UNIVERSITÉ D'OTTAWA • UNIVERSITY OF OTTAWA

Sciences • Science

Mathématiques et statistique
Mathematics and Statistics

MAT 1341A Mid Term Test

October 21, 2003. Duration: 80 minutes.

Instructor: Barry Jessup.

Family Name:_____

First Name:_____

Student number:_____

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PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators**, cell phones, pagers or any text storage or communication device **is not permitted.**
3. Read each question carefully -you will save yourself time and unnecessary grief later on.
4. Questions 1 to 3 are multiple choice. These questions are worth 2 points each and no part marks will be given. Please record your answers in the space provided above.
5. Questions 4 – 6 require a complete solution, and are worth 6 points each, so spend your time accordingly. **The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
6. Where it is possible to check your work, do so.
7. Good luck! Bonne chance!

1. If the augmented matrix $[A|b]$ of a linear system is row equivalent to $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$, which of the following statements is true?

A. The system is inconsistent
B. $(s, 1, 2)$ is the solution for any value of s
C. $(1, 1, 2)$ is the unique solution of the system
D. $(s, 1 - s,\ 2)$ is a solution for any value of s
E. $(s, 1 - s, 2 - s)$ is the solution for any value of s
F. $(0, 1, 2)$ is the unique solution to the system

2. Which of the following is a basis for the subspace $\{(x,\ y,\ z) \mid 2x - y + 3z = 0\}$ of $\mathbf{R}^3$?

A. $\{(1,\ 2,\ 0),\ (0,\ 3,\ 1)\}$
B. $\{(1,\ 0,\ 0),\ (0,\ 1,\ 0),\ (0,\ 0,\ 1)\}$
C. $\{(1,\ 2,\ 0)\}$
D. $\{(1,\ 0,\ 0),\ (1,\ 2,\ 0)\}$
E. $\{(3,\ 0,\ -2)\}$
F. $\{(-3,\ 0,\ 2),\ (1,\ 0,\ 0)\}$

3. Suppose X is a subspace of the vector space $\mathbf{M}_{2\,2}$ of 2×2 real matrices, that $X \neq \{0\}$ and that $X \neq \mathbf{M}_{2\,2}$. Which of the following statements are true?

I. X has a spanning set consisting of 4 vectors.
II. X has a linearly independent subset consisting of 4 vectors.
III. $1 \leq \dim X \leq 3$.
IV. X has a basis that spans $\mathbf{M}_{2\,2}$.
V. For all vectors u, v, w in X, $au + bv + cw = 0$ implies $a = b = c = 0$.

A. III & II
B. I & III
C. II & IV
D. III & V
E. I & IV
F. I & V

4. Suppose $a, c \in \mathbf{R}$ and consider the linear system in x, y and z:

$$\begin{array}{rcrcrcl} x & & & - & z & = & 2 \\ -x & + & y & + & z & = & a-2 \\ 2x & + & 3y & + & az & = & 3a+c \end{array}$$

a) If $[\,A\,|\,b\,]$ is the augmented matrix of the system above, find $\operatorname{rank} A$ and $\operatorname{rank}[\,A\,|\,b\,]$ for all values of a and c.

b) Using part (a), find all values of a and c so that this system has

(i) a unique solution,
(ii) infinitely many solutions, or
(iii) no solutions.

c) In case b(ii) above, give a geometric description of the set of solutions.

5. Let $\mathbf{F}[-\frac{\pi}{2}, \frac{\pi}{2}] = \{f \mid f : [-\frac{\pi}{2}, \frac{\pi}{2}] \to \mathbf{R}\}$ be the vector space of real-valued functions defined on $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Define four functions in $\mathbf{F}([-\frac{\pi}{2}, \frac{\pi}{2}])$ by

$$f(x) = 1, \quad g(x) = \cos x, \quad h(x) = \cos 2x, \text{ and } k(x) = \sin x, \quad \text{for all } x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

and let

$$W = \text{span}\{f, g, h\}.$$

a) Show that $\{f, g, h\}$ is linearly independent.

b) Give a basis for W and hence find $\dim W$.

c) Show that $k \notin W$. (Hint: Suppose it is and evaluate both sides at 2 conveniently chosen points.)

d) Find the dimension of $\text{span}\{f, g, h, k\}$.

6. State whether the following are true or false. You must justify your answer.

i) If V is a vector space and $\{v_1, v_2, v_3\} \subset V$ is linearly independent, then $\{v_1, v_1 + v_2 + v_3, v_3\}$ is also linearly independent.

ii) $\{(x, y) \in \mathbf{R}^2 \mid xy \geq 0\}$ is a subspace of $\mathbf{R}^2$.

iii) There is a linear system of 3 equations in 4 unknowns which has a unique solution.