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CHG 2312-Assignement #3-Solutions

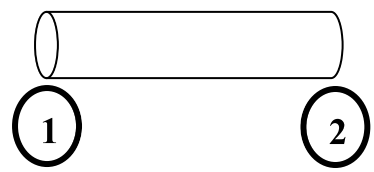
Question 1:

Given:

- Liquid drug with viscosity and density of water: μ = 0.890x10-3 Pa.s, ρ = 1000 kg/m^3
- Smooth think capillary of D = 0.25 mm & Δx = 50 mm

Assumptions:

- Steady state systems
- Incompressible flow (liquid with water properties
- Newtonian fluid (liquid with water properties)



Find:

a) Maximum volume flow rate to stay in the laminar region
b) Pressure drop
c) Wall shear stress

Solution:

a) Flow is required to be laminar inside the needle, therefore the Reynolds number: Re < 2000

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{Q}{A}\frac{\rho D}{\mu} = \frac{Q}{\left(\pi D^2/4\right)}\frac{\rho D}{\mu} = \frac{4Q\rho}{\mu\pi D} < 2000$$

$$Q < \frac{(2000)\mu\pi D}{4\rho} = \frac{(2000)(0.89x10^{-3}Pa.s)(3.14)(0.00025m)}{4(1000kg/m^3)} = 3.5x10^{-7}\frac{m^3}{s}$$

b) The needle is a straight piece of pipe thus:

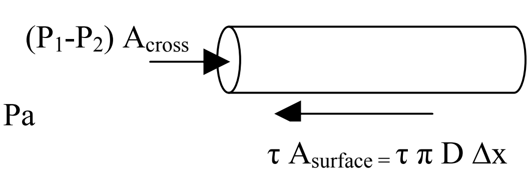
$$\boldsymbol{F} = \frac{-\Delta P}{\rho} = Q\Delta x\frac{\mu}{\rho}\frac{128}{\pi D_0^4}$$

$$P_1 - P_2 = Q\Delta x\mu\frac{128}{\pi D_0^4} = (3.5x10^{-7}\frac{m^3}{s})(0.05m)(0.89x10^{-3}Pa.s)\frac{128}{(3.14)(0.00025m)^4} = 162{,}535.5Pa$$

c) Fluid flowing in the laminar regime inside the needle, thus the pressure forces acting on both sides will counter balance the sheer force

$$\tau = \text{shear stress at } r = \frac{-r\left(P_1 - P_2\right)}{2\Delta x}$$

$$\tau = \frac{-(0.00025/2m)(162535.5Pa)}{2(0.05m)} = 203.169 \text{ Pa}$$



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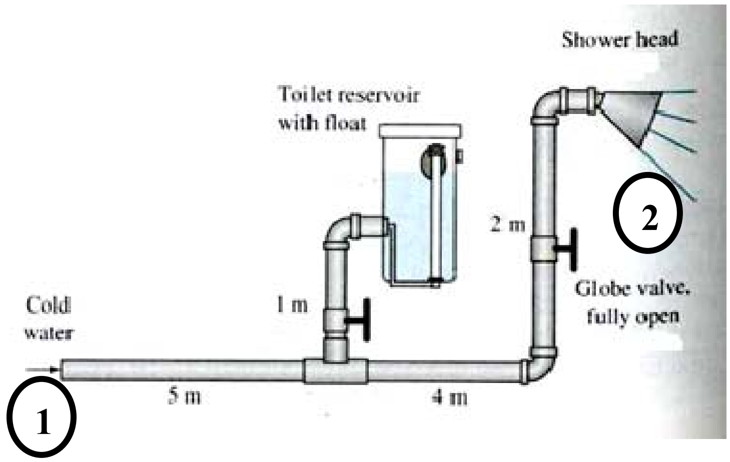
Question 2:



Given:

- All pipe diameters and length, P_1 = 200 kPa, K_{shower} = 12, ρ = 998 kg/m^3 and μ = 1.002x10^{-3} kg/m.s

Assumptions:

- Steady state systems
- Incompressible flow (water in system)
- Newtonian fluid (water)

Find:

- Volumetric flow ate of water in through the shower head (Point 2)

