ECOR 1101 - Lecture Notes

Lecture #2 - Vectors I (Chapter 2 - Sections 2.1 - 2.4)

Scalars and Vectors

- Scalar: A scalar is completely described by its magnitude (can be positive or negative)
  - Examples mass, volume, length, time
- Vector: A vector has more than one attribute or characteristic to describe it.
  - Magnitude (size), Direction (Angle), Sense(Tension/Compression), Point of Application (Location)

Vector

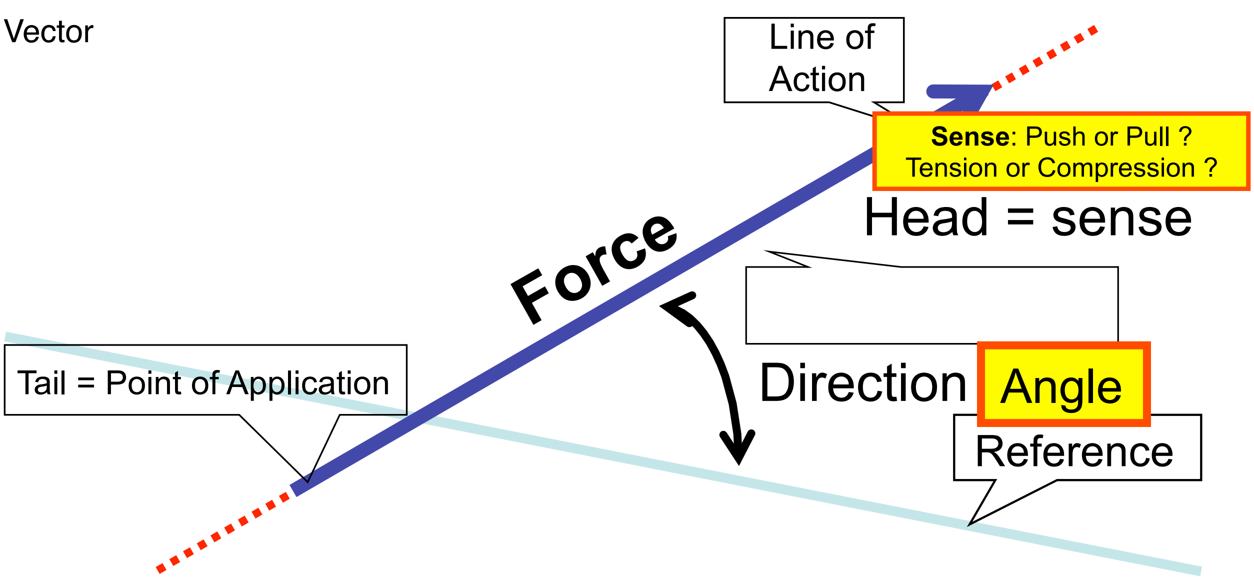
Magnitude = Size

Direction = Angle

P of A = Location

Sense = T or C

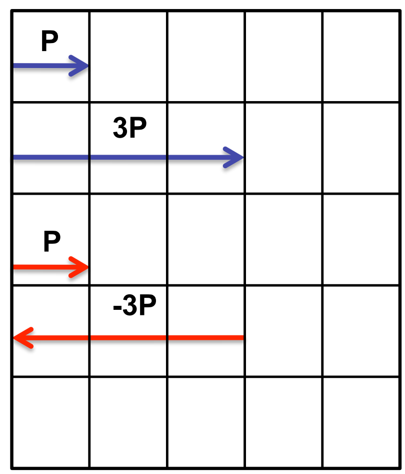
Vector



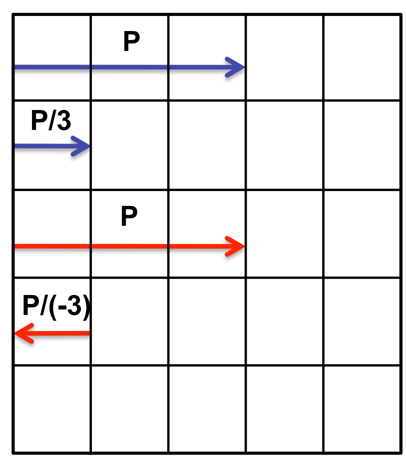
Vector Notation

- Vectors can be represented by a variety of notations:
  - Bold face type lettering (A)
  - Letter with an arrow over it (A) (With Arrow) →
  - Underlined letters (A) (With Underline)
  - Two letters denoting origin and end and an arrow over top (AB) →

Vector Operations



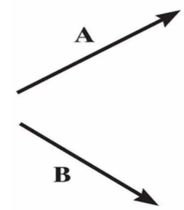
- Multiplication of a vector by a scalar
  - Multiplication by a +ve scalar increases magnitude by scalar value. Sense remains unchanged.
  - Multiplication by -ve scalar increases magnitude by scalar value. Sense changes.

