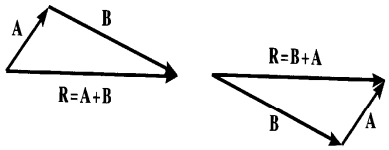
## VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:




Triangle method
(always 'head to tail'):




How do you subtract a vector?

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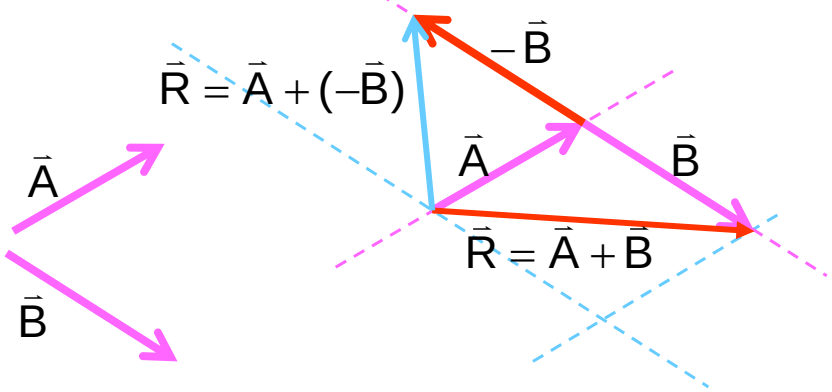
## Subtraction

$$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

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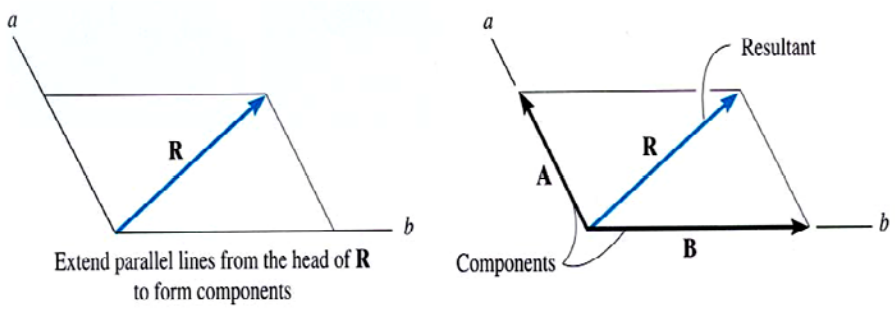
## Resolution of a vector

"Resolution" of a vector is breaking up a vector into components.
It is like using the parallelogram law in reverse.

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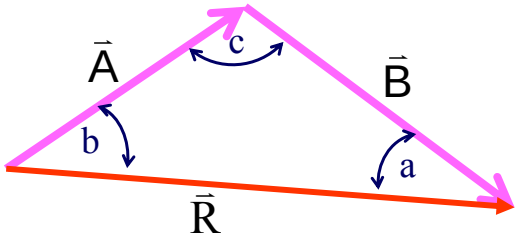
## Resultant of two Forces

$$\frac{A}{\text{Sin a}} = \frac{B}{\text{Sin b}} = \frac{R}{\text{Sin c}}$$

Sine Law




Cosine Law

$$R^2 = A^2 + B^2 - 2AB\,\text{Cos c}$$

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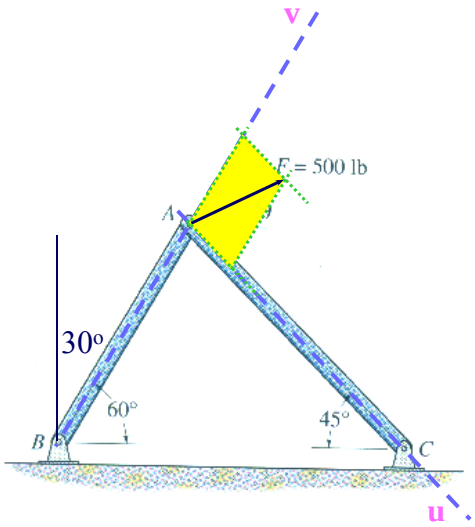
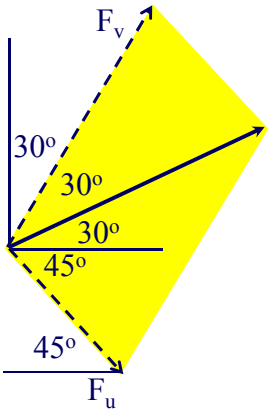
# Example

Find the components of the 500 lb force in the direction of AB and AC for θ = 30°

