Part I: Solution to Multiple choice questions.

1) $H_0: \mu = 50$
$H_a: \mu \neq 50$
$n = 9$, ; normally destributed popl^n
$\bar{x} = 61$; $S = 21$
⇒ Small sample with normally distributed
under H_0

$$t_{obs} = \frac{\bar{x} - 50}{S/\sqrt{n}} = \frac{61 - 50}{21/\sqrt{9}} \sim t_{(9-1)}$$

$$= \frac{11 \times 3}{21} = 1.57$$

$P.value = 2P(|t| > 1.57)$
$= 2P(t > 1.57)$.
$P(t > 1.57) = \frac{1}{2} P.value.$
⇒ $P.value > 0.1$

Answer: (a)

(2) Answer: (c)

(3) Answer: (d.)

(4) Answer: (a) with normally assumption

(5) If $Z_{score} = \frac{x-\mu}{\sigma} > 3$, then that observation is outlier.
~~Answer~~: $Z_{score} = \frac{4-0}{1} = 4 > 3$

Answer: (d)

(6) length of an interval $= l = 2 Z_{\alpha/2} \sigma/\sqrt{n}$

$$l/3 = 2 Z_{\alpha/2} \frac{\sigma}{3\sqrt{n}}$$

$$= 2 Z_{\alpha/2} \frac{\sigma}{\sqrt{9n}}.$$

∴ New sample size should be $9n$.
$= 9 \times 90 = \cancel{270}\ 810$

Answer! ~~(b)~~ (c)

(7) Answer! (c).

(8) Answer! (c)

(9) $X \sim \text{Poisson}(2.5)$

$$P(X = x) = \frac{e^{-\mu} \mu^{x}}{x!}$$

$$P(X = 0) = \frac{e^{-2.5} 2.5^{0}}{0!} = e^{-2.5}$$

$$= 0.0821$$

Answer! (b)

(10) Let X be the time

$X \sim N(15, 4)$.

$P(X \geq x) = 0.04$

$P\left(Z \geq \frac{x - 15}{2}\right) = 0.04.$

$$P\left(Z \le \frac{x-15}{2}\right) = 0.96.$$

$$\frac{x-15}{2} = 1.75$$

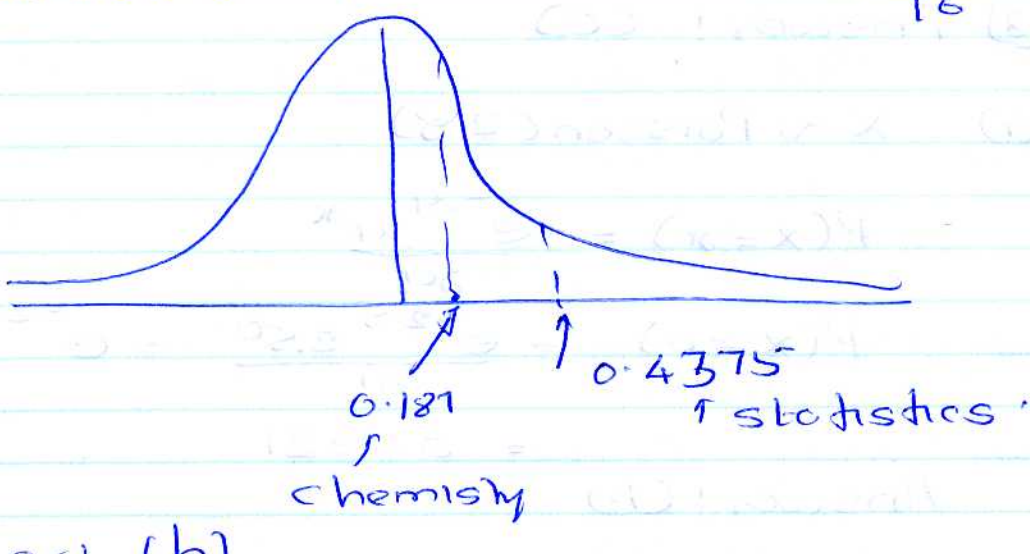
$$x = 18.5 \text{ min.}$$

Answer : (d)

(11) Chemistry marks : $X \sim N(90, 64^2)$

Statistics marks : $Y \sim N(70, 16^2)$.