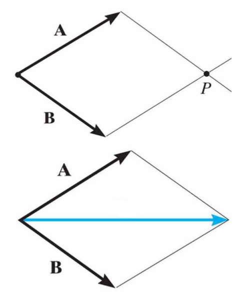


- Division of a vector by a scalar
  - Division by a +ve scalar decreases magnitude by scalar value. Sense remains unchanged
  - Division by a -ve scalar decreases magnitude by scalar value. Sense changes.

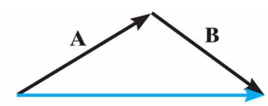
Vector Operation - Addition

- Vectors obey the parallelogram law of vector addition.
- When two vectors, A and B are added they form a resultant vector C.
- The general rule is:
  - Join the tails of A and B at a point (O) to make them concurrent
  - From the head of B draw a line parallel to A. Then draw a line from head of A parallel to B so they intersect at point (P) and form a parallelogram.
  - The diagonal of the parallelogram (O-P) represents the resultant vector C.
  - Vectors can also be added by using the triangle rule - a special case of the parallelogram law of vector addition.
  - For the two vectors, A and B, the general rule is:
    - Draw vector A and then from the head of A, draw vector B in a "head-to-tail" fashion
    - The resultant vector C extends from the tail to the head of B.

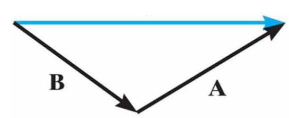




= **A** + **B**
Parallelogram law



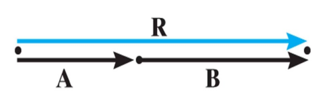
= **A** + **B**
Triangle rule



= **B** + **A**
Triangle rule

Collinear Vectors

- If two vectors A and B are collinear, i.e. both have the same line of action, the parallelogram law of vector addition reduces to algebraic sum (or scalar sum) of their magnitudes



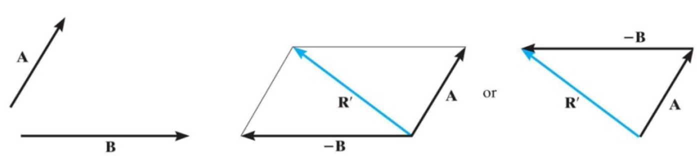
$R = A + B$

Addition of collinear vectors
fig02_05.jpg

Vector Operations - Subtraction

- The resultant R' of the difference between two vectors A and B can be expressed as:

R' = A - B = A + (-B)

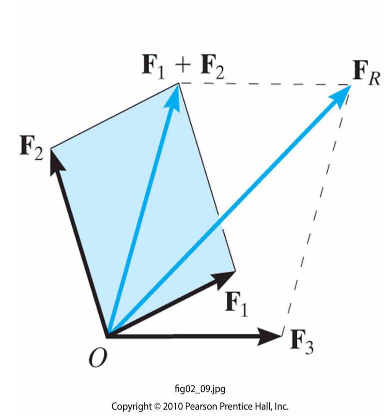


Parallelogram law   Triangle construction
Vector subtraction
fig02_06.jpg

Forces

- A Force is a vector quantity.
  - It has a specified magnitude, sense, direction and point of application as do vectors.
- In statics, we often have two forces (or components of a force) and are required to find their resultant (force)
- Or we have a resultant force and are required to resolve the force into its component forces

