

- A. Which of the following can cause a project to have multiple IRRs?
- (a) The project has a large initial outlay.
 - (b) A ten-year project has a negative cash flow in the last year of the project's life.**
 - (c) A project has negative cash flows in the first three years, but positive cash flows thereafter.
 - (d) Whenever project cash flows are conventional.
 - (e) With mutually exclusive investments.
- B. A business formed by two or more individuals or entities is called a(n):
- (a) corporation.
 - (b) sole proprietorship.
 - (c) partnership.**
 - (d) closed receivership.
 - (e) open structure.
- C. The possibility of conflict of interest between the stockholders and management of the firm is called:
- (a) the shareholders' conundrum.
 - (b) corporate breakdown.
 - (c) the agency problem.**
 - (d) corporate activism.
 - (e) legal liability.

IRR and Non-conventional Cash Flows

- ▶ When the cash flows change sign more than once, there is more than one IRR
- ▶ When you solve for the IRR, you are solving for the root of an equation. When you cross the x-axis more than once, there will be more than one return that solves the equation
- ▶ If you have more than one IRR, how do you make your decision? (get back to NPV)

Another Example – Non-conventional Cash Flows

- ▶ Suppose an investment will cost \$90,000 initially and will generate the following cash flows:
 - ▶ Year 1: 132,000
 - ▶ Year 2: 100,000
 - ▶ Year 3: **-150,000** (change sign)
- ▶ The required return is 15%.
- ▶ Should we accept or reject the project?
- ▶ $NPV = -90,000 + 132,000 / 1.15 + 100,000 / (1.15)^2 - 150,000 / (1.15)^3 = 1,769.54 > 0$
- ▶ Calculator: $CF_0 = -90,000$; $C01 = 132,000$; $F01 = 1$; $C02 = 100,000$; $F02 = 1$; $C03 = -150,000$; $F03 = 1$; $I = 15$; $CPT\ NPV = 1769.54$
- ▶ If you compute the IRR on the calculator, you get **10.11%** < 15% because it is the first one that you come to.
- ▶ So, if you just blindly use the calculator without recognizing the uneven cash flows, **NPV** would say to accept and **IRR** would say to reject.

▶ 9-25

▶ 9-26

The Three Types of Firms in Canada

	Advantages	Disadvantages
Partnership (A business formed by two or more co-owners)	<ul style="list-style-type: none">• Two or more owners• More human and financial capital available• Relatively easy to start• Income <u>taxed once</u> as personal income	<ul style="list-style-type: none">• <u>Unlimited liability</u>• General partnership• Limited partnership• Partnership dissolves when one partner dies or wishes to sell• Difficult to transfer ownership• Possible disagreements between partners

The Agency Problem

- ▶ Agency relationship
 - ▶ Principal hires an agent to represent their interests
 - ▶ Stockholders (principals) hire managers (agents) to run the company
- ▶ Agency problem
 - ▶ The conflicts of interest between shareholders and management
- ▶ Managerial goals may be different from shareholder goals
 - ▶ Expensive perquisites
 - ▶ Survival
 - ▶ Independence

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- D. The process of making regular payments that reduce the principal loan balance is called:
- (a) annuitizing the loan.
 - (b) amortizing the loan.**
 - (c) depleting the loan.
 - (d) discounting the loan.
 - (e) compounding the loan.
- E. Capital budgeting is defined as the:
- (a) mix of debt and equity used by a firm to finance its operations.
 - (b) management of a firm's long-term investments.**
 - (c) process of determining the optimal types and amounts of inventory to keep on hand.
 - (d) determination of the total amount of money which a firm should borrow.
 - (e) management of a firm's net working capital.
- F. Which one of the following is correct concerning the annual percentage rate (APR)?
- (a) The APR is greater than the effective annual rate.
 - (b) The APR formula for rate disclosure is $[1 + (r/m)]^m - 1$.
 - (c) The APR is the rate which lenders are required to disclose.**
 - (d) The APR is best used to compare offers from various lenders.
 - (e) The APR considers all the effects of compounding.