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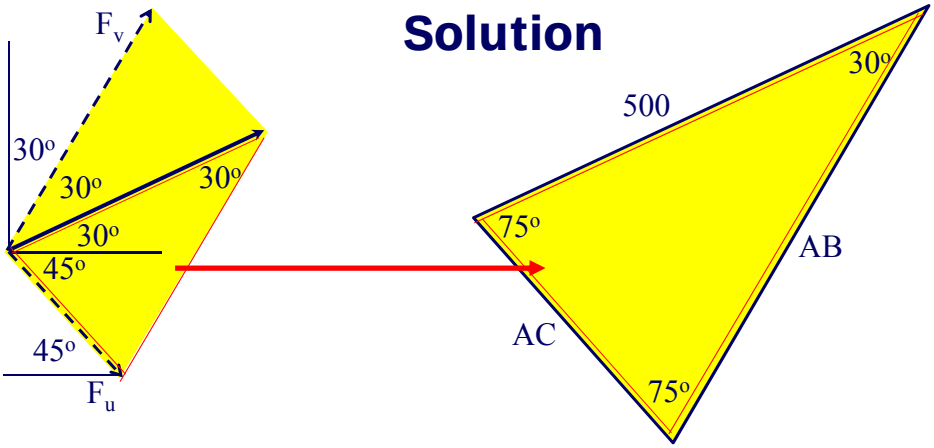
# Solution




$$\frac{500}{\text{Sin } 75^\circ} = \frac{AB}{\text{Sin } 75^\circ} = \frac{AC}{\text{Sin } 30^\circ}$$ Sine Law

$$AC^2 = AB^2 + 500^2 - 2(AB)(500)\text{Cos } 30^\circ$$
$$AB^2 = AC^2 + 500^2 - 2(AC)(500)\text{Cos } 75^\circ$$ Cosine Law

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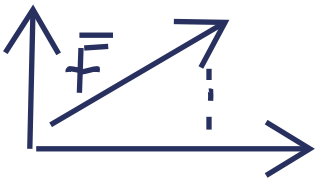
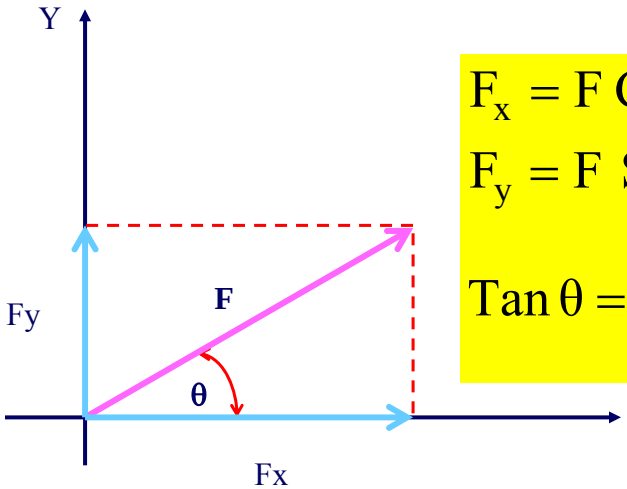



## Special Case

(Rectangular Components)




$$F_x = F\,\mathrm{Cos}\,\theta$$

$$F_y = F\,\mathrm{Sin}\,\theta$$

$$\mathrm{Tan}\,\theta = \frac{F_y}{F_x}$$

projection vector

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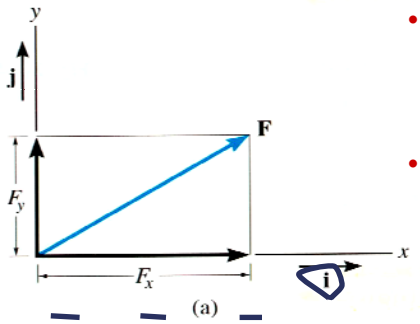
## CARTESIAN VECTOR NOTATION
## (Section 2.4)




- We 'resolve' vectors into components using the x and y axes system.
- Each component of the vector is shown as a magnitude and a direction.

F = fx i + fy j

- The directions are based on the x and y axes.
- We use the "unit vectors" ***i*** and ***j*** to designate the x and y axes.

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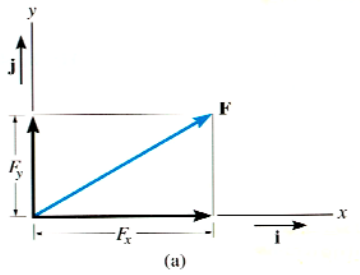
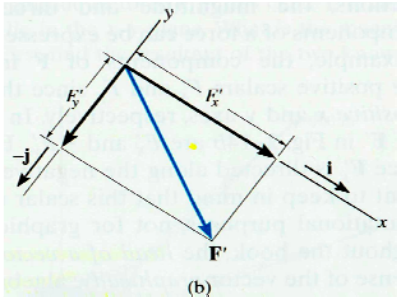
For example,

$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$ **or** $\boldsymbol{F'} = F'_x\,\boldsymbol{i} - F'_y\,\boldsymbol{j}$




- The x and y axes are always perpendicular to each other.
- Together, they can be directed at any inclination.