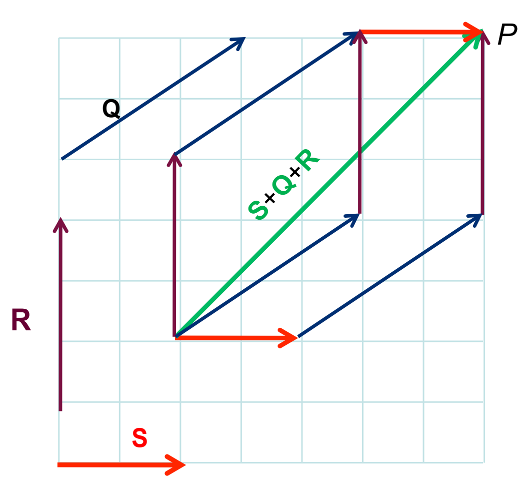
Addition of Multiple Forces



- Forces can be added and subtracted in the same manner as vector by the parallelogram law or triangle rule
- If more than two forces (vectors) are to be added, successive applications of the parallelogram law can be used.
- $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = ((\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3)$ Does $F_R = F_2 + F_3 + F_1$?

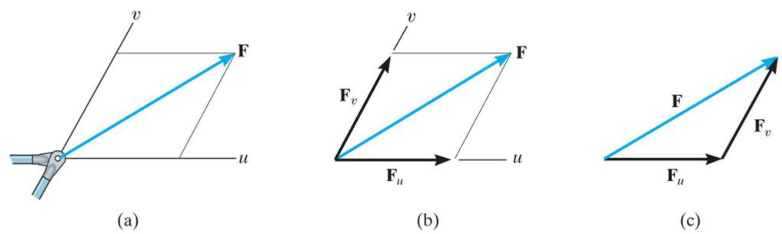
Sample Problem

- Given vectors Q, R and S. Show that :
- (S + Q + R) = (R + Q + S) = (Q + R + S)



System of Coplanar Forces

- An arbitrary force, F, can be resolved into two coplanar components, Fu and Fv, along any two axes u and v by the parallelogram law of vector addition or the triangle rule



$$\mathbf{F} = \mathbf{F}_u + \mathbf{F}_v$$

Sine and Cosine Laws

Using the triangle rule of vector addition, the sine and cosine laws can be used to determine the direction and magnitude of the resultant force, respectively

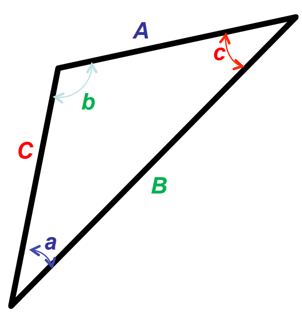
Sine Law

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine Law

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

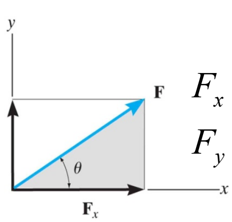
$$A = \sqrt{B^2 + C^2 - 2BC\cos a}$$



Scalar Notation

- When the forces are resolved along the x and y axes the components are called rectangular
- Scalar Notation:

The components of the force can be expressed in Scalar Notation using the parallelogram law. F can be resolved along x and y axes. The magnitudes of the components can be calculated with the angle (theta) or by using the slope of the force



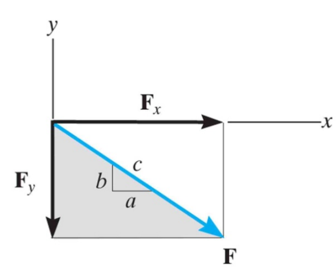
(a)

$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

$$F_x = F\left(\frac{a}{c}\right)$$

$$F_y = -F\left(\frac{b}{c}\right)$$



(b)

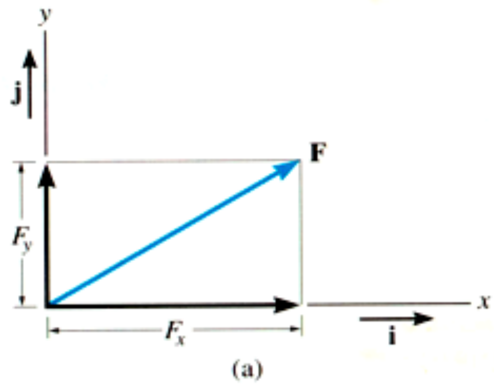
Cartesian Vector Notation

- Components of a Forces can also be expressed in Cartesian Vector Notation using unit vectors i and j ain the x and y axes, respectively.
- In Cartesian Vector Notation, F can be expressed as a vector as:

F = Fx i + Fy j

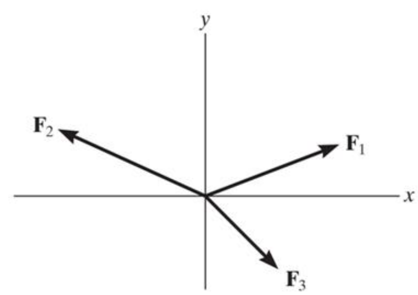
Fx and Fy are scalar quantities representing the magnitude of the components of the force, F.

