MAT 1341 Test 1

Summer 2007

June 4

Instructor: Charles Starling

Family Name:______________________

First Name:________________________

Student Number: ___________________

Question	Response	Points
1		
2		
3		
4		
5		
6	–	
7	–	
Total	–	

PLEASE READ THESE INSTRUCTIONS VERY CAREFULLY.

1. You have 80 minutes to complete this exam.
2. This is a closed book exam, and no notes of any kind are allowed. **Do not use your own scrap paper! Use the last page or the backs of pages for rough work**.
3. The use of calculators, cell phones, pagers or any text storage or communication device is not permitted.
4. Questions 1 through 5 are multiple choice. They are worth 2 points each and no part marks will be given. Please record your answers in the space provided above.
5. Questions 6 and 7 require a complete solution, and are worth 6 points each, so spend your time accordingly. **The correct answer requires justifcation written legibly and logically: you must convince me that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
6. Where it is possible to check your work, do so.
7. Good luck! Bonne chance!

1. Let $W = \{(x, y, z) \in \mathbb{R}^3 \mid xy = xz = 0\}$. Then,

A. $(0, 0, 0) \in W$ but W is not closed under multiplication by scalars.
B. W is closed under addition and W is closed under multiplication by scalars.
C. W is closed under addition but W is not closed under multiplication by scalars.
D. W is not closed under addition but W is closed under multiplication by scalars.
E. $(0, 0, 0) \notin W$ but W is closed under addition.
F. None of the other statements is true.

2. Which of the following are subspaces of $\mathbb{R}^3$?

(1) $\{(x, y, z) \mid 2x - y + 3z = 0\}$
(2) $\{(x, y, z) \mid xy = 0\}$
(3) $\{(x, y, z) \mid 2x = 5z\}$
(4) $\{(x, y, z) \mid (x/2) = (y + 3)/5 = 7z\}$

A. (1) and (2).
B. (1), (3) and (4).
C. (3) and (4).
D. (1) and (3).
E. (2) and (4).
F. (2) and (3).

3. Which two of the following are subspaces of $\mathbb{F}(\mathbb{R}) = \{f \mid f : \mathbb{R} \longrightarrow \mathbb{R}\}$?

$S = \{f \in \mathbb{F}(\mathbb{R}) \mid f(0) = 1\}$
$T = \{f \in \mathbb{F}(\mathbb{R}) \mid f(1) = f(2)\}$
$U = \{f \in \mathbb{F}(\mathbb{R}) \mid f(1)f(2) = 0\}$
$V = \{f \in \mathbb{F}(\mathbb{R}) \mid f(x) = f(-x) \text{ for all } x \in \mathbb{R}\}$

A. S and V.
B. T and U.
C. S and T.
D. T and V.
E. S and U.
F. V and U.

4. Suppose we know that Y is a subspace of $\mathbb{R}^9$ that can be spanned by 8 vectors, and that Y contains a linearly independent set with 6 vectors. Then,

A. $\dim Y < 6$
B. $\dim Y > 6$
C. $6 < \dim Y \leq 8$
D. $6 \leq \dim Y < 8$
E. $6 \leq \dim Y \leq 8$
F. None of the above is true.

5. Which of the following statements is true?

(1) If every vector in a vector space W is a linear combination of u and x, then $\{u, v, w, x\}$ spans W.
(2) A set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent if, for $a = b = c = d = 0$, we have $av_1 + bv_2 + cv_3 + dv_4 = 0$.
(3) A set of vectors $\{u, v, w\}$ is linearly independent if none of u, v or w is a multiple of one of the others.
(4) In a vector space V, a spanning set of V is always linearly dependent.
(5) If $\dim U = 1$, then every linearly independent set in U is a basis of U.

A. (2)
B. (3)
C. (1) and (2)
D. (4)
E. (1)
F. (1) and (5)

6. Let $v = (0, 1, -2)$ and $U = \{u \in \mathbb{R}^3 \mid \mathrm{proj}_v u = 0\}$.

a) Show that if $u = (x, y, z) \in \mathbb{R}^3$, then $\mathrm{proj}_v u = \frac{y-2z}{5}(0, 1, -2)$.
b) Find a cartesian equation for U. (i.e. find an equation that every $u = (x, y, z)$ in U satisfies.)
c) Give a complete geometric description of U. Is U a subspace of $\mathbb{R}^3$?
d) Find a spanning set for U.

7. State whether the following are true or false. You must justify your answer: if true, explain why, if not, give an example to show it is false.

a) If V is a vector space and $\{v_1, v_2, v_3\} \subseteq V$ is linearly dependent, then $\{v_1, v_2\}$ is also linearly dependent.

b) If u_1, u_2 and u_3 are non-zero vectors in a vector space V, and $U = \operatorname{span}\{u_1, u_2, u_3\}$ then $\dim U = 3$.

c) If W and X are subspaces of $\mathbb{R}^2$, then their intersection $W \cap X$ is also a subspace of $\mathbb{R}^2$.

d) Every linear system of 2 equations in 2 variables has a unique solution.

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