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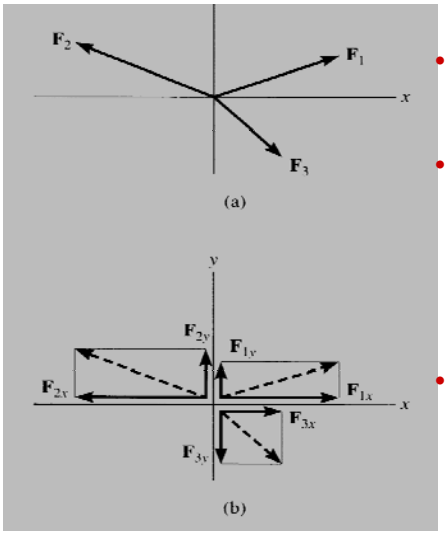
## ADDITION OF SEVERAL VECTORS




- Step 1 is to resolve each force into its components
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

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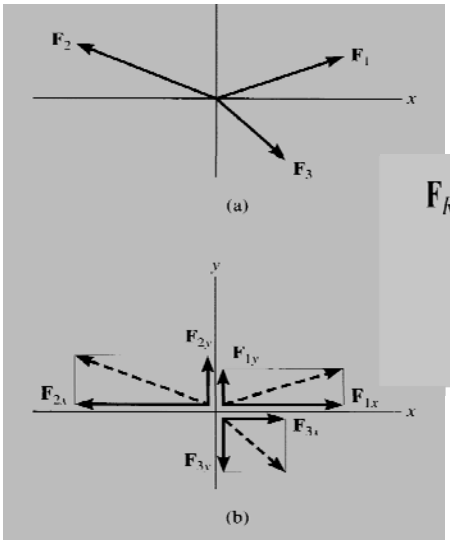


Example of this process,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

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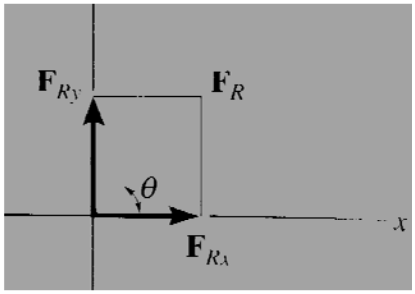
You can also represent a 2-D vector with a magnitude and angle.




$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

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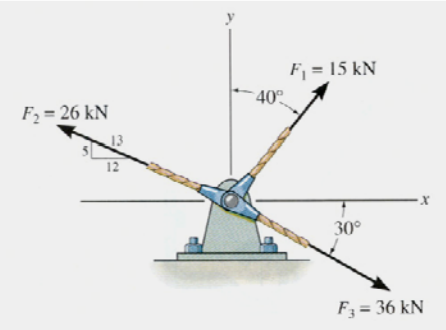
## EXAMPLE




**Given:** Three concurrent forces acting on a bracket.

**Find:** The magnitude and angle of the resultant force.

**Plan:**

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

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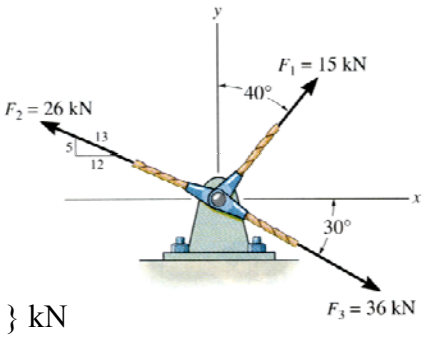
## EXAMPLE (continued)




$\boldsymbol{F_1}$ = { 15 sin 40° ***i*** + 15 cos 40° ***j*** } kN

= { 9.642 ***i*** + 11.49 ***j*** } kN

$\boldsymbol{F_2}$ = { -(12/13)26 ***i*** + (5/13)26 ***j*** } kN

= { -24 ***i*** + 10 ***j*** } kN

$\boldsymbol{F_3}$ = { 36 cos 30° ***i*** – 36 sin 30° ***j*** } kN

= { 31.18 ***i*** – 18 ***j*** } kN

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## EXAMPLE
(continued)