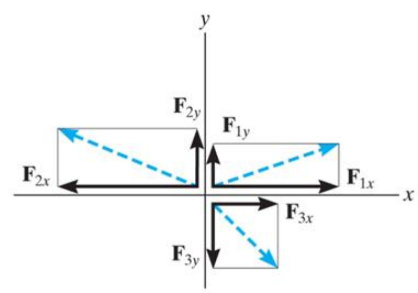
The unit vectors i and j designates the directions along x and y axes.



Resultant of Coplanar Forces

- Using either the Scalar or Cartesian Vector notation, the resultant of forces can be determined
- First resolve forces into their components (rectangular)
- Using scalar algebra, sum collinear vectors





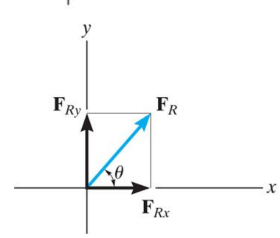
$\mathbf{F}_R = (F_{1x} - F_{2x} + F_{3x})\mathbf{i} +$
$(F_{1y} + F_{2y} - F_{3y})\mathbf{j}$
$= (F_{Rx})\ \mathbf{i} + (F_{Ry})\ \mathbf{j}$

(+) $F_{Rx} = F_{1x} - F_{2x} + F_{3x}$
(+) $F_{Ry} = F_{1y} + F_{2y} - F_{3y}$

$\mathbf{F}_R = F_{Rx} + F_{Ry}$

Resultant of Coplanar Forces

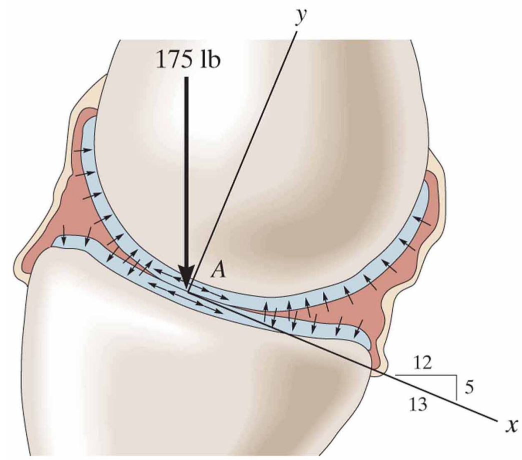
- The magnitude of the resultant as: $$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$



- While the direction of the resultant force is given as: $$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$$

Sample Problem - 2.35

The contact point between the femur and tibia bones of the leg is at A. If the vertical force of 175 lb is applied at this point, determine the components along the x and y axes



