

STAT 3502B

Test # 2

Last Name: _____ first Name: _____

Student No: _____

1. The density function of the time Z in minutes between calls to an electrical supply store is given by

$$f(z) = \begin{cases} \frac{1}{10}e^{-z/10} & 0 < z < \infty \\ 0 & \text{otherwise.} \end{cases}$$

- a. What is mean time and variance in the time between calls?
 b. What is probability that the time between calls exceeds the mean?

Sol:

This is exponential distribution with $\lambda = \frac{1}{10}$

- a) $E(X) = \frac{1}{\lambda} = 10, V(X) = \frac{1}{\lambda^2} = 100$
 b) $P(X > 10) = e^{-10/10} = e^{-1} = 0.3678794$

2. Given the joint density function

$$f(x, y) = \begin{cases} \frac{x(1+3y^2)}{4} & 0 < x < 2, 0 < y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find $f_Y(y), f(x|y)$, and evaluate $P(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3})$

Sol:

$$f_Y(y) = \int_0^2 \frac{x(1+3y^2)}{4} dx = \left. \frac{x^2}{8} + \frac{3x^2y^2}{8} \right|_0^2 = \frac{1}{2} + \frac{3}{2}y^2 = \frac{1+3y^2}{2} \quad 0 < y < 1$$

$$f(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{\frac{x(1+3y^2)}{4}}{\frac{1+3y^2}{2}} = \frac{1}{2}x$$

$$P(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{1}{3}) = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{2}x dx = \left. \frac{x^2}{4} \right|_{\frac{1}{4}}^{\frac{1}{2}} = \frac{3}{64}$$

3. [5] Suppose that a random sample of size n is drawn from the Bernoulli distribution

$$f(x; p) = \begin{cases} p^x(1-p)^{1-x} & x = 0, 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find a maximum likelihood estimator of p .

Solution:

$$f(x_1, \dots, x_n; p) = p^{\sum x_i} (1-p)^{\sum (1-x_i)}$$

$$\ln f(x_1, \dots, x_n; p) = \sum x_i(\ln p) + \sum(1 - x_i)(\ln(1 - p))$$

$$\frac{\partial \ln f(x_1, \dots, x_n, p)}{\partial p} = \frac{\sum x_i}{p} - \frac{\sum(1-x_i)}{1-p} = 0 \rightarrow \frac{\sum x_i}{p} = \frac{n - \sum x_i}{1-p} \rightarrow \hat{p} = \frac{\sum x_i}{n} = \bar{X}$$

4. The life of components of a certain type is normally distributed with a mean of 100 hours and a standard deviation of 10 hours.

(a) {5 marks}. What is the probability that a randomly selected component will last more than 85 hours?

Sol:

$$P(X > 85) = P(Z > \frac{85-100}{10}) = P(Z > -1.5) = 1 - P(Z < -1.5) = 1 - 0.0668 = 0.9332$$

(b) {5 marks}. What is the probability that the average life of a random sample of 36 components is greater than 103 hours?

Sol:

$$\bar{X} \text{ is normally distributed with } E(X) = 100 \text{ and } V(X) = \frac{100}{36}, \text{ so } P(\bar{X} > 103) = P(Z > \frac{103-100}{10/6}) = P(Z > 1.8) = 1 - P(Z < 1.8) = 1 - 0.9641 = 0.0359$$