Amortized Loans

- ▶ The with a pure discount or interest-only loan, **the principal is repaid all at once.**
- ▶ An alternative is **an amortized loan**, with which the lender may require the borrower to **repay parts** of the loan amount **over time.**
- ▶ The process of providing for a loan to be paid off by making regular principal reductions is called *amortizing* the loan.

What Is Corporate Finance?

- ▶ Corporate Finance addresses the following three questions:
 - ▶ 1. What long-term investments should the firm engage in?
 - ▶ The capital budgeting decision: the process of planning and managing a firm's investment in long-term assets.
 - ▶ E.g. product or service, equipment replacement.
 - ▶ 2. How can the firm raise the money for the required investments or assets?
 - ▶ The capital structure decision: the mixture of debt and equity.
 - ▶ 3. How much short-term cash flow does a company need to finance its daily operations?
 - ▶ The working capital decision: planning and managing the firm's current assets and liabilities.
 - ▶ E.g. sell on credit? how much inventory should we carry?

Annual Percentage Rate

- ▶ This is the **annual rate** that is quoted by law
- ▶ The amount of simple interest earned in one year
- ▶ By definition **APR** = period rate times the number of periods per year
- ▶ Consequently, to get the period rate we rearrange the APR equation:
 - ▶ **Period rate** = **APR** / number of periods per year
- ▶ You should **NEVER** divide the effective rate by the number of periods per year - it will **NOT** give you the period rate

- G. Which of the following is an advantage of ownership of a corporation compared to that of a sole proprietorship?
- (a) The owners of the corporation have unlimited liability for the firm's debts.
 - (b) It is the simplest to start.
 - (c) The corporation has an unlimited life.**
 - (d) Dividends received by the corporation's shareholders are tax-exempt.
 - (e) It is more difficult to transfer ownership in a corporation.
- H. A pure discount loan is one where the borrower receives money today and repays the loan with:
- (a) a single lump sum in the future.**
 - (b) equal monthly payments called annuity payments.
 - (c) partial monthly payments followed by a balloon payment.
 - (d) even monthly payments that fully amortize the principal balance.
 - (e) monthly interest only payments followed by a balloon payment.
- I. As the discount rate increases, the present value of \$500 to be received six years from now:
- (a) remains constant.
 - (b) also increases.
 - (c) decreases.**
 - (d) becomes negative.
 - (e) will vary but the direction of the change is unknown.

The Three Types of Firms in Canada

	Advantages	Disadvantages
<p>Corporation</p> <p>(A business created as a distinct <u>legal entity</u> owned by one or more individuals or entities)</p>	<ul style="list-style-type: none"> • <u>Limited liability</u> • Unlimited life • Transfer of ownership is easy • <u>Easier to raise capital</u> • Separation of ownership and management • Benefit from ownership in several different businesses (<u>diversification</u>) • The expertise of others (<u>comparative advantage</u>) • Easier to <u>transfer ownership</u> 	<ul style="list-style-type: none"> • <u>Double taxation</u> (income is taxed at the corporate rate and then dividends are taxed at the personal rate) • Separation of ownership and management <ul style="list-style-type: none"> • <u>Agency problems</u> if management goals and owner goals are not aligned. • <u>More expensive form of organization</u> to establish and maintain (corporate taxes have to be filed every year). • Lenders sometimes require the owners of small corporations to make <u>personal guarantees on any credit/loans extended to the corporation</u>, thereby eliminating the limited liability benefit.

