Université d'Ottawa
Faculté de génie

Département de
Génie Chimique

University of Ottawa
Faculty of Engineering

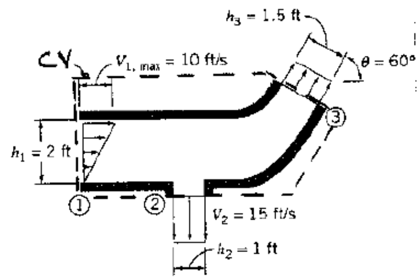
Department of
Chemical & Biological Engineering

CHG 2312 – Fluid Flow

Solution-Assignment #2

Question 1

Given: Two-dimensional reducing bend as shown.



Assumptions:

1- Steady flow
2- Incompressible flow
3- Uniform flow at 2 and 3

Find: Magnitude and direction of uniform velocity at section 3.
Solution: Apply conservation of mass using control volume shown.

$$\frac{dm_{C.V.}}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0 \qquad \text{(at steady state)}$$

$$0 = \rho \dot{Q}_{in} - \rho \dot{Q}_{out}$$

$$0 = \dot{Q}_1 - \dot{Q}_2 - \dot{Q}_3 \qquad \text{(assuming that the flow at point 3 is outward)}$$

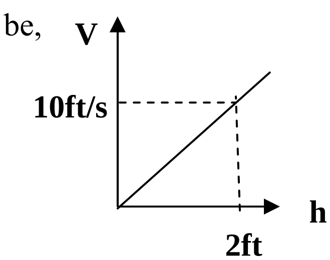
Then, by considering assumption #3, we can write,

$$0 = \int_{Area} V_1 dA_1 - V_2 A_2 - V_3 A_3$$

<u>Note that at point 1, since the velocity is changing with respect to height and thus area, the integral form of the Q is used. At point 2 and 3 the fluid velocity is assumed uniform.</u>

At point 1, as stated in the question, the velocity has a linear profile which will be,

V = slope h → slope = $V_{1,max}/h_1$



Therefore, substitute this back into the equation above, and knowing that $dA_1 = w.dh$, where w is the width of the channel.

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$$0=\int_0^{h_1}\frac{V_{1,\max}}{h_1}h\ w\,dh\ -V_2\ \ w\,h_2-V_3\ w\,h_3\ =\left(\frac{V_{1,\max}}{h_1}\right)\left[\frac{h^2}{2}\right]_0^{2\,ft}-V_2\ \ w\,h_2-V_3\ w\,h_3$$

$$V_3=\frac{\left(\frac{V_{1,\max}}{h_1}\right)\left[\frac{h^2}{2}\right]_0^{2\,ft}-V_2\ \ h_2}{h_3}$$

$$V_3=\frac{\left(\frac{10\,ft/s}{2\,ft}\right)\left(\frac{4\,ft^2}{2}\right)-(15\,ft/s)(1\,ft)}{1.5\,ft}$$

$$V_3=-3.3\,ft/s$$

<u>Since the velocity was found to be negative then the flow at point 3 must be into the control volume.</u>

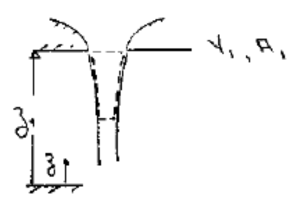
Question 2:

<u>Given:</u>

1- Liquid stream leaving a nozzle pointing downward as shown
2- Frictionless flow
3- Uniform flow

<u>Assumption:</u>

1- Steady flow
2- Incompressible flow
3- Flow along a streamline
4- $P=P_1=P_{atm}$



<u>Find:</u> Variation in jet area for z, z_0

<u>Solution:</u>

(Note: Both velocity and projected area (Normal vector) are vectors in the following equations)

$$\frac{P_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{P}{\rho}+\frac{V^2}{2}+gz$$

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0 (Steady state)

$$0 = \frac{\partial}{\partial t}\int_{cv} \rho \, dV + \int_{cs} \rho \, V . dA$$