A student's grade
Chemistry : 102 $\Rightarrow$ Zscore $= \frac{102-90}{64} = 0.1875$
Statistics : 77 $\Rightarrow$ Zscore $= \frac{77-70}{16} = 0.4375$



Answer: (b)

(12) $n = 100$, median $= 23$, mean $= 20$
Standard deviation $= 5$, range $= 35$
Tchebysheff's Thm

$k = 2$, $\bar{x} \pm 2s \Rightarrow 20 \pm 2 \times 5$
$\Rightarrow (10, 30)$ should contain
atleast 75% of observations.

$x = 3. \quad \bar{x} \pm 3s \Rightarrow 20 \pm 3 \times 5$
$\Rightarrow (5, 35).$

$\sigma \simeq \frac{\text{Range}}{4} = \frac{35}{4} \simeq 8.75 \not\approx 5.$

. Answer! (b).

(13) $P(A) = 0.5, \quad P(B) = 0.7$
$P(A \cap B) = 0.3.$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.5 + 0.7 - 0.5 = 0.9.$
$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.7} = 0.42.$

Answer! None.

(14) $P(A) = 0.3, \quad P(B) = 0.4$
$P(A|B) = 0.6 = \frac{P(A \cap B)}{P(B)}$

$\therefore P(A \cap B) = 0.6 \times 0.4 = 0.24$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.4 - 0.24 = 0.46$

(15)

I		II		III	
G	G	G	S	S	S

$P(\text{I}) = P(\text{II}) = P(\text{III}) = 1/3$

$P(G|\text{I}) = 1, \quad P(G|\text{II}) = 1/2, \quad P(G|\text{III}) = 0$

$$P(I/G) = \frac{P(G|I)P(I)}{P(G|I)P(I) + P(G|II)P(II) + P(G|III)P(III)}$$

$$= \frac{1 \times 1/3}{1 \times 1/3 + 1/2 \times 1/3 + 0}$$

$$= 6/9 = 2/3$$

Answer: (b).

Long answer questions (solutions)

1) $X \sim$ Men's height

$$X \sim N(174, 6^2).$$

(a) $P(170 \leq X \leq 179)$

$= P\left(\frac{170-174}{6} \leq Z \leq \frac{179-174}{6}\right)$

$= P(-2/3 \leq Z \leq 5/6) = P(Z \leq 5/6) - P(Z \leq -2/3)$

$= P(Z \leq \cancel{0.8\ldots}\ 0.83) - P(Z \leq -0.67)$

$= 0.7967 - 0.2514 = 0.5453$

(b) $P(X > a) = 0.05$
where a - ceiling height.

$P(X \leq a) = 1 - 0.05 = 0.95$

$P\left(Z \leq \frac{a-\mu}{\sigma}\right) = 0.95$

$\Rightarrow P\left(Z \leq \frac{a-174}{6}\right) = 0.95$

$\therefore \frac{a-174}{6} = 1.64$

$a = 174 + 1.64 * 6$

(c) $P(\bar{X} \geq 176)$, $n = 49$

$X \sim N(174, 6^2) \Rightarrow \bar{X} \sim N(174, 6^2/49)$

$\therefore P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \geq \frac{176-174}{6/7}\right) = P(Z \geq 14/6)$

$= 1 - P(Z \le 2.33)$

$= 1 - 0.9901 \quad = \underline{0.009}$

(2) Popl^n 1 : Marriott
Popl^n 2 : Radisson.