Pure Discount Loans 6.4 - Example

- ▶ Treasury bills are excellent examples of pure discount loans. **The principal amount is repaid at some future date, without any periodic interest payments.**
- ▶ If a T-bill promises to repay \$10,000 in 12 months (FV) and the market interest rate is 4 percent, how much will the bill sell for in the market (Today)?
 - ▶ $PV = 10,000 / 1.04^1 = \$9,615.38 < \$10,000$
 - ▶ (Discounted due to time value of money, "cheaper"),
 - ▶ The value of an investment is the present value of expected future cash flows.
 - ▶ 1 N; 10,000 FV; 4 I/Y; CPT PV = -9,615.38

Present Value – Important Relationship I

- ▶ For a given interest rate – the longer the time period, the lower the present value
 - ▶ $PV = FV / (1 + r)^t$
- ▶ What is the present value of \$500 to be received in 5 years? 10 years? The discount rate is 10%
 - ▶ $PV = 500 / (1 + 0.10)^5 = 310.46$
 - ▶ $PV = 500 / (1 + 0.10)^{10} = 192.77$
 - ▶ Notice that the 10-year present value is lower than the 5-year present value
 - ▶ $N=5, I/Y=10, FV=500, CPT PV = -310.46$
 - ▶ $N=10, I/Y=10, FV=500, CPT PV = -192.77$

J. A perpetuity differs from an annuity because:

- (a) perpetuity payments vary with the rate of inflation.
- (b) perpetuity payments vary with the market rate of interest.
- (c) perpetuity payments are variable while annuity payments are constant.
- (d) perpetuity payments never cease.**
- (e) annuity payments never cease.

Perpetuities and Annuities 6.2

- ▶ **Perpetuity:** $PV = \frac{CF}{r}$
- ▶ **Growing perpetuity:** $PV = \frac{CF_1}{r-g}$
- ▶ **Ordinary annuity:** $PV = \sum_{t=1}^n \frac{CF}{(1+r)^t} = CF \frac{1 - \frac{1}{(1+r)^n}}{r}$
- ▶ **Annuity due:** $PV = CF \frac{1 - \frac{1}{(1+r)^n}}{r} (1+r)$
 $= CF * PVIFA(r;n) * (1+r)$
 $= CF + CF \frac{1 - \frac{1}{(1+r)^{n-1}}}{r}$
- ▶ **Growing annuity:** $PV = \sum_{t=1}^n \frac{CF_1(1+g)^{t-1}}{(1+r)^t} = \frac{CF_1}{r-g} \left(1 - \left(\frac{1+g}{1+r} \right)^n \right)$

Question 2 (18 Marks)

The required return is 12% for the stock of Company A, Company B, and Company C.

- (a) Company A just paid a dividend of \$0.50 per share. The company plans to increase its dividend by 20% next year and then reduce its dividend growth rate by 6% per year until it reaches the industry average of 8% dividend growth, after which it is expected the company will maintain a constant growth rate forever. What will a share of Company A stock sell for today? (6 marks)

$$D_1 = .50(1 + .2) = \$0.60$$

$$g_2 = .2 - .06 = 14\%$$

$$D_2 = .60(1 + .14) = \$0.68$$

$$g_3 = .14 - .06 = 8\%$$

$$D_3 = .68(1 + .08) = \$0.74$$

$$P_2 = \frac{.74}{.12 - .08} = \$18.47$$

$$P = \frac{.60}{(1 + .12)} + \frac{.68 + 18.47}{(1 + .12)^2} = \$15.80$$

- (b) Company B just paid a dividend of \$0.90 per share. The company plans to increase its dividend by 10% per year for the next 5 years before leveling off at 8% into perpetuity. What will a share of Company B stock sell for today? (6 marks)

$$P_5 = \frac{.90(1 + .1)^5(1 + .08)}{.12 - .08} = \$39.14$$

$$P = \frac{.90(1 + .1)}{.12 - .1} \left[1 - \left(\frac{1 + .1}{1 + .12} \right)^5 \right] + \frac{39.14}{(1 + .12)^5} = \$26.47$$

