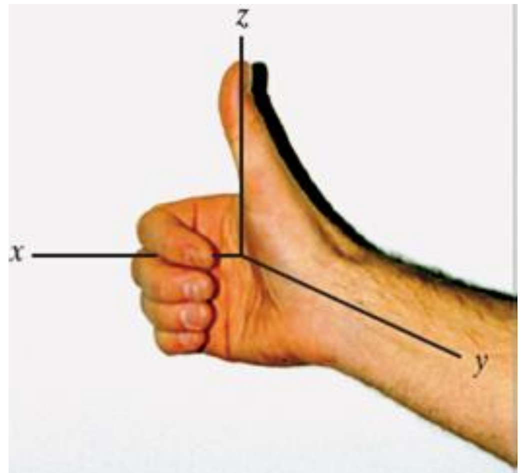
ECOR 1101 - Lecture Notes

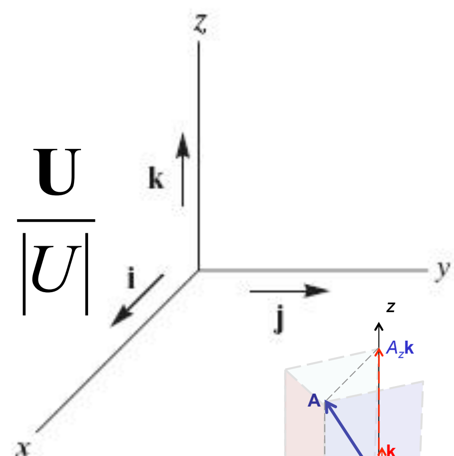
Lecture #3 - Vectors II (Chapter 2: Sections 2.5 - 2.8)

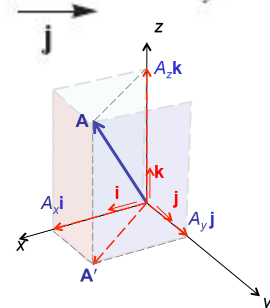
Cartesian Vectors in 3D



- 3D vectors are best represented in Cartesian vector notation
- 3D Coordinate System:
  - Right-handed CS
  - Thumb +ve z-axis
  - Fingers curled around z-axis, sweeps from x-axis to y-axis
- Cartesian Unit Vectors
  - Cartesian unit vectors i, j, k designate vectors in the x,y,z directions
  - The positive directions of the unit vectors are as shown in fig.

$$\mathbf{u} = \frac{\mathbf{U}}{|U|}$$





Cartesian Vector Representation

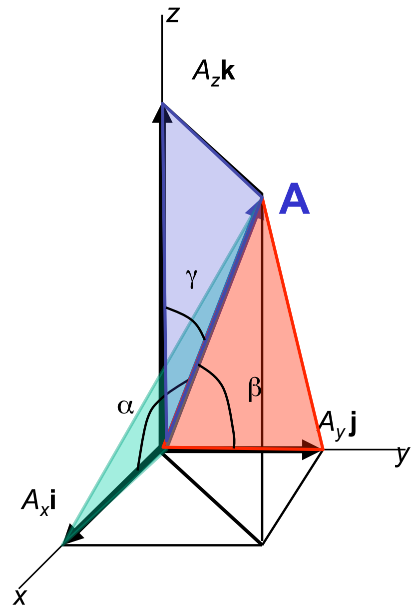
- Resolution of A into the Cartesian unit vectors will require two successive application of parallelogram law

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

- Magnitude of Cartesian vector

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction of Cartesian Vector



- The direction of the Cartesian vector A is defined by the angles for 3 angles it makes with the x, y, and z axes, respectively.