$H_0 : \mu_1 - \mu_2 = 0$

$H_a : \mu_1 - \mu_2 \neq 0$

$\bar{x}_1 = 170 \qquad \bar{x}_2 = 145 \qquad n_1 = 50$

$s_1 = 15 \qquad s_2 = 10 \qquad n_2 = 50$

Under Ho, test statistic. (large sample $n_1 > 30$, $n_2 > 30$)

$$Z_{obs} = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

$$= \frac{170 - 145 - 0}{\sqrt{15^2/50 + 10^2/50}} = \frac{25}{2.55}$$

$$= 9.80$$

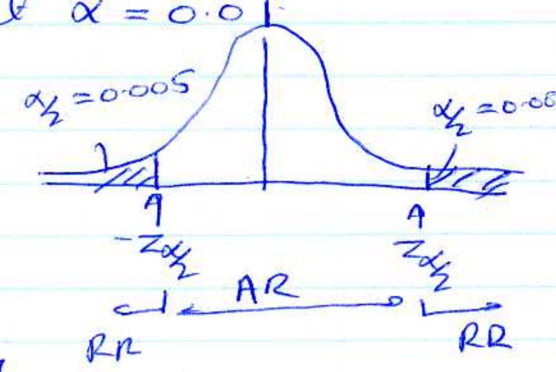
~~Pvalue~~ Critical points at $\alpha = 0.01$

$\alpha/2 = 0.005$

$Z_{0.005} = 2.57$

$-Z_{0.005} = -2.57$.

Reject Ho, if $Z_{obs} > Z_{0.005}$
or $Z_{obs} < -Z_{0.005}$



$Z_{obs} = 9.80 > Z_{0.005} = 2.57$.

$\therefore$ Reject Ho. $\Rightarrow$ There is a difference in the average room rates for Marriott & Radisson

(b) $P.value = P(|Z| > Z_{obs})$

$= \cancel{2P(Z > 2.37)}\ 2P(Z > 13.72)$

≈ 0

$P.value = 0 < \alpha = 0.005 \Rightarrow$ We reject Ho.

(3)

	1	2	3	4	5
Before	500	475	525	490	530
After	510	480	525	495	533
d_i	-10	-5	0	-5	-3.

Paired-difference as two samples are dependet

$\cancel{Ho}\ \mu_d = \mu_{before} - \mu_{after}$.

$H_0 : \mu_d = 0$ (Not effective)

$H_a : \mu_d < 0$ (effective).

$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i = \frac{1}{5}\cancel{(-10-5+0} = \frac{1}{5}[(-10)+(-5)+(0)+(-5) + (-3)]$

$= -4.6.$

$S_d^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} d_i^2 - \frac{(\sum d_i)^2}{n}\right]$

$= 13.3.$

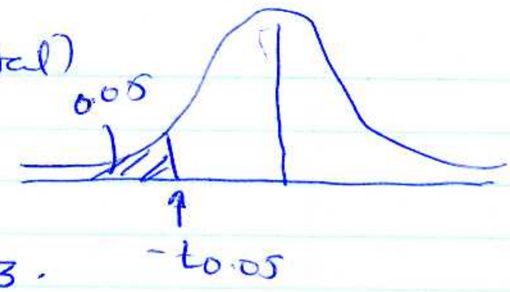
$\Rightarrow S_d = 3.64.$

Test statistic under Ho

$t_{obs} = \frac{\bar{d} - \cancel{0}}{S_d/\sqrt{n}} = \frac{-4.6 - 0}{3.64/\sqrt{5}} = -2.825$

Critical value (left-tailed test)

$-t_{0.05}(4) = -2.13.$



As $t_{obs} = -2.825 < -t_{0.05} = -2.13.$

We reject H_0.
$\therefore$ We conclude that the time management program is effective.

(4) Preservative A : $n_A = 10$
Preservative B : $n_B = 10$

μ_A : Average time until spoilage begins when treated with A
μ_B : Average time until spoilage begins when treated with B.

(a) $H_0 : \mu_A - \mu_B = 0$
$H_a : \mu_A - \mu_B \neq 0$

$\frac{S_B^2}{S_A^2} = \frac{11.5^2}{9.5} < 3.$

$\therefore$ Equal variance assumption is valid. Under normality assumption, Pooled variance