- (c) Company C plans to pay the following dividends per share over the next three years: \$0.25, \$0.50, and \$0.75. Afterward, it is expected the company will increase its dividend by 8% per year indefinitely. What is the dividend yield for Company C stock? (6 marks)

$$P_3 = \frac{.75(1.08)}{.12 - .08} = \$20.25$$

$$P = \frac{.25}{(1 + .12)} + \frac{.50}{(1 + .12)^2} + \frac{.75 + 20.25}{(1 + .12)^3} = \$15.57$$

$$DY = \frac{.25}{15.57} = 1.61\%$$

Example – Non-constant Dividend Growth

- ▶ Suppose a firm is expected to increase dividends by 20% in **1 year** and by 15% in **2 years**. After that dividends will increase at a rate of 5% per year **indefinitely**. If the last dividend was \$1 and the required return is 20%, what is the price of the stock?
- ▶ Remember that we have to find the PV of all expected future dividends. Compute the dividends until growth levels off
 - ▶ $D_0 = \$1.00$
 - ▶ $D_1 = 1(1+0.2) = \$1.20$
 - ▶ $D_2 = 1.20(1+0.15) = \$1.38$
 - ▶ $D_3 = 1.38(1+0.05) = \$1.449$
- ▶ Find the expected future price (growing perpetuity)
 - ▶ $P_2 = D_3 / (R - g) = 1.449 / (.2 - .05) = 9.66$
 - ▶ Only discount the first cash flow to the last period.
- ▶ Find the present value of the expected future cash flows
 - ▶ $P_0 = 1.20 / (1.2) + (1.38 + 9.66) / (1.2)^2 = 8.67$

Using the Constant DGM to Find R

- ▶ Start with the constant DGM:
- ▶ $P_0 = \frac{D_0(1+g)}{R-g} = \frac{D_1}{R-g}$
- ▶ Rearrange and solve for R
- ▶ $R = \frac{D_0(1+g)}{P_0} + g = \frac{D_1}{P_0} + g$
- ▶ This shows the components of the **required return** (total return)
- ▶ $\frac{D_1}{P_0}$ is the **dividend yield**.
- ▶ g is dividend growth rate which is also the rate at which the stock price grows. So it is also the **capital gains yield**, the rate at which the value of the investment grows.

Question 3 (12 Marks)

On your friend's 25th birthday, she deposits \$1,000 into a retirement savings account paying a 6% APR with quarterly compounding. She plans to continue depositing \$1,000 on the first day of each month up until her planned retirement date 30 years from now. Her final deposit will be made 1 month before her planned retirement date. During her retirement years, she plans to make a 1-time charitable donation on her 75th birthday and annual withdrawals beginning with a \$40,000 withdrawal on her 56th birthday. She believes the amount of the annual withdrawals will need to increase by 3% each year. If she wants to be able to make the annual withdrawals for 35 years (i.e. her final withdrawal will be made on her 90th birthday), what is the most she will be able to donate to charity on her 75th birthday?

$$EMR = \left(1 + \frac{.06}{4}\right)^{4/12} - 1 = .5\%$$

$$FV \text{ of Savings Account}_{30} = 1,000 \left[\frac{(1 + .005)^{30(12)} - 1}{.005} \right] (1 + .005) = \$1,003,786.78$$

$$EAR = \left(1 + \frac{.06}{4}\right)^4 - 1 = 6.14\%$$

$$PV \text{ of Annual Withdrawals}_{30} = \frac{40,000}{.0614 - .03} \left[1 - \left(\frac{1 + .03}{1 + .0614} \right)^{35} \right] = \$828,999.18$$

$$Surplus Savings_{30} = 1,003,786.78 - 828,999.18 = \$174,787.60$$

$$FV \text{ of Surplus Savings}_{50} = 174,787.60(1 + .0614)^{20} = \$575,167.06$$

Question 4 (5 Marks)

A company has bonds on the market with 15 years to maturity and a YTM of 4%. The bond pays a 6% coupon and makes semiannual payments. What is the current bond price?