- These angles are called coordinate direction angles

$$\left.\begin{aligned} \cos\alpha &= \frac{A_x}{A} \\ \cos\beta &= \frac{A_y}{A} \\ \cos\gamma &= \frac{A_z}{A} \end{aligned}\right\} \textbf{Direction cosines of } \mathbf{A}$$

- A vector A can be represented using unit vectors as:

A = Aua, where UA is a unit vector in the direction of A

$$\mathbf{u_A} = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

$$\mathbf{u_A} = \cos\alpha\mathbf{i} + \cos\beta\mathbf{j} + \cos\gamma\mathbf{k}$$

- The magnitude of a unit vector is 1. Therefore if any two coordinate angles (direction cosines) are known we can easily find the third.

$$u_A = \sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma} = 1$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

2nd Method

- The direction of **A** can be found from two angles.

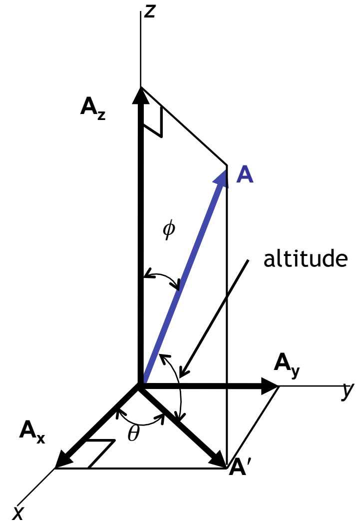
$A_z = A\cos\phi$

$A' = A\sin\phi$

$A_x = A'\cos\theta = A\sin\phi\cos\theta$

$A_y = A'\sin\theta = A\sin\phi\sin\theta$

A= $A\sin\phi\cos\theta$ **i** + $A\sin\phi\sin\theta$ **j** + $A\cos\phi$ **k**



Addition of Cartesian Vectors

- Given two vectors A and B:

$A = A_x i + A_y j + A_z k$

$B = B_x i + B_y j + B_z k$

- We can add A and B using Cartesian components as follows:**R** = **A** + **B** = (A_x+B_x)**i** + (A_y+B_y)**j** + (A_z+B_z)**k**

- We can also subtract A and B using Cartesian components as follows:**R** = **A** – **B** = $(A_x - B_x)$**i** + $(A_y - B_y)$**j** + $(A_z - B_z)$**k**

- General formulation : R= sumF = SumFxI + SumFyJ + SumFzK

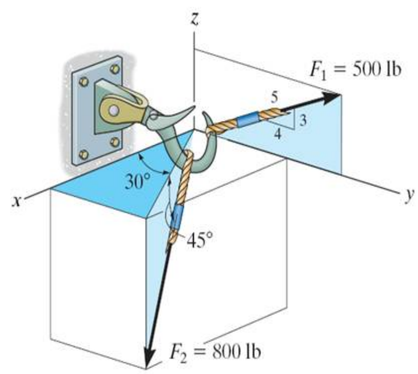
Sample Problem

Determine the resultant force acting on the hook

Solution Procedure

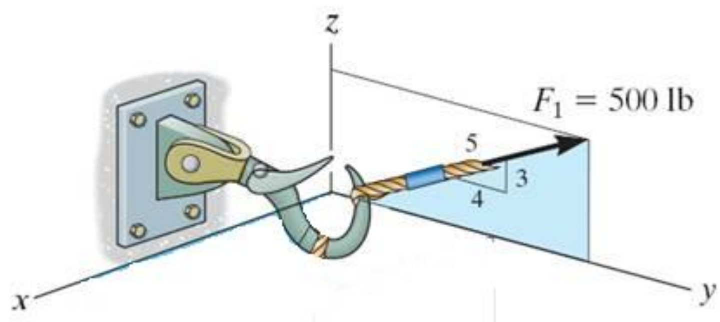
1) Using geometry and trigonometry, write F1 and F2 in Cartesian vector form

2) Then add the two forces (by adding x and y components of the forces)



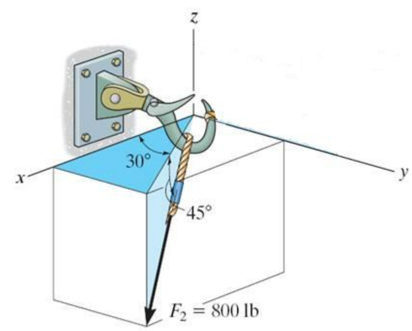
Resolve force F1

$F_{1x} = 0 = 0$ lb
$F_{1y} = 500\ (4/5) = 400$ lb
$F_{1z} = 500\ (3/5) = 300$ lb



