[7] **1)** Solve the differential equation $y' = \dfrac{x^2 + xy}{xy + y^2}$.

[6] **2)** Solve the differential equation $y'' + 3y' + 2y = 0$, $y(0) = 1$, $y'(0) = 2$.

[7] **3)** Solve the Bernoulli equation $y' + \dfrac{y}{x} = 3xy^2$.

[5] **4.a)** Verify that the differential equation

$$(e^x \sin y + 2x)dx + (e^x \cos y + 2y)dy = 0$$

is exact and solve it.

[5] **4.b)** Show that the differential equation $(4x + 3y^2)\,dx + (2xy)\,dy = 0$ is not exact. Find an integrating factor of the form $I = I(x)$ that makes it exact. Write down the new exact differential equation.
(DO NOT SOLVE THE NEW DIFFERENTIAL EQUATION.)

[7] **5)** Solve the differential equation $xy' + (2 + 3x)y = xe^{-3x}$.

[7] **6)** Solve the differential equation $9x^2y'' + 15xy' + 5y = 0$, $x > 0$.

[15] **7)** Determine whether the following series converges absolutely, converges conditionally, or diverges. Justify your answer.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ **b)** $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^e}$ **c)** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\sqrt[3]{n}}{n+1}$

[10] **8)** Find the radius of convergence and the interval of convergence of the

series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}.$$

[7] **9)** Solve the differential equation $y'' + 4y = 3\cos(2x)$.

[8] **10)** Find the Taylor series for the function $f(x) = \cos x$ at $a = \pi/3$.

[7] **11)** Use the binomial series to expand the function $\dfrac{1}{\sqrt[3]{8+x}}$ as a power series.

[4] **12.a)** Suppose that the differential equation $x^2y'' + xy' + x^2y = 0$ has a power series solution of the form $y = \sum_{n=0}^{\infty} c_n x^n$. Find a **recurrence relation** that determines c_n, $n = 0, 1, 2, 3, \ldots$.
(Do not solve the differential equation)

[5] **12.b)** Use the recurrence relation

$$c_1 = c_2 = 0, \quad c_{n+1} = \frac{c_{n-2}}{n+1} \text{ for } n = 2, 3, 4, \ldots$$

to solve the differential equation $y' - x^2y = 0$, by power series solution, where $y = \sum_{n=0}^{\infty} c_n x^n$.