(b) $$S_p^2 = \frac{(n_A - 1)S_A^2 + (n_B - 1)S_B^2}{(n_A + n_B - 2)}$$

$$= \frac{(10-1) \times 9.5^2 + (10-1)11.5^2}{(10+10-2)}$$

$$= 111.25$$

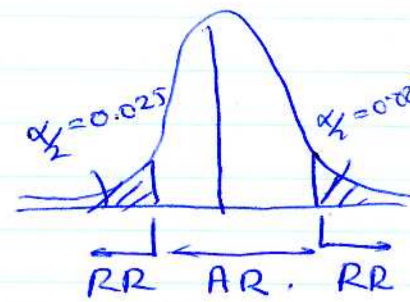
Test statistic under Ho

$$t_{obs} = \frac{\bar{x}_A - \bar{x}_B - 0}{\sqrt{S_p^2\left(\frac{1}{n_A}+\frac{1}{n_B}\right)}}$$

$$= \frac{108.7 - 98.7}{\sqrt{111.25\left(\frac{1}{10}+\frac{1}{10}\right)}} = 2.11.$$

$df = n_A + n_B - 2 = 18$

(c) Critical value at $\alpha = 0.05$



$$t_{0.025\,(18)} = 2.101.$$

If ~~tabs~~ $t_{obs} > t_{0.025}$ or $t_{obs} < -t_{0.025}$ then reject Ho.

$\therefore t_{obs} = 2.11 > t_{0.025(18)} = 2.101$

$\therefore$ We reject Ho.

There is a significant difference ~~when~~ bet^n two preservatives.

(5) Large sample confidence interval for quantitative mean.

$n = 225, \quad \bar{x} = 12.5, \quad S = 5.4.$

$\therefore$ 95% large sample confidence interval

$$\bar{x} \pm Z_{0.025}\, S/\sqrt{n}$$

$$= 12.5 \pm 1.96 \left(\frac{5.4}{\sqrt{225}}\right)$$

$$12.5 - 1.96\left(\frac{5.4}{15}\right) \le \mu < 12.5 + 1.96\left(\frac{5.4}{15}\right)$$

(6) $n = 140$,

X is # of products sold.

$X \sim \text{Binomial}(140, 0.05)$.

$np = 140 \times 0.05 = 7 \leq 7$

Poisson approximation to binomial

$Y \sim \text{Poisson}(7)$

$P(X \geq 2) = 1 - P(X \leq 1)$
$\approx 1 - P(Y \leq 1)$

$= 1 - 0.007$ (from poisson table)
$= \underline{\underline{0.993}}$

(7) Proportion of improperly documented reports $= P = 0.1$
$n = 200$

$X \sim \text{Binomial}(200, 0.1)$

$np = 200 \times 0.1 = 20 > 5$

$nq = 200(1-0.1) = 200 \times 0.9 = 18 > 5$

Normal approximation to Binomial with continuity correction.

$$P(X > 40) = 1 - P(X \leq 40)$$

$$= 1 - P\left(Z \leq \frac{40+0.5-20}{\sqrt{npq}}\right)$$

$$= 1 - P\left(Z \leq \frac{40.5-20}{\sqrt{200 \times 0.1 \times 0.9}}\right)$$

$$= 1 - P(Z \leq 4.83)$$

$$= 1 - 1 = 0$$

(8) $\bar{y} = \frac{1}{n}\sum y_i$, $\bar{x} = \frac{1}{n}\sum x_i$

$\bar{y} = 80$, $\bar{x} = 3.125$

$$S_x = \sqrt{\frac{1}{n-1}\left[\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right]} = 1.457$$

$$S_y = \sqrt{\frac{1}{n-1}\left[\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right]} = 38.172$$

$$S_{xy} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{n-1}$$

$$= 54.285$$

$$r = \frac{S_{xy}}{S_x S_y} = 0.975$$

$$\text{Slope} = r\frac{S_y}{S_x} = 0.975 \times \frac{38.172}{1.452}$$

$$b = \bar{y} - a\bar{x} = 80 - (0.975 \times \frac{38.172}{1.452} \times 3.125)$$

$$\therefore y = 0.975\left(\frac{38.172}{1.452}\right)x + (80 - 0.975 \times \frac{38.172}{1.452} \times 3.125)$$

$$= 25.546x + 0.1681$$

$x = 6$

$$y_6 = 25.546 * 6 + 0.1681$$

$$= 153.44$$