$$\text{Semiannual } C = \left(\frac{.06}{2}\right) 1,000 = \$30$$

$$\text{Semiannual YTM} = \left(\frac{.04}{2}\right) = 2\%$$

$$P = 30 \left[\frac{1 - \frac{1}{(1 + .02)^{15(2)}}}{.02} \right] + \frac{1,000}{(1 + .02)^{15(2)}} = \$1,223.96$$

Example – Semiannual Coupons

- ▶ Most bonds in Canada make **coupon** payments **semiannually**.
- ▶ Suppose you have an **8% semiannual-pay** bond with a face value of **\$1,000** that matures in 7 years. If the yield is 10%, what is the price of this bond?
 - ▶ The bondholder receives a **payment of \$40 every six months** (a total of \$80 per year)
 - ▶ The market automatically assumes that the **yield** is **compounded semiannually** (Natural ESAR = (10% per year)/2 = **5%**)
 - ▶ The number of semiannual periods is **14** (=7 years * 2)

Example – Semiannual Coupons continued

▶ 7-12

$$\text{Bond Price} = 40 \times \frac{\left[1 - \frac{1}{1.05^{14}}\right]}{0.05} + \frac{1,000}{1.05^{14}} = 901.01$$

- ▶ Or PMT = 40; N = 14; I/Y = 5; FV = 1000; CPT PV = -901.01

▶ 7-13

Question 5 (10 Marks)

You applied for a \$500,000 mortgage to a bank quoting 3%. You want a 25-year amortization period.

(a) How much will your weekly payment be? (5 marks)

$$EWR = \left(1 + \frac{.03}{2}\right)^{2/52} - 1 = .06\%$$

$$500,000 = C \left[\frac{1 - \frac{1}{(1 + .0006)^{25(52)}}}{.0006} \right]$$

$$C = \$545.53$$

(b) Prepare an amortization schedule for the first 3 weeks of the mortgage. (5 marks)

Week	Beginning Balance	Total Payment	Interest Payment	Ending Balance
1	\$500,000	\$545.53	\$286.40	\$499,740.87
2	\$499,740.87	\$545.53	\$286.25	\$499,481.59
3	\$499,481.59	\$545.53	\$286.10	\$499,222.16

Amortized Loan with Fixed Principal Payment - Example (Student loan amortization)

Year	Beginning Balance	Interest Payment	Fixed Principal Payment	Total Payment	Ending Balance	
1	20,000	1,234 (20,000*EAR)	5,000	6,234 (1,234+5,000)	15,000 (20,000 -5,000)	EAR= 0.0616 77812
2	15,000	925	5,000	5,925	10,000	(1+0.06 /12)^(1 2/1)-1
3	10,000	617	5,000	5,617	5,000	EMR= 0.06/1 2
4	5,000	308	5,000	5,308	0	
	Total =	3,084	20,000	23,084		

Amortized Loan with Fixed Payment - Example (Student loan amortization)

Month	Beginning Balance	Fixed Total Monthly Payment	Interest Paid	Principal Paid	Ending Balance (principal)	
0	--	--	--	--	20,000.00	
1	20,000.00	250	100.00	150.00	19,850.00	EMR=6%/12=0.5%
2	19,850.00	250	99.25	150.75	19,699.25	
3	19,699.25	250	98.50	151.50	19,547.75	
4	19,547.75	250	97.74	152.26	19,395.48	
5	19,395.48	250	96.98	153.02	19,242.46	
6	19,242.46	250	96.21	153.79	19,088.67	
7	19,088.67	250	95.44	154.56	18,934.12	
8	18,934.12	250	94.67	155.33	18,778.79	
9	18,778.79	250	93.89	156.11	18,622.68	
10	18,622.68	250	93.11	156.89	18,465.80	
11	18,465.80	250	92.33	157.67	18,308.13	
12	18,308.13	250	91.54	158.46	18,149.67	

Question 6 (6 Marks)

What is the present value of \$5,000 per year, at a discount rate of 6%, if the first payment is received 11 years from now and the last payment is received 25 years from now?

