MAT 1341E Assignment 1, 2012
Due: 5pm, 3-October, 2012.
Instructor: Aziz Khanchi

Last Name: ______________________________

First Name: ______________________________

Student # ______________________________

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Family Name:______________________
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(For the marker's use only →)

1	
2	
3	
4	
5	
6	
7	
[Bonus] 8	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 6 to 8, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 5 are worth 1 point each, and no part marks will be given. However, you must show some work to obtain the point. Simply writing the correct answer will earn you 0.
3. Questions 6 and 7 and are worth 6 points each, and part marks can be earned. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.** Question 8 is a bonus question and is worth 3 points. Earning points here will be much more difficult than in questions 1-7.
4. You have to submit this assignment by putting into the box marked **"MAT1341 Section E"** in the cabinet in the foyer of the math building (KED) on the due date, by 5pm.

1. Which of the following subsets of $\mathbf{R}^4$ are closed under (the standard operation of) multiplication by scalars?A. $\{(a,\ b,\ c,\ d)|\ a\ b\ c = 0\}$

B. $\{(a,\ b,\ c,\ d)|\ a = 1, b = 0\ \text{ and } c + d = 0\}$
C. $\{(a,\ b,\ c,\ d)|\ a > 1 \text{ and } b < 1\}$
D. $\{(a,\ b,\ c,\ d)|\ a > 0 \text{ and } b > 0\}$
E. $\{(a,\ b,\ c,\ d)|\ a - b + 2c = 0\}$

ANSWER

2. Which of the following are subspaces of $\mathbf{F}[\mathbf{R}] = \{f \mid f\ \mathbf{R} \to \mathbf{R}\}$?

$U = \{f \in \mathbf{F}[\mathbf{R}] \mid f(-1)f(1) = 0\}$ $V = \{f \in \mathbf{F}[\mathbf{R}] \mid f(1) + f(2) = 0\}$ $S = \{f \in \mathbf{F}[\mathbf{R}] \mid f(x) = f(-x), \forall x \in \mathbf{R}\}$ $T = \{f \in \mathbf{F}[\mathbf{R}] \mid f(1) \leq 0\}$

ANSWER

3. Which two of the following statements are true?

I. The span of any vector in $\mathbf{R}^3$ is a line through the origin.

II. The span of any two distinct vectors in $\mathbf{R}^2$ is all of $\mathbf{R}^2$.

III. A set of vectors $\{u, v, w\}$ in a vector space spans V if every vector in V is a linear combination of $u, u+v$ and $u+v+w$.

IV. The set $\{(1,1),(2,3)\}$ spans $\mathbf{R}^2$.

ANSWER	

4. For which value of s does the vector (6, 3, s) belong to the subspace of $\mathbf{R}^3$ spanned by $(1,2,3)$ and $(0,1,2)$?

ANSWER	

5. If we give $X = \mathbf{R}^2$ the *non-standard* operations

$$(x,\ y,) \oplus (x',\ y') = (x + x' - 1,\ y + y' + 2) \quad \text{(vector addition)}$$

and

$$k \odot (x,\ y) = (kx - k + 1,\ ky + 2k - 2) \quad \text{(multiplication by scalars)},$$

then X *is* a real vector space.

- What is the zero vector of X?
- If $\mathbf{v} = (x,\ y)$ is in X then what is $-\mathbf{v}$?

ANSWER

$0 =$	$-\mathbf{v} =$

6. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid x - y + z = 0\}$

a) Is W a subspace of $\mathbf{R}^3$?

b) Find a spanning set for W.

c) Give a complete geometric description of W.

(You must justify your answers.)

7. Consider the vector space $\mathbf{F}(\mathbf{R}) = \{f \mid f : \mathbf{R} \to \mathbf{R}\}$, with the standard operations. Recall that the zero of $\mathbf{F}(\mathbf{R})$ is the function that has the value 0 for all $x \in \mathbf{R}$.

Let $U = \{f \in \mathbf{F}(\mathbf{R}) \mid f(1) = f(-1)\}$ be the subspace of functions which have the same value at $x = -1$ and $x = 1$. Define functions g, h, j and $k \in \mathbf{F}[\mathbf{R}]$ by

$$g(x) = 2x^3 - x^2 - 2x + 1, \quad h(x) = x^3 + x^2 - x + 1, \tag{1}$$

$$k(x) = -x^3 + 5x^2 + x + 1 \quad \text{and} \quad j(x) = x^3 - x, \qquad \forall x \in \mathbf{R}. \tag{2}$$

a) Show that g and h belong to U.
b) Show that $k \in \operatorname{span}\{g, h\}$.
c) Show that $j \notin \operatorname{span}\{g, h\}$.
d) Show that $\operatorname{span}\{g, h\} \neq \operatorname{span}\{g, h, j\}$.

(You must justify your answers.)

8. [Bonus] Let $\mathbf{E} = \{\text{“}ax + by + cz = d\text{”} \mid a, b, c, d \in \mathbf{R}\}$ be the set of linear equations with real coefficients in the variables x, y and z. Equip $\mathbf{E}$ with the usual operations on equations that you learned in high school: addition of equations, denoted here by "$\oplus$" and multiplication by scalars, denoted here by "$\otimes$", as follows:

$$\text{“}ax + by + cz = d\text{”} \oplus \text{“}ex + fy + gz = h\text{”} = \text{“}(a + e)x + (b + f)y + (c + g)z = d + h\text{”}$$

and

$$\forall k \in \mathbf{R}, \quad k \otimes \text{“}ax + by + cz = d\text{”} = \text{“}ka\,x + kb\,y + kc\,z = k\,d\text{”}.$$

You may assume without proof that $\mathbf{E}$ is a vector space.

Find a spanning set for $\mathbf{E}$. *(You must justify your answer.)*