Write F1 in Cartesian vector form (don't forget the units!). $\mathbf{F_1}$ = {0**i** + 400**j** + 300**k**} lb

Resolve force F2



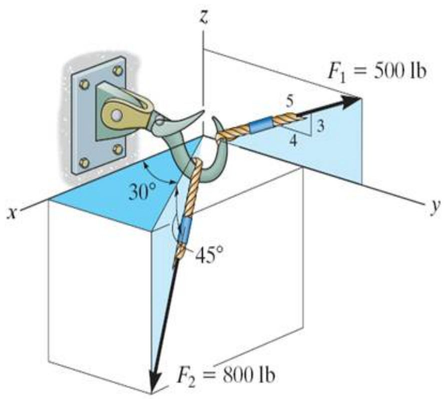
We are given only two angles. So we need to resolve F2 into z-axis and the xy-plane. Be careful with your positive and negative directions.

$$F_{2z} = F_2 \sin 45^o = 800 \times \sin 45^o = 565.69 \text{ lb.}$$
$$F_{2xy} = F_2 \cos 45^o = 800 \times \cos 45^o = 565.69 \text{ lb.}$$

$$F_{2x} = F_{2xy} \cos 30^o = 565.69 \times \cos 30^o = 489.90 \text{ lb.}$$
$$F_{2y} = F_{2xy} \sin 30^o = 565.69 \times \sin 30^o = 282.84 \text{ lb.}$$

$$\mathrm{F_2} = \left\{489.90\mathrm{i} + 282.84\mathrm{j} - 565.69\mathrm{k}\right\} \text{ lb}$$

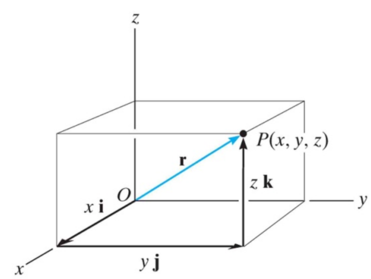
The + and - only enter the equation when you write the Force using Cartesian vector coordinates!



$$F_2 = \{489.90\text{i} + 282.84\text{j} - 565.69\text{k}\}\ \text{lb}$$

$$F_R = F_1 + F_2 = \{(489.90)\text{i} + (400 + 282.84)\text{j} + (300 - 565.69)\text{k}\}\ \text{lb}$$
$$F_R = \{490\text{i} + 683\text{j} - 266\text{k}\}\ \text{lb.}$$

Position Vectors

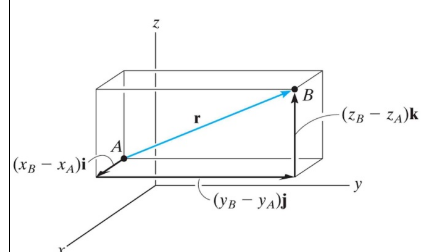


(b)

fig02_35b.jpg

- A position vector r is a fixed vector which defines a point in space relative to another point e.g. a point P relative to the origin O
- O (0,0,0) and P (x,y,z) —— r = xi + yj + zk
- In general if a position vector is directed from point A (Xa, Ya, Za) to B(Xb, Yb, Zb), then $r_{AB} = r_B - r_A$

$$= (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$
$$= (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$$



(b)

fig02_36b.jpg

- The magnitude of the Position vector rAB is given as:

$$r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

- The direction of the position vector rAB is given by the direction cosines of unit vector of rAB, $\cos\alpha$, $\cos\beta$, and $\cos\gamma$

$$\mathbf{u} = \frac{\mathbf{r_{AB}}}{r_{AB}} = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$