From the Bernoulli equation

$$V^2 = V_1^2 + 2g(z_1 - z)$$

From the continuity equation

$$0 = \int_{cs} \rho V . dA = -\{\rho V_1 A_1\} + \{\rho V A\}$$

And

$$V_1 A_1 = VA \qquad \text{or} \qquad V = V_1 \frac{A_1}{A}$$

Thus

$$V_1^2 \left(\frac{A_1}{A}\right)^2 = V_1^2 + 2g(z_1 - z)$$

Solving for A

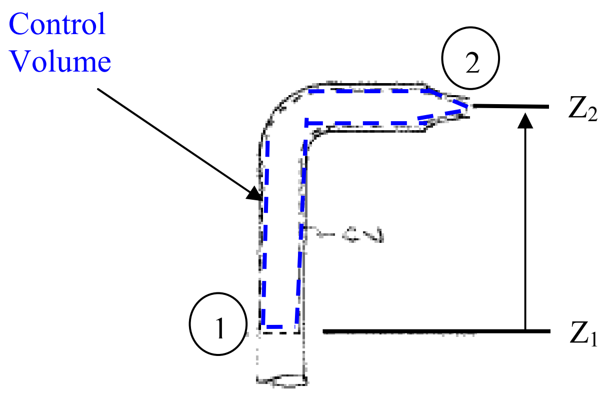
$$A = A_1 \sqrt{\frac{1}{1 + \frac{2g(z_1 - z)}{V_1^2}}}$$

Note: Jet area decreases as z decreases, owing to the higher velocity. You can observe such behaviour from any water tap in kitchens or bathrooms.

Question 3:

Given:

D_1=0.1 m
D_2=0.05 m
P_2=P_{atm}
V_2=20m/s
Z_1=0 m
Z_2 = 4m



Find:

1) Gauge Pressure, P_1
2) Gauge Pressure, P_1, if device was inverted

Assumptions:

1) Steady Flow

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Solution-Assignment #2

2) Incompressible Fluid
3) Frictionless Flow
4) P_2 gauge = 0
5) z_1=0 m
6) no external work

Solution:

Apply continuity to CV shown to determine V_1; the Bernoulli equation is then applied along a streamline from 1 to 2 to determine P_1.

Basic Equations:

$$\frac{dm_{cv}}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0 \qquad \text{(Continuity)}$$

$$\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_1 - z_2) = \frac{dW}{dm} - F \qquad \text{(Bernoulli)}$$

From the continuity equation:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \Rightarrow V_1 A_1 = V_2 A_2 \text{ or } V_1 = \left(\frac{D_2}{D_1}\right)^2 \cdot V_2$$

From the Bernoulli equation:

$$P_1 = \rho\left(\frac{V_2^2 - V_1^2}{2} + gz_2\right) = \rho\left(\frac{V_2^2}{2}\left(1 - \frac{V_1^2}{V_2^2}\right) + gz_2\right) = \rho\left(\frac{V_2^2}{2}\left(1 - \left(\frac{D_2}{D_1}\right)^4\right) + gz_2\right)$$

$$P_1 = (999\frac{kg}{m^3})\left(\frac{(20\frac{m}{s})^2}{2}\left(1 - \left(\frac{0.05m}{0.1m}\right)^4\right) + 9.81\frac{m}{s^2} \cdot 4m\right) = 227\frac{kN}{m^2} = 227kPa\,(gauge)$$

If the device is inverted, z_2= -4 m with z_1=0 m

$$P_1 = 999\frac{kg}{m^3}\left(\frac{(20\frac{m}{s})^2}{2}\left(1 - \left(\frac{0.05m}{0.1m}\right)^4\right) + (9.81\frac{m}{s^2}) \cdot (-4m)\right) = 148\frac{kN}{m^2} = 148\,kPa\,(gauge)$$

