Student Name (last) ______________________________________________

(first) ______________________________________________

Student Number ____________________  Section (A = Bryant, C = Wallace) ______

**Question 1 (8 marks)**

You have been told that the function below has a root somewhere between 8 and 10.

$$f(x) = 60 - 4^{(x/3)}$$

i) Confirm that there is in fact a root in this range by performing the appropriate check.

(2 marks) f(8) = 19.68, f(10) = -41.59, as there is a sign change a root should exist. Actually finding the root is not "performing the appropriate check" and students who do this should get zero marks.

ii) Show your understanding the bisection search process by completing the following table:

| **Step** | **$X_{LOW}$** | **$X_{HIGH}$** | **$X_{ROOT}$** | **$f(x_{ROOT})$** | **$E_{MAX}$** |
|---|---|---|---|---|---|
| 1 | 8 | 10 | 9 | -4 | 1 |
| 2 | 8 | 9 | 8.5 | 9.203166 | 0.5 |
| 3 | 8.5 | 9 | 8.75 | XXXX | 0.25 |

(3 marks) One mark for picking $X_{ROOT}$ in the middle (twice), one mark for moving the correct wall (twice), and one mark for getting $E_{MAX}$ right.

(iii) How many ADDITIONAL guesses would it take to reduce EMAX to less than 1e-6?

(1 mark) The formula says that the desired accuracy will be achieved on the 21st guess (20.93 rounded upwards). This means 18 additional guesses. Check: each guess reduces the error by a factor of two : 0.25/2^18 = 0.95e-6.

(iv) If a Regula Falsi search were used instead of a bisection search what would the first guess at $X_{ROOT}$ be?

(1 mark) 8.642

(v) Suppose that at some point in another bisection search $X_{LOW}$ is 50 and $E_{MAX}$ is 0.006. What is $X_{HIGH}$?

(1 mark) 50.012, from ($X_{LOW}$ + 2 * $E_{MAX}$)

**Question 2 (3 marks)**

Convert all of the following word problems into root finding problems (i.e. put each of them into root finding form).

i) The load on some component (in Newtons) is given by function $L(x)$. What value of x will make the load equal to 120N?

answer: L(x) - 120 = 0 (1 mark)

ii) Assume two functions: $f_1(x)$ and $f_2(x)$. What value of $x$ makes $f_1(x)$ twice as big as $f_2(x)$?

answer: $f_1(x) - 2 * f_2(x) = 0$ (1 mark)

iii) What is ln(8)? Recall that $Y = e^{\ln(Y)}$.

answer: $e^x - 8 = 0$ (1 mark)

**Question 3 (2 marks).**

Is the following Matlab function vector friendly (i.e. will the output be a vector if the input is a vector)? If not, show how the problem can be corrected. Note: You will lose marks if you make more than the minimum required number of changes.

```
f = @(t)   ((t + 4) + ( t * 6))^5   -  4 / (t  / 5) -  (t^2   * (4 + sqrt(t)))
```

solution:
```
f = @(t)   ((t + 4) + ( t * 6)).^5   -  4 ./ (t  / 5) -  (t.^2   .* (4 + sqrt(t)))
```

+0.5 marks for each of the four dots that should be added
-0.5 marks for each dot added to +, -, or the function (hard and fast errors)
-0.5 marks for each dot added to * or / where this is not required

minimum mark = zero (students cannot go negative)

**Question 4 (3 marks)**

Assume that we are dealing with a system of equations of the form *Ax=b* and the Gaussian elimination has produced the following augmented matrix:

| | | | |
|---|---|---|---|
| 2 | 5 | 1 | 21 |
| 0 | 4 | 2 | 18 |
| 0 | 0 | 3 | 15 |

What is the solution vector *x*?

x(1) = 3 , x(2) = 2, x(3) = 5

1 mark for each calculation. Do not penalize cumulative errors. If a student somehow slips up on x(3) (unlikely!) but the calculated values for x(1) and x(2) are correct given the wrong value for x(3) the student should get 2 marks.

**Question 5 (7 Marks)**

The function below has a single positive root.

$f(x) = x^3 - 2x - 21$

(i) Suppose that we would like to use the Newton-Raphson method to locate this root. Give the appropriate iterative formula (i.e. the general formula customized for this particular problem).

x(i+1) = x(i) - (x(i)^3 - 2*x(i) -21) / (3*x(i)^2 - 2)

3 marks: 3 = perfect, 2 = a minor slip, 1 = a major mistake but shows some signs of understanding

(ii) Use your formula to complete two search iterations starting with $x_0 = 5$.

| i | $X_i$ | $E_{ABS}$ |
|---|---|---|
| 0 | 5 | XXXX |
| 1 | 3.712329 | 1.287671 |
| 2 | 3.134444 | 0.577884 |

2 marks: 1 mark for applying whatever formula the student gave in part (i)
1 mark for having the approximate error equal to the changes in x(i)

(iii) Give a particularly bad choice for $x_0$ and explain why it doesn't work.

x = sqrt(2/3) = 0.816497

The denominator because zero and the search fails immediately.
2 marks: One for the value and one for the reason

**Question 6 (7 Marks)**

The function $f(x) = -(4^{-x})(x^2)$ has a minimum somewhere between 1 and 2.

(i) Illustrate your understanding of the golden section search technique by completing the following table:

| $x_L$ | $x_U$ | $x_1$ | $x_2$ | $f(x_1)$ | $f(x_2)$ |
|---|---|---|---|---|---|
| *1* | *2* | *1.618034* | *1.381966* | *-0.277857* | *-0.281170* |
| *1* | *1.618034* | *1.381966* | *1.236068* | *-0.281170* | *-0.275357* |
| *1.236068* | *1.618034* | XXXXXX | XXXXX | XXXXXX | XXXXX |

(ii) Assuming that the search is ended at this point

What is the best estimate of the location of the minimum? $x_{MIN}$ = ________1.427051___

What is the maximum possible error in this estimate? $E_{MAX}$ = ______0.190983________

(iii) What would $E_{MAX}$ be if the search was ended after a total of ten function evaluations (i.e. assuming that we are only allowed to evaluate to function ten times, what is the best $E_{MAX}$ that can be achieved)?

The function must be evaluated twice before the first wall movement can take place. After that each function evaluation allows a wall to be moved. Therefore 10 function evaluations allow 9 wall movements. Using the formula with N = 9 and delta x zero = 1 gives EMAX = 6.5778e-3.

(i) 4 marks. One mark for correctly calculating the initial x1 and x2. One mark for moving the correct wall ($X_U$). One mark for x2 becoming x1 and a correct new x2 being calculated. One mark for moving the correct wall ($X_L$). No marks for the function evaluations (these are just there for the student's convenience and to allow you to make a better judgment of what the student has or has not done correctly). Use discretion and do not unduly penalize a student for getting off on the wrong foot (i.e. a small slip in calculating of the two initial test points) if it appears that they understand the process.

(ii) 1 mark. Award this mark is the estimate is midway between the final $X_L$ and $X_U$.

(iii) 1 mark. Award this mark if the error is half the distance between the final $X_L$ and $X_U$.

(iv) 1 mark. Award a half mark if a student overlooks the fact that two function evaluations are required on the first iteration.