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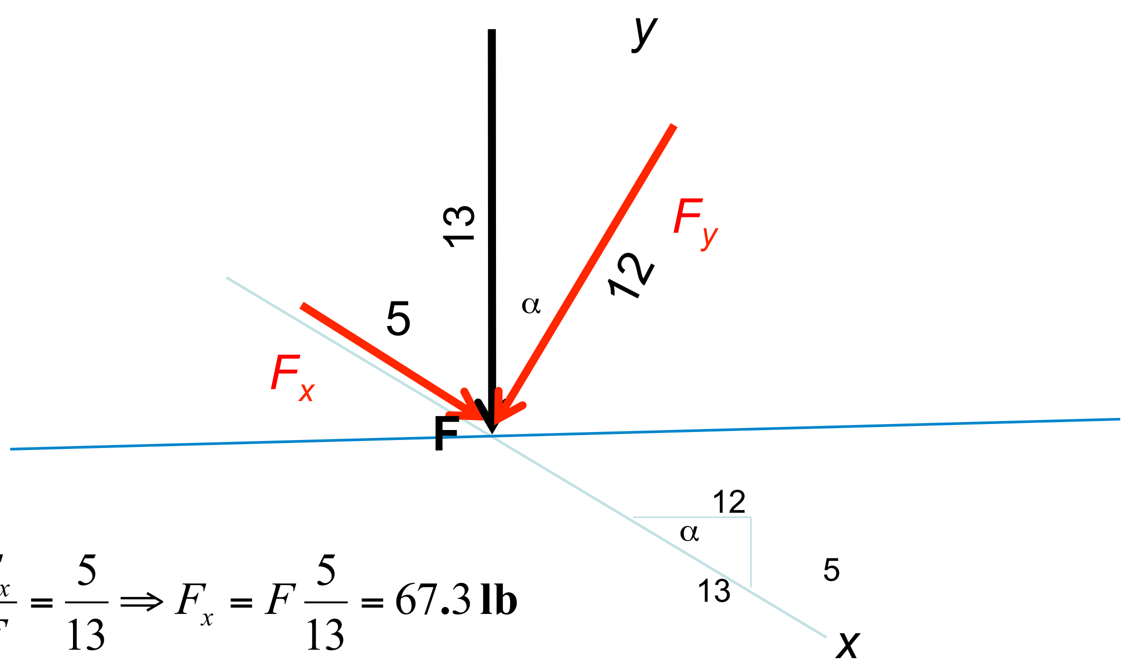
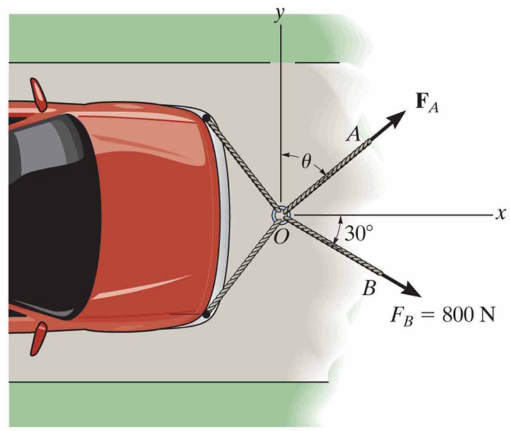


$$\frac{F_x}{F} = \frac{5}{13} \Rightarrow F_x = F\frac{5}{13} = 67.3\ \mathbf{lb}$$

$$\frac{F_y}{F} = \frac{12}{13} \Rightarrow F_y = F\frac{12}{13} = 162\ \text{lb}$$

Sample Problem - 2.48

Determine the magnitude and direction measured counterclockwise from the positive x axis of the resultant force acting on the ring at O if Fa = 750 N and angle is 45 degrees



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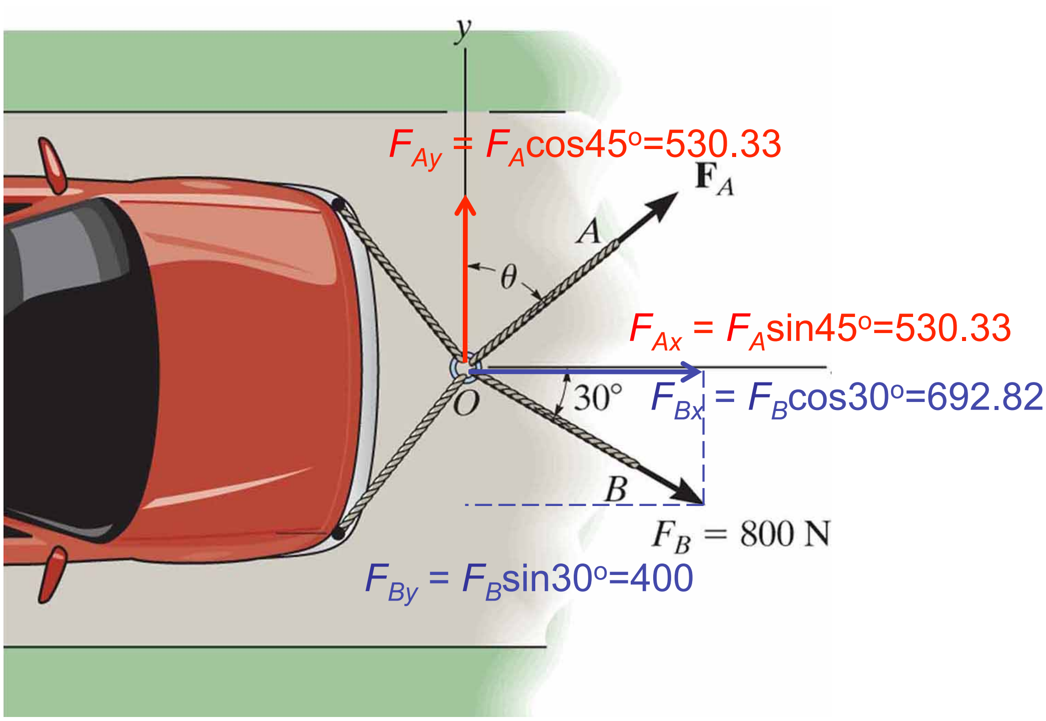
Solution 1 : Cartesian Vector Notation