Solution:

Applying B.E. from points 1 (inlet of the water to the pipe) to point 2 (end of shower head)

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{(V_2^2 - V_1^2)}{2} = \frac{dW}{dm} - F$$

$$P_2 = 0 \qquad \textbf{\textit{open to atmospher}}$$

$$\frac{dW}{dm} = 0$$

Then apply mass balance from point 1 to 2 and assuming the shower head has the same diameter as the pipe, then

$$A_1 V_1 = A_2 V_2 \quad \text{and } V_1 = V_2$$

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$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) = -F$$

$$F = h_m + h_l$$

$$h_m = 4f\frac{\Delta X}{D}\frac{V^2}{2} = 4f\left[\frac{(5+4+2)m}{0.015\,m}\right]\frac{V^2}{2} = 1466.65\,fV^2$$

$$\frac{\varepsilon}{D} = \frac{0.00085\ ft}{0.015\,m\,\frac{1\,ft}{0.3048\,m}} = 0.01727$$

$$h_l = \sum k \cdot \frac{V^2}{2} = (12 + 0.4 + 2x0.74 + 6.3)\frac{V^2}{2} = 10.09V^2$$

$$\frac{0 - 200{,}000\text{Pa}}{1000\,\frac{kg}{m^3}} + (9.8\frac{m}{s^2})(2m) = -(1466.65\,fV^2 + 10.09V^2)$$

$$180 = (1466.65\,fV^2 + 10.09V^2)$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(1000)(V)(0.015\,m)}{1.002\text{x}10^{-3}\ \text{kg/m.s}} = (14{,}970)V$$

This is an iterative problem now, so start with guessing an initial friction factor which could be the minimum you see on the moody chart, and then calculate the velocity from equation above, then Reynolds number and finally the new friction fact. You can enter the equation for friction factor as per lecture notes for f(new) in table below. You can set up this in Excel and here would be the results you would obtain.

Iteration #1	*f(old)*	*V (m/s)*	*Re*	*ε/D*	*f(new)*
1	0.0020000000	3.7177118874	4608734	0.0172720000	0.0110247888
2	0.0110247888	2.6181406431	3245629	0.0172720000	0.0110256367
3	0.0110256367	2.6180786491	3245552	0.0172720000	0.0110256368
4	0.0110256368	2.6180786442	3245552	0.0172720000	0.0110256368
5	0.0110256368	2.6180786442	3245552	0.0172720000	0.0110256368

Thus, V = 2.618 m/s and then volumetric flow, Q = V. (π D^2/4) = 0.000462 m^3/s.

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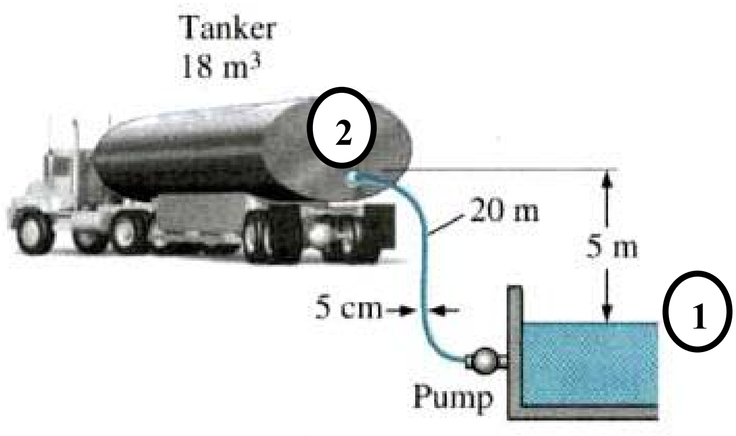
Question 3 :



Given:

- All pipe diameters and length, $P_1 = P_2 = 0$ (gauge), $\rho = 920\ kg/m^3$ and $\mu = 0.045$ kg/m.s, 2 elbows and one sudden expansion where the pipe attaches to the tanker, h = 5m, D = 0.05m

Assumptions:

- Steady state systems
- Incompressible flow (oil)
- Newtonian fluid (oil)
- Smooth pipe
- 100% pump efficiency

Find:

- Pressure at the outlet of the pump

Solution:

Applying B.E. from points 1 (top surface of the oil reservoir) to point 2 (where the hose is connected to the tanker)

$Q = (18\ m^3/\ 30\ min) = 0.6\ m^3/min = 0.01\ m^3/s$

$$V_2 = \frac{Q}{A} = \frac{0.01\,\frac{m^3}{s}}{\pi\,\frac{(0.05\,m)^2}{4}} = 5.093\,m/s$$

$$\text{Re} = \frac{\rho V_2 D_2}{\mu} = \frac{(920\,kg/m^3)(5.093\,m/s)(0.05\,m)}{(0.045\,Pa.s)} = 5206$$

Since Re > 4000, then it is a turbulent flow.

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Since it is a **smooth pipe**, pipe ***relative roughness = 0.000001***

$$.f = 0.001375\left[1+\left(20{,}000\frac{\varepsilon}{D}+\frac{10^6}{\mathrm{Re}})^{1/3}\right)\right] = 0.0093$$

Assuming $V_1 << V_2$

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{(V_2^2 - V_1^2)}{2} = \frac{dW}{dm} - F$$

$$g(z_2 - z_1) + \frac{(V_2^2)}{2} = \frac{dW}{dm} - F$$

$$F = h_m + h_l$$

$$F = \left(4f\frac{\Delta X}{D} + \sum k\right)\frac{V^2}{2} = \left(4(0.0093)\left[\frac{(20)m}{0.05m}\right] + (2x0.74+1)\right)\frac{(5.093\,m/s)^2}{2}$$

$$F = 225.147\frac{m^2}{s^2}$$

$$\frac{dW}{dm} = g(z_2 - z_1) + \frac{(V_2^2)}{2} + F$$

$$\frac{dW}{dm} = (9.8m/s^2)(5m) + \frac{(5.093\,m/s)^2}{2} + 225.147\frac{m^2}{s^2} = 287.12\frac{m^2}{s^2}$$

If you write a B. Equation just around the pump, knowing that the inlet and outlet of pump are at the same height and the inlet and outlet diameter of the pump is the same, and pump is frictionless:

$$\frac{P_{out} - P_{in}}{\rho} + g(z_{out} - z_{in}) + \frac{(V_{out}^2 - V_{in}^2)}{2} = \frac{dW}{dm} - F$$

$$\frac{P_{out} - P_{in}}{\rho} = \frac{dW}{dm}$$

Now we can substitute the ***dw/dm*** found above in the pump equation, and knowing that the pump inlet is very close to the top of reservoir we can assume $P_{inlet} = 0$ gauge, the we can find the $P_{outlet:}$

$$\frac{P_{out} - 0}{920\frac{kg}{m^3}} = 287.12\frac{m^2}{s^2}$$

$$P_{outlet} = 264150\ Pa = 264.2kPa$$

Note that you could have also solved the problem from the beginning by taking the inlet as the outlet of the pump and the outlet as the top of the tank. That way, there was no work term in the B. E. and you could have solved for P at the outlet of the pump.

Question 4:

Given:

- Crude oil:
    - $T = 140$ ˚F, S.G.= 0.93, $\mu = 16.75$ cP, $P_{vapour} = 50$ psi
    - Q = 1.6 Mbpd
- Pipeline of
    - ID = 48 inch, roughness of galvanized iron ($\varepsilon = 0.006$ inch),
    - Allowable pressure : 1200 psi

Assumptions:

- Steady state systems
- Incompressible flow
- Newtonian fluid

Find:

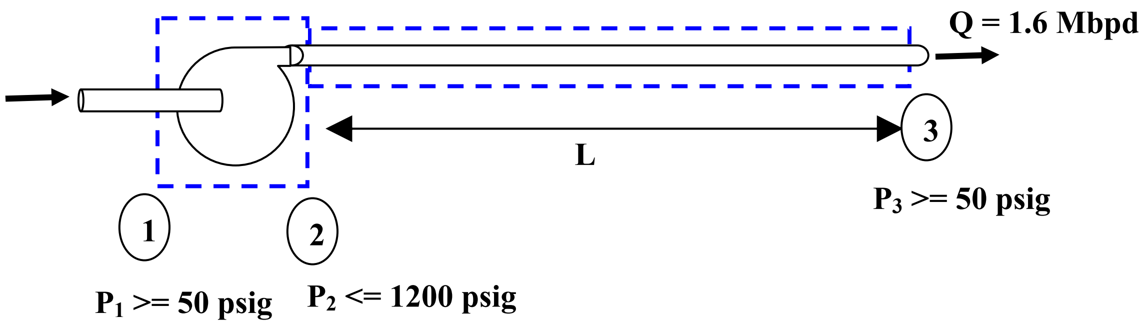
a) Maximum possible spacing between pumping stations
b) Pump power

Solution:

As shown in diagram below, we assume that the Alaskan pipeline is made up of repeating pump-pipe sections.

Point 1 is at the suction (inlet) of the pump, point 2 is at the discharge (outlet) of the pump and point 3 is downstream where the pipe approaches the second pumping station.

As the oil flows from point 2 to point 3 inside the pipe, the pressure drops. Thus the lowest pressure point between points 2 and 3 will be point 3. As we know the pressure drop of the liquid should not fall below its vapour pressure to avoid cavitations. Therefore, the pressure at point 3 has to be >= of vapour pressure of oil which is 50 psig.



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First apply B.E. from point 2 to 3, where it is a straight piece of pipe with fluid going through at constant density and viscosity.

$$\frac{P_3 - P_2}{\rho} + g(z_3 - z_2) + \frac{(V_3^2 - V_2^2)}{2} = \frac{dW}{dm} - F$$

$$\frac{P_3 - P_2}{\rho} = -F = 4f\frac{L}{D}\frac{V^2}{2}$$

$$L = \frac{D}{2fV^2}\frac{P_3 - P_2}{\rho}$$

$$Q = (1.6x10^6 \frac{bbl}{day})(\frac{42\ USG}{1bbl}\frac{0.003785\text{m}^3}{1\ \text{USG}})(\frac{1day}{24*3600\ s}) = 2.94\frac{m^3}{s}$$

$$V = \frac{Q}{A} = \frac{2.94\frac{m^3}{s}}{\pi\frac{(\frac{48\,in}{39.37\,in/m})^2}{4}} = 2.5m/s$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(0.93*1000\,kg/m^3)(2.5m/s)(48in/39.37\,inch/m)}{(16.75*0.001\,Pa.s)} = 1.7x10^5$$

Now from the Moody diagram you can find the friction factor at above Reynolds number and ε/D = 0.000125 to be *f =0.0041 (you can also use the equation representing the lines in Moody diagram and you would get f = 0.004168)*

$$L = \frac{D}{2fV^2}\frac{P_3 - P_2}{\rho} = \frac{(\frac{48\,in}{39.37\,in/m})(1200\ psi - 50\ psi)(\frac{101325\ Pa}{14.7\ psi})}{2(0.0041)(2.5m/s)^2(0.93*1000\ kg/m^3)}$$

$$\boldsymbol{L = 2.0277x10^5\,m = 202.77km}$$

Now we can move to apply B.E. from point 1 to 2:

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{(V_2^{\,2} - V_1^{\,2})}{2} = \frac{dW}{dm} - F$$

Assuming that the inlet and outlet of the pump have similar diameters then based on the mass balance:

$$V_1 = V_2$$

Also it is usually assumed that the pump friction is negligible, F = 0

$$\frac{P_2 - P_1}{\rho} = \frac{dW}{dm}$$

$$W_{pump} = \frac{\dot{m}\,\Delta P}{\rho} = Q\Delta P$$

Pump efficiency is usually defined as the ratio of the power required for the system over the imput power to the pump

$$\eta = \frac{W_{pump}}{W_{in}}$$

$$W_{pump} = Q\Delta P = (2.94\frac{m^3}{s})(1200psi - 50psi)(\frac{101325Pa}{14.7psi}) = 23304750W = 2330.5kW$$

Then the required power input to the pump then will be:

$$W_{input} = \frac{W_{pump}}{\eta} = \frac{2330.5kW}{0.85} = 27417.35kW = 27.4MW$$