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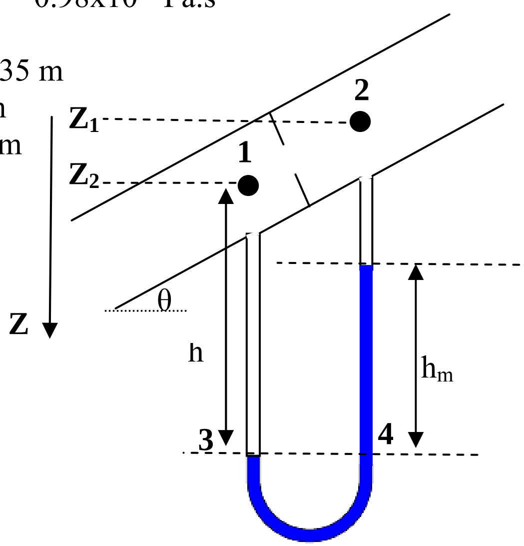
Question 4:

$\rho_w = 999\ kg/m^3$, $\mu_w = 0.98 \times 10^{-3}$ Pa.s
$\rho_m = 13600\ kg/m^3$
$h_m = 135$ mm = 0.135 m
$D_2 = 6$ cm = 0.06 m
$D_1 = 10$ cm = 0.10 m
$\theta = 30°$



Assumptions: Steady-state flow, Incompressible fluid, inviscid fluid
Mass balance from point 1 to 2:

$$\frac{dm_{C.V.}}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0$$

$$\rho V_1 A_1 = \rho V_2 A_2 \qquad \Rightarrow \quad V_1 = \frac{A_2}{A_1} V_2 = \frac{D_2^2}{D_1^2} V_2$$

Energy balance, B.E.:

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) - W + F = 0 \qquad where\ W = 0,\ F = 0$$

$$\Delta\left(\frac{P}{\rho} + gz + \frac{V^2}{2}\right) = 0$$

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^2 - V_1^2}{2} = 0$$

$$\frac{P_2 - P_1}{\rho} + g(z_2 - z_1) + \frac{V_2^2}{2}(1 - \frac{D_2^4}{D_1^4}) = 0$$

$$V_2^2 = 2\left(\frac{P_2 - P_1}{\rho} + g(z_2 - z_1)\right) / (1 - \frac{D_2^4}{D_1^4})$$

$$V_2 = \left(\frac{2\frac{P_1 - P_{21}}{\rho} + 2g(z_1 - z_2)}{(1 - \frac{D_2^4}{D_1^4})}\right)^{1/2}$$

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Solution-Assignment #2

Determine $P_1 - P_2$ from manometer reading:

$$P_1 + \rho_w g h - \rho_m g h_m - \rho_w g (h - h_m + (z_2 - z_1)) - P_2 = 0$$
$$P_1 - P_2 = \rho_m g h_m - \rho_w g h_m + \rho_w g (z_2 - z_1))$$

Mass flow rate in the orifice is:

$$\dot{m} = \rho V_2 A_2 = \rho \pi \frac{D_2^2}{4} C_v \left(\frac{2\frac{P_1 - P_2}{\rho} + 2g(z_1 - z_2)}{(1 - \frac{D_2^4}{D_1^4})} \right)^{1/2} = \rho \pi \frac{D_2^2}{4} C_v \left(\frac{\frac{2\rho_m g h_m - 2\rho g h_m + 2\rho g (z_2 - z_1)}{\rho} + 2g(z_1 - z_2)}{(1 - \frac{D_2^4}{D_1^4})} \right)^{1/2}$$

$$\dot{m} = C_v \rho \pi \frac{D_2^2}{4} \left(\frac{2 g h_m (\rho_m - \rho)}{\rho (1 - \frac{D_2^4}{D_1^4})} \right)^{1/2}$$

Determine C_v:

1. find V_2 without C_v

$$V_2 = \left(\frac{\frac{2\rho_m g h_m - 2\rho g h_m}{\rho}}{(1 - \frac{D_2^4}{D_1^4})} \right)^{1/2} = \left(\frac{2\,(9.8)\,(0.135m)(13600 - 999)}{(999)(1 - (\frac{0.06}{0.1})^4)} \right)^{1/2} = 6.2 m/s$$

2. find Reynolds:

$$\text{Re} = \frac{\rho D_2 V_2}{\mu} = \frac{(999)(0.06)\,(6.2)}{(0.89x10^{-3})} = 417559$$

3. read C_v from graph
   $C_v = 0.61$
4. Calculate new V_2 by including C_v, and then new Re number:
   $V_2 = 3.8$ m/s
   Re = 243249
   New $C_v = 0.61$
   Since new Cv and old Cv are similar then no more calculation is required and $V_2 = 3.8$ m/s