$$PV_{10} = 5,000 \left[\frac{1 - \frac{1}{(1 + .06)^{15}}}{.06} \right] = \$48,561.24$$

$$PV = \frac{48,561.24}{(1 + .06)^{10}} = \$27,116.35$$

Question 7 (3 Marks)

An investment offers a 9% total return over the coming year. An investor believes the total real return on this investment will only be 7%. What must the investor believe the inflation rate will be over the next year?

$$1 + .09 = (1 + .07)(1 + h)$$

$$h = 1.87\%$$

The Fisher Effect

- ▶ The Fisher Effect defines the relationship between real rates, nominal rates and inflation
- ▶ Exact relationship is $(1 + R) = (1 + r)(1 + h)$, where:
 - ▶ R = nominal rate
 - ▶ r = real rate
 - ▶ h = expected inflation rate
 - ▶ In the preceding example, the nominal rate was 15.5% and the inflation rate was 5%. What was the real rate?
 - ▶ $r = (1+0.155)/(1+0.05) = 10\%$
- ▶ Approximation of the above relationship is:
 - ▶ $R = r + h$
 - ▶ If the expected inflation rate is high, then there can be a substantial difference ($r \times h$).

Question 8 (10 Marks)

Bond A, Bond B, and Bond C each have 15 years to maturity.

- (a) Bond A is a zero coupon bond with a required return of 5%. What is the current price of Bond A? (2 marks)

$$P = \frac{1,000}{(1 + .05)^{15}} = \$481.02$$

- (b) Bond B is a 4% coupon bond that makes semiannual payments and currently sells for \$895.35. What is Bond B's YTM? (4 marks)

Hint: The correct answer will round to ".00%" or ".50%".

$$\text{Semiannual } C = \left(\frac{.04}{2}\right) 1,000 = \$20$$

$$895.35 = 20 \left[\frac{1 - \frac{1}{(1 + YTM)^{15(2)}}}{YTM} \right] + \frac{1,000}{(1 + YTM)^{15(2)}}$$

Use trial and error to solve for semiannual YTM = 2.5%

$$YTM = 2(.025) = 5\%$$

- (c) Today, you purchased Bond C, which is a 4% coupon bond that makes annual payments. If you sell the bond 3 years from now for \$1,200 and the holding period yield is 3%, how much did you pay for the bond? (4 marks)

$$P = 40 \left[\frac{1 - \frac{1}{(1 + .03)^3}}{.03} \right] + \frac{1,200}{(1 + .03)^3} = \$1,211.31$$

Question 9 (5 Marks)

You have a \$10,000 balance on your credit card, which has an interest rate of 19% compounded monthly. How many years will it take you to pay off the balance by making monthly payments of \$500?

$$EMR = \left(1 + \frac{.19}{12}\right)^{12/12} - 1 = .02\%$$

$$10,000 = 500 \left[\frac{1 - \frac{1}{(1 + .0002)^t}}{.0002} \right]$$

$$t = 24.24 \text{ months}$$

$$\text{Years to Pay Off Balance} = \frac{24.24}{12} = 2.02$$

Question 10 (17 Marks)

A company is considering a project with the following expected cash flows:

Year	Cash Flow
0	-\$85,000
1	\$36,000
2	\$36,000
3	\$36,000

The company requires a 12% return on investment.

- (a) Compute the NPV for the project. Should the company accept or reject the project based on this decision rule? (4 marks)

$$NPV = -85,000 + 36,000 \left[\frac{1 - \frac{1}{(1 + .12)^3}}{.12} \right] = \$1,465.93$$

$$NPV = \$1,465.93 > \$0 \rightarrow \text{Accept}$$

Project Example Information

- ▶ We will use the **following example** for each of the decision rules. We then can compare the different rules and see that conflicts can arise.
- ▶ You are looking at a new project and you have estimated the following cash flows:
 - ▶ Year 0: CF = -165,000
 - ▶ Year 1: CF = 63,120; NI = 13,620
 - ▶ Year 2: CF = 70,800; NI = 3,300
 - ▶ Year 3: CF = 91,080; NI = 29,100
 - ▶ Average Book Value = 72,000
- ▶ Your required return for assets of this risk is 12%.