Force Vector Along a Line

- In 3-dimensional statics, the force F can be specified by two points, A and B through which passes the line of action of F
- F can also be represented by the position vector, r from point A to B.
- Hence

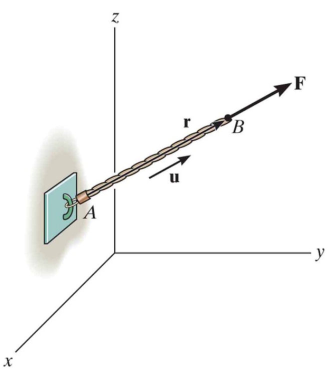


fig02_38.jpg

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

Problem F2-22

Express the force as a cartesian vector.

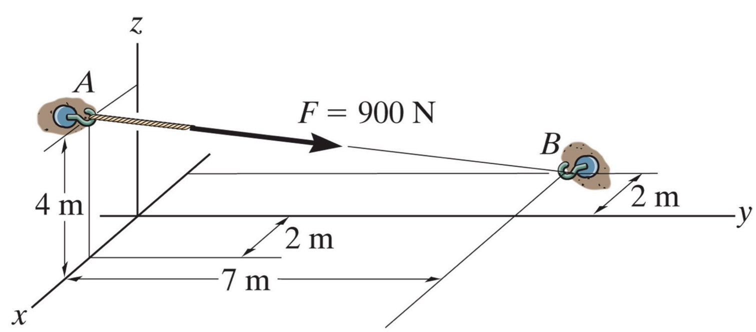


Figure: 02_FP022

Problem F2-23

Determine the magnitude of the resultant force at A.

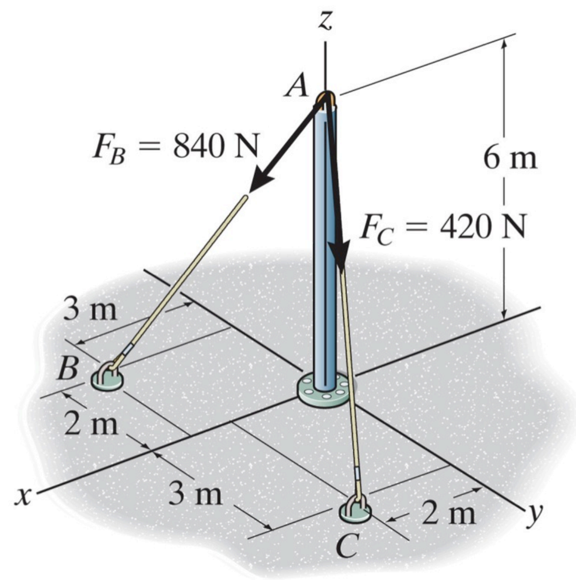
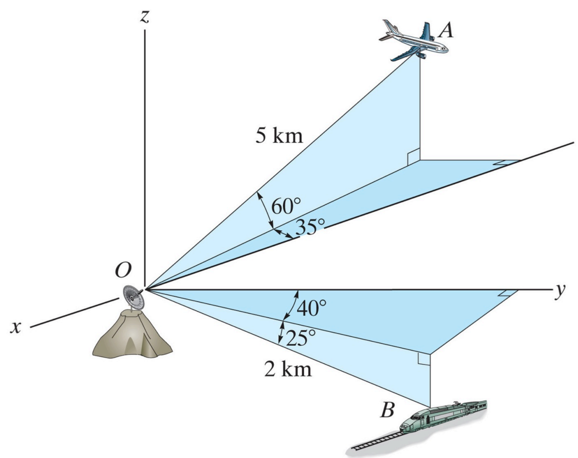


Figure: 02_FP023

Problem 2-95

At a given distance, the position of a plane at A and a train at B are measured relative to a radar antenna at O. Determine the distance d between A and B at this instant. To solve the problem, formulate a position vector, directed from A to B, and then determine its magnitude.



Problem 2-105

The pipe is supported at its end by a cord AB. If the cord exerts a force of F= 12 lb on a pipe at A, express this force as a Cartesian vector.