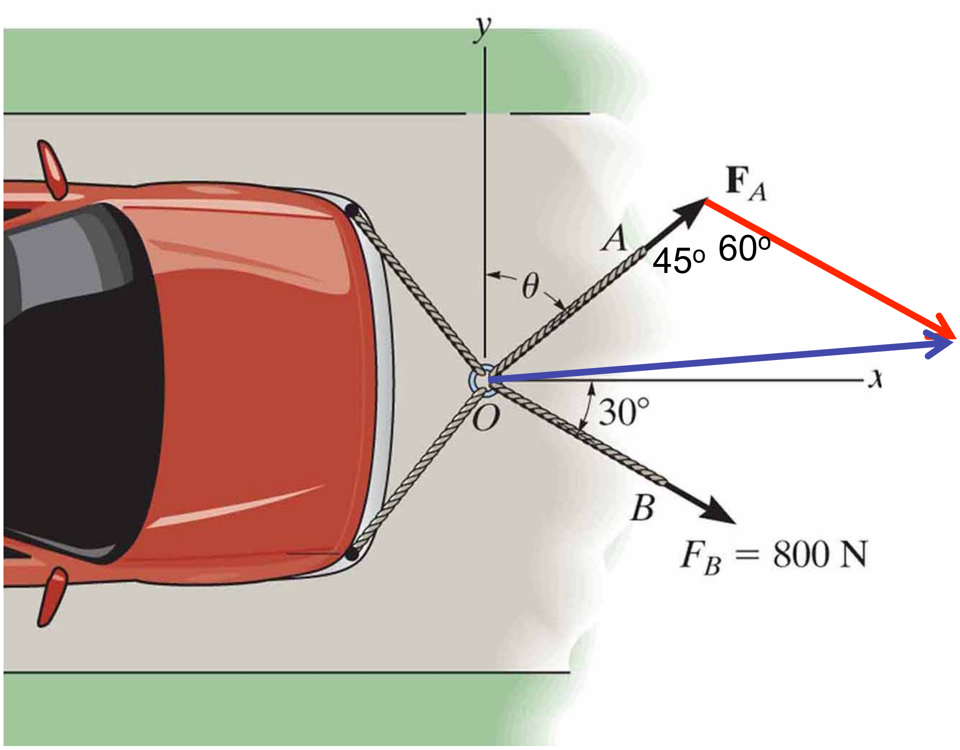
$$\mathbf{F} = (F_{Ax} + F_{Bx})\mathbf{i} + (F_{Ay} + F_{By})\mathbf{j}$$

$$\mathbf{F} = (530.33 + 692.82)\mathbf{i} + (530.33 - 400.00)\mathbf{j} = 1223.15\mathbf{i} + 130.33\mathbf{j}$$

$$F = \sqrt{1223.15^2 + 130.33^2} = 1230.1 \text{ N} = 1.23\times10^3 \text{ N} = 1.23 \text{ kN}$$

$$\beta = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08^o$$

Solution 2 - Sine and Cosine Laws



$$F = \sqrt{F_A^2 + F_B^2 - 2 \cdot F_A \cdot F_B \cos 105^o} = 1230.1 \text{ N} = 1.23 \text{ kN}$$

$$\frac{F_B}{\sin B} = \frac{F}{\sin 105^o} \Rightarrow B = \arcsin\left(\frac{800 \times \sin 105^o}{1230.1}\right) = 38.92^o$$

$$\beta = 45 - 38.92 = 6.08^o$$