Test 1 Solutions MATH 1004H, Winter, 2014

Last name: First name:
Student no.:

This test has two parts, the first part has 5 multiple choice questions (3 marks each) and the second part has 3 long answer questions.
Part 1 Circle the correct answer. No partial marks.
Answers:
1: c
2: c
3: c
4: c
5: d

1) The formula for $f^{-1}(x)$ the inverse of the function $f(x) = \dfrac{7x-3}{2x+4}$ is

a) $\dfrac{x-5}{7x-2}$ b) $\dfrac{2x+4}{7x-3}$ **c)** $\dfrac{4x+3}{7-2x}$ d) $\dfrac{7-3x}{2+4x}$.

2) The $\lim\limits_{x\to 2^-} \dfrac{2-x}{|x^2-4|}$ is

a) $-\infty$ **b)** $+\infty$ c) $\dfrac{1}{4}$ d) $-\dfrac{1}{4}$.

3) The $\lim\limits_{x\to\infty} \dfrac{10x^4-100x+20}{3-2x^4}$ is

a) 0 b) $-\infty$ c) -5 d) 5.

4) Let $f(x) = \dfrac{x^3+4x}{x^2+5x}$. What is $f'(1)$?

a) $\dfrac{-5}{36}$ b) $\dfrac{3}{49}$ c) $\dfrac{7}{36}$ d) $\dfrac{77}{36}$.

5) Using the Squeeze theorem the $\lim\limits_{x\to 0} 3x^2 \cos\dfrac{5}{2x}$ is

a) $\dfrac{5}{2}$ b) $\dfrac{15}{2}$ c) doesn't exist d) 0.

Part 2: long answer questions

6-[5 marks]: Use the definition of derivative, ONLY, to find the derivative of $f(x) = \sqrt{x}$ at $x = 9$.
Answer:

$$f'(9) = \lim_{x\longrightarrow 9} \frac{f(x)-f(9)}{x-9} = \lim_{x\longrightarrow 9} \frac{\sqrt{x}-3}{x-9} \quad \textbf{2 marks}$$

$$= \lim_{x\longrightarrow 9} \frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}-3)} = \lim_{x\longrightarrow 9} \frac{1}{(\sqrt{x}+3)} = \frac{1}{6} \quad \textbf{3 marks}$$

Using the following definition is right too:

$$f'(9) = \lim_{h\longrightarrow 0} \frac{f(9+h)-f(9)}{h} = \lim_{h\longrightarrow 0} \frac{\sqrt{9+h}-3}{h} \quad \textbf{2 marks}$$

$$= \lim_{h\longrightarrow 0} \frac{(\sqrt{9+h}-3)}{h} \frac{(\sqrt{9+h}-3)}{(\sqrt{9+h}+3)} = \lim_{h\longrightarrow 0} \frac{9+h-9}{h(\sqrt{9+h}+3)} = \frac{1}{6} \quad \textbf{3 marks}$$

7-[5 marks]: Find the equation of the tangent line to the curve $y = x^3$ at the point $(2, 8)$.
Answer:

$$f(x) = x^3 \Longrightarrow f'(x) = 3x^2 \Longrightarrow f'(2) = 12 \quad \textbf{2 marks}$$

$$y - 8 = 12(x - 2) \quad \textbf{3 marks}$$

or:

$$y = 12x - 16 \quad \textbf{2 marks}$$

8-[3+3 marks]: Let y be a function of x given by $x^5 + y^4 + x - y = 2$.
a: Use implicit differentiation to find y'.
b: Find the equation of the tangent line to the curve $x^5 + y^4 + x - y = 2$ at the point (1,1).
Answer:
a:

$$5x^4 + 4y'y^3 + 1 - y' = 0 \Longrightarrow y'(4y^3 - 1) = -5x^4 - 1 \Longrightarrow y' = \frac{-5x^4 - 1}{4y^3 - 1}$$

b:

$$y' = \frac{-5x^4 - 1}{4y^3 - 1} = -6/3 = -2$$

tangent line: $y - 1 = -2(x - 1)$