Computing NPV for the Project

▶ 9-6

- ▶ Using the formulas:
 - ▶ $NPV = -165,000 + 63,120/(1.12) + 70,800/(1.12)^2 + 91,080/(1.12)^3 = 12,627.42$
 - ▶ Do we accept or reject the project?
 - ▶ Since $12,627.42 > 0$ (**add value to the firm**), we accept it.
- ▶ Using the calculator:
 - ▶ Use the Cash Flow and NPV functions on the TI BAII Plus
 - ▶ $CF_0 = -165,000$; $C01 = 63,120$; $F01 = 1$; $C02 = 70,800$; $F02 = 1$; $C03 = 91,080$; $F03 = 1$; NPV; $I = 12$; CPT NPV = 12,627.41

- (b) Compute the discounted payback period for the project. The company will accept the project if it pays back within 3 years. Should the company accept or reject the project based on this decision rule? (4 marks)

Year	Remaining at Beginning	PV of Cash Flow	Remaining at End
1	\$85,000.00	\$32,142.86	\$52,857.14
2	\$52,857.14	\$28,698.98	\$24,158.16
3	\$24,158.16	\$25,624.09	(\$1,465.93)

$$DPP = 2 + \frac{24,158.16}{25,624.09} = 2.94 \text{ years}$$

$$DPP = 2.94 < 3 \rightarrow \text{Accept}$$

Computing Discounted Payback for the Project

- ▶ (Same example) You are looking at a new project and you have estimated the following cash flows (cash flows occur at the end of the year):
 - ▶ Year 0: CF = -165,000
 - ▶ Year 1: CF = 63,120; NI = 13,620
 - ▶ Year 2: CF = 70,800; NI = 3,300
 - ▶ Year 3: CF = 91,080; NI = 29,100
 - ▶ Average Book Value = 72,000
 - ▶ Your required return for assets of this risk is 12%.
- ▶ Assume we will accept the project if it pays back on a discounted basis in **2 years**.
- ▶ Compute the PV for each cash flow and determine the payback period using discounted cash flows
 - ▶ Year 1: $-165,000 + 63,120/1.12^1 = -108,643$
 - ▶ Year 2: $-108,643 + 70,800/1.12^2 = -52,202 < 0$ (not break even yet !)
 - ▶ Year 3: $-52,202 + 91,080/1.12^3 = 12,627$ project pays back in year 3
- ▶ **Do we accept or reject the project? Reject**

▶ 9-13

Computing Payback For The Project

- ▶ (Same example) You are looking at a new project and you have estimated the following cash flows (cash flows occur at the end of the year):
 - ▶ Year 0: CF = -165,000
 - ▶ Year 1: CF = 63,120; NI = 13,620
 - ▶ Year 2: CF = 70,800; NI = 3,300
 - ▶ Year 3: CF = 91,080; NI = 29,100
 - ▶ Average Book Value = 72,000
 - ▶ Your required return for assets of this risk is 12%.
- ▶ Now assume we will accept the project if it pays back **within two years (preset limit)**.
 - ▶ Year 1: $-165,000 + 63,120 = -101,880$ still to recover
 - ▶ Year 2: $-101,880 + 70,800 = -31,080$ **still to recover**
 - ▶ Year 3: $-31,080 + 91,080 = 60,000$ *project pays back in year 3*
- ▶ **If the preset limit is 3 years, do we accept or reject the project?**
- ▶ If we assume that the cash flows occur evenly throughout the year, then the project pays back in 2.34 years.
 - ▶ $PB = 2 + \frac{0 - (-31,080)}{60,000 - (-31,080)} = 2.34 \text{ years}$