Determine mass flow rate:

$$\dot{m} = \rho \pi \frac{D_2^2}{4} V_2 = (999)\, \pi\, (0.06)^2\, (3.8)/4 = 10.73 kg/s$$

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Solution-Assignment #2

Question 5

Given:

Fluid: Air
$D_2 = 3$ inch
$D_1 = 6$ inch
$P_1 = 60$ psi
$T_1 = T_2 = 68$ °F = 20°C

Assumptions:

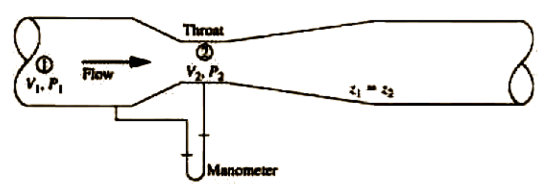
1- Density change is negligible
2- Ideal gas
3- Steady state system
4- Frictionless Flow

Find: Maximum flow rate for incompressible flow; Pressure reading

Solution:

In order to determine the mass flow rate of the air in the venturi meter, we could apply the B.E. equation. However, for any gas to be counted as an incompressible flow (important criteria for using B.E. equation), its velocity has to be low. In your text book this limit has been set at 200 ft/s = 63 m/s, whereas other literature use velocity of 100 m/s. The later number has been used here.

Looking at a venturi meter arrangement and again for applying B.E., then the maximum gas velocity in the system will be 100 m/s. But now the question would be where would the gas has a maximum velocity (i.e., at upstream of piping or at the throat).



As we know the highest velocity will be at the throat of the venturi since it has the smallest diameter. Therefore, $V_2 = 100$ m/s. Therefore, we can easily calculate the maximum mass flow rate as:

$$\dot{m} = \rho V_2 A_2$$

$$\rho = \frac{PM}{RT} = \frac{(60\,psi\,\frac{101325 Pa}{14.7\,psi})(29 kg/kmol)}{(8.314\times10^3\,\frac{m^3 Pa}{K.kmol})(20+273)K} = 4.9 kg/m^3$$

$$\dot{m} = (4.9 kg/m^3)(100 m/s)(\pi \times (3 inch\,\frac{1m}{39.37 inch})^2/4) = 2.23\,\frac{kg}{s}$$

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The pressure drop (i.e. reading on the manometer) can be found from the velocity equation of the venturi meter:

$$V_2 = C_v \sqrt{\frac{2(P_1 - P_2)/\rho_{air}}{\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}}$$

$$(P_1 - P_2) = \frac{\rho_{air} V_2^2 \left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}{2C_v^2}$$

Since we assumed air is incompressible flow in this problem, then its density will stay constant at what was calculated above.

We need to find the C_v factor, in which Reynolds number needs to be calculated. Viscosity of air at 68 ˚F = 20 ˚C is found to be about $1.9\text{x}10^{-5}$ Pa.s.

$$\mathrm{Re} = \frac{\rho V_1 D_1}{\mu} = \frac{\rho(\frac{V_2 A_2}{A_1})D_1}{\mu} = \frac{(4.9\frac{kg}{m^3})(100m/s(\frac{3inch}{6inch})^2(6inch\frac{1m}{39.37inc})}{1.9\times10^{-5}\,Pa.s} = 9.8\times10^5$$

From charts of C*v* vs. Reynolds number, you can find the Cv to be about 0.985.

$$(P_1 - P_2) = \frac{(4.9\frac{kg}{m^3})(100\frac{m}{s})^2\left(1 - \left(\frac{3inch}{6inch}\right)^4\right)}{2(0.985)^2} = 23673.63Pa = 23.7kPa$$

Therefore, the manometer reading (Δh) will be :

$$\Delta \mathrm{P} = \rho_{\mathrm{Hg}} g \Delta h$$

$$\Delta h = \frac{\Delta P}{\rho_{\mathrm{Hg}} g} = \frac{23673.63Pa}{(13600kg/m^3)(9.8m/s^2)} = 0.177m$$