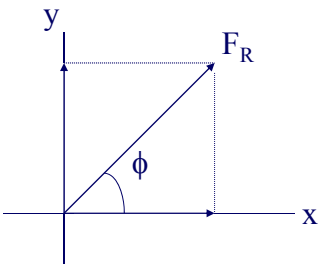
Summing up all the ***i*** and ***j*** components respectively, we get,

$\boldsymbol{F_R} = \{ (9.642 - 24 + 31.18)\, \boldsymbol{i} + (11.49 + 10 - 18)\, \boldsymbol{j} \}$ kN

$= \{ 16.82\, \boldsymbol{i} + 3.49\, \boldsymbol{j} \}$ kN

$F_R = ((16.82)^2 + (3.49)^2)^{1/2} = 17.2$ kN

$\phi = \tan^{-1}(3.49/16.82) = 11.7°$

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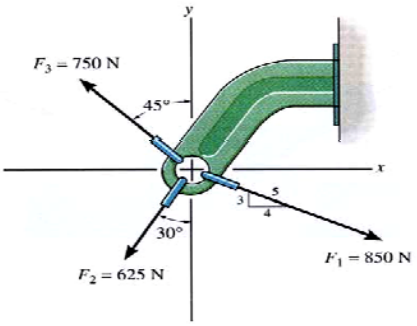
## PROBLEM




**Given:** Three concurrent forces acting on a bracket

**Find:** The magnitude and angle of the resultant force.

**Plan:**

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

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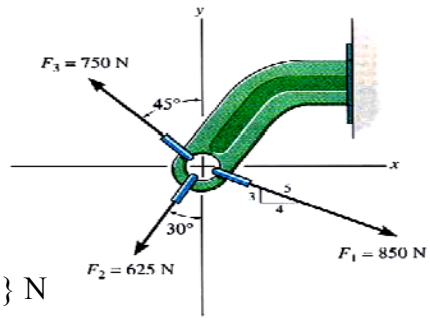
**PROBLEM** (continued)




$\boldsymbol{F_1}$ = { (4/5) 850 ***i*** - (3/5) 850 ***j*** } N

= { 680 ***i*** - 510 ***j*** } N

$\boldsymbol{F_2}$ = { -625 sin(30°) ***i*** - 625 cos(30°) ***j*** } N

= { -312.5 ***i*** - 541.3 ***j*** } N

$\boldsymbol{F_3}$ = { -750 sin(45°) ***i*** + 750 cos(45°) ***j*** } N

{ -530.3 ***i*** + 530.3 ***j*** } N

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**PROBLEM** (continued)

Summing up all the ***i*** and ***j*** components respectively, we get,

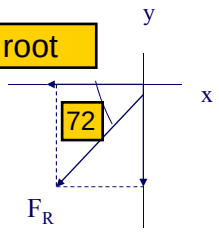
$\boldsymbol{F_R}$ = { (680 – 312.5 – 530.3) ***i*** + (-510 – 541.3 + 530.3) ***j*** }N

= { - 162.8 ***i*** - 521 ***j*** } N

$F_R = ((162.8)^2 + (521)^2)^{1/2} = 546$ N = sqr root

$\phi = \tan^{-1}(521/162.8) = 72.64°$ or

From Positive x axis $\theta = 180 + 72.64 = 253°$

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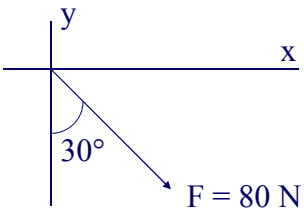
## ATTENTION QUIZ

1. Resolve $\boldsymbol{F}$ along x and y axes and write it in vector form. $\boldsymbol{F}$ = { _____ } N




A) 80 cos (30°) $\boldsymbol{i}$ - 80 sin (30°) $\boldsymbol{j}$

B) 80 sin (30°) $\boldsymbol{i}$ + 80 cos (30°) $\boldsymbol{j}$

C) 80 sin (30°) $\boldsymbol{i}$ - 80 cos (30°) $\boldsymbol{j}$

D) 80 cos (30°) $\boldsymbol{i}$ + 80 sin (30°) $\boldsymbol{j}$

2. Determine the magnitude of the resultant ($\boldsymbol{F_1} + \boldsymbol{F_2}$) force in N when $\boldsymbol{F_1}$ = { 10 $\boldsymbol{i}$ + 20 $\boldsymbol{j}$ } N and $\boldsymbol{F_2}$ = { 20 $\boldsymbol{i}$ + 20 $\boldsymbol{j}$ } N .

A) 30 N B) 40 N C) 50 N

D) 60 N E) 70 N

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