▶ 9-9

- (c) Compute the profitability index for the project. Should the company accept or reject the project based on this decision rule? (4 marks)

$$PV \text{ of Cash Inflows} = 36,000 \left[\frac{1 - \frac{1}{(1 + .12)^3}}{.12} \right] = \$86,465.93$$

$$PI = \frac{86,465.93}{85,000} = 1.02$$

$$PI = 1.02 > 1 \rightarrow \text{Accept}$$

Profitability Index 9.5

- ▶ Measures the benefit per unit cost, based on the time value of money
- ▶ $NPV = -PV(\text{Costs}) + PV(\text{Cash flows or benefits})$
- ▶ $PI = \frac{PV(\text{Cash flows})}{PV(\text{Costs})} = \frac{PV(\text{"Output"})}{PV(\text{"Input"})}$
- ▶ A profitability index of 1.1 implies that for every \$1 of investment, we create an additional \$0.10 in value
- ▶ This measure can be very useful in situations where we have limited capital
- ▶ PI looks at the payoff per unit of cost (both in present value terms), thus ignoring scale/size of the initial outlays.

- (d) Compute the IRR for the project. Should the company accept or reject the project based on this decision rule? (4 marks)

Hint: The correct answer will round to “.00%” or “.50%”.

$$NPV = 0 = -85,000 + 36,000 \left[\frac{1 - \frac{1}{(1 + IRR)^3}}{IRR} \right]$$

Use trial and error to solve for IRR = 13%

IRR = 13% > 12% → Accept

- (e) Which of the above decision rules is the preferred approach to assess the profitability of a project? (1 mark)

NPV

Computing IRR For The Project

▶ (Same example) You are looking at a new project and you have estimated the following cash flows (cash flows occur at the end of the year):

- ▶ Year 0: CF = -165,000
- ▶ Year 1: CF = 63,120; NI = 13,620
- ▶ Year 2: CF = 70,800; NI = 3,300
- ▶ Year 3: CF = 91,080; NI = 29,100
- ▶ Average Book Value = 72,000
- ▶ Your required return for assets of this risk is 12%.

▶ $NPV = 0 = -165,000 + \frac{63,120}{(1+IRR)^1} + \frac{70,800}{(1+IRR)^2} + \frac{91,080}{(1+IRR)^3}$

▶ If you do not have a financial calculator, then this becomes a trial and error process (the required rate of return 12 % could be a reference point)

▶ Calculator

- ▶ Enter the cash flows as you did with NPV
- ▶ Press IRR and then CPT
- ▶ **IRR = 16.13%** > 12% required return. **Accept the project.**

▶ 9-21

That is why we end up choosing the **NPV as our ultimate decision rule**. (E.g., not unique for non-conventional projects)

▶ 9-23

Question 11 (4 Marks)

Bank A charges an interest rate of 6.00% compounded weekly. Bank B offers an interest rate of 6.05% compounded semiannually. Which bank charges the favourable lending rate?

$$EAR^A = \left(1 + \frac{.06}{52}\right)^{52} - 1 = 6.18\%$$

$$EAR^B = \left(1 + \frac{.0605}{2}\right)^2 - 1 = 6.14\% \text{ (Favourable)}$$

Mortgages - Example 1

- ▶ Theodore D. Kat is applying to his friendly, neighborhood bank for a mortgage of \$200,000. The bank is quoting 6% (**APR with semi-annual compounding**). He would like to have a 25-year **amortization period** and wants to make **payments monthly**. What will Theodore's payments be?

- ▶ First, calculate the EAR $EAR = \left[1 + \frac{0.06}{2}\right]^2 - 1 = 6.09\%$

- ▶ Second, calculate the effective monthly rate (**make payments monthly**)

$$\text{Effective Monthly Rate} = \left[1 + 0.0609\right]^{1/12} - 1 = 0.4939\%$$

- ▶ Then, calculate the monthly payment $200,000 = \frac{C}{0.004939} \left[1 - \frac{1}{1.004939^{300}}\right]$
 $C = 1,279.61$

- ▶ OR, 300 N, 0.4939 I/Y, 200,000 PV, CPT PMT