SYSC-3200 Fall 2014 Assignment 3 Solutions

Question 1 [10 marks]

[Marking: 6 formulation, 3 computer solution, 1 final statement]

To formulate as a network flow problem we inject one unit of flow at the start node (so node A is a source of exactly one unit of flow), and extract one unit of flow at the end point (so node K) is a sink of exactly one unit of flow. Otherwise there is simple flow balance at the other nodes.

If we follow the usual (sum of the outflows) – (sum of the inflows) = RHS convention, then the constraints are all as shown in the LINDO formulation below. The one exception is node K, which should be -gk - jk - lk = -1 but is coded for LINDO input as gk + jk + lk = 1.

Variables are named for the start and end nodes, e.g. *ab* represents the flow in the arc starting at node A and ending at node B. All variables have a lower bound of zero and an upper bound of infinity.

The LINDO formulation is:

```
min time) 21ab + 12ac + 33ad +23bg + 17be + 10cb + 21ce + 20cf + 11dc
  + 16df + 14di + 14eg + 15ej + 14eh +19fh + 12fi + 23gk + 10gj + 14hj
  + 10hi + 16ij + 16il +18jk + 20jl + 27lk
subject to
nodea) ab + ac + ad = 1
nodeb) bg + be - ab - cb = 0
nodec) cb + ce + cf - ac - dc = 0
noded) dc + df + di - ad = 0
nodee) eg + ej + eh - be - ce = 0
nodef) fh + fi - cf - df = 0
nodeg) gk + gj - bg - eg = 0
nodeh) hj + hi - eh - fh = 0
nodei) ij + il - hi - fi - di = 0
nodej) jk + jl - gj - ej - hj - ij = 0
nodek) gk + jk + lk = 1
nodel) lk - jl - il = 0
```

And the LINDO solution is:

```
        OBJECTIVE FUNCTION VALUE
     TIME)      66.00000

  VARIABLE        VALUE          REDUCED COST
      AB        0.000000           0.000000
      AC        1.000000           0.000000
      AD        0.000000           0.000000
      BG        0.000000           0.000000
      BE        0.000000           5.000000
      CB        0.000000           1.000000
      CE        1.000000           0.000000
```

CF	0.000000	12.000000
DC	0.000000	32.000000
DF	0.000000	29.000000
DI	0.000000	15.000000
EG	0.000000	3.000000
EJ	1.000000	0.000000
EH	0.000000	13.000000
FH	0.000000	5.000000
FI	0.000000	0.000000
GK	0.000000	1.000000
GJ	0.000000	6.000000
HJ	0.000000	0.000000
HI	0.000000	12.000000
IJ	0.000000	0.000000
IL	0.000000	0.000000
JK	1.000000	0.000000
JL	0.000000	20.000000
LK	0.000000	9.000000

This indicates that the fastest route is ACEJK, which requires 66 minutes.

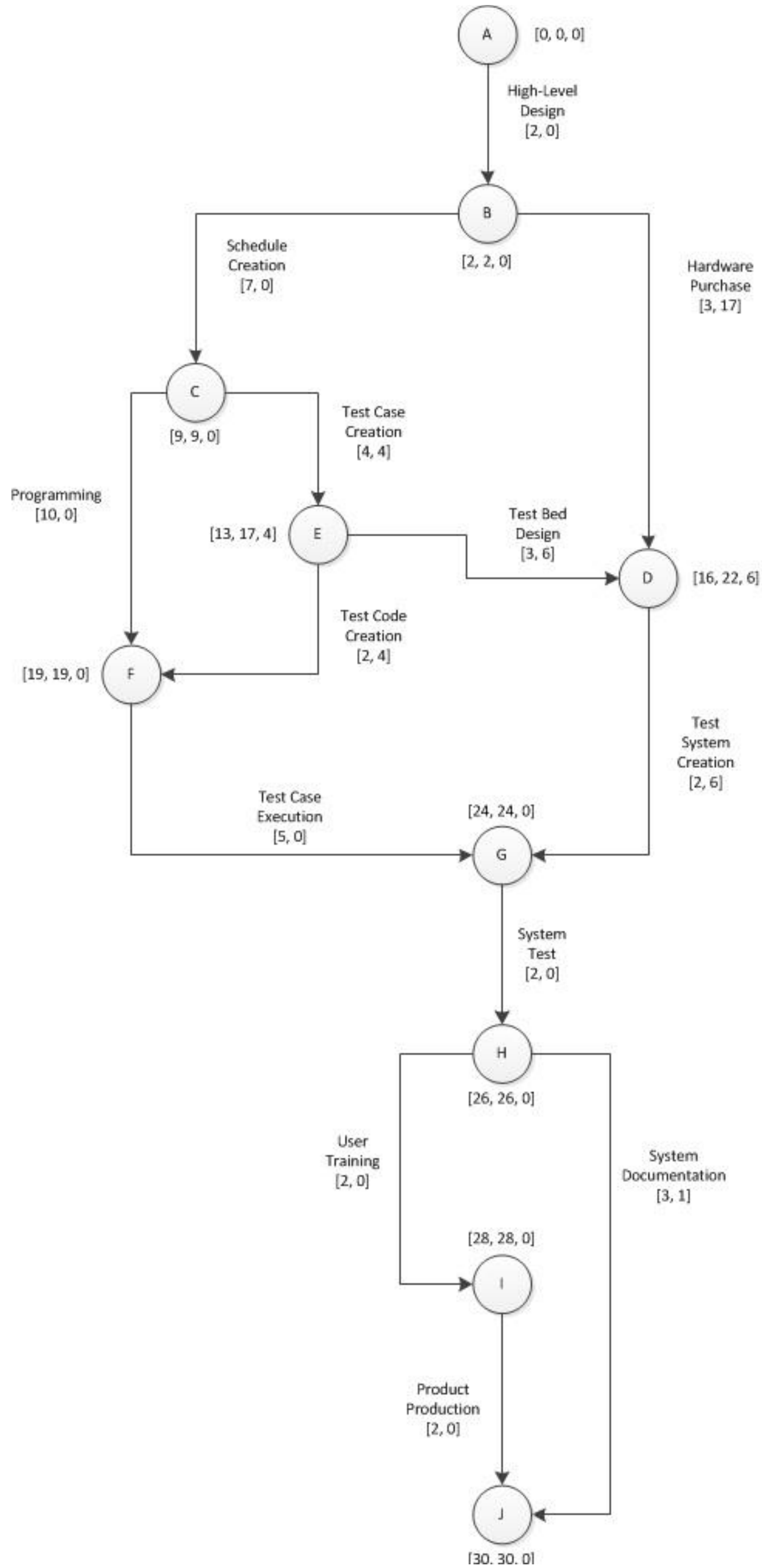
Question 2 [10 marks]

[marking: 2 construction of the PERT chart, 3 all steps in finding the critical path, 2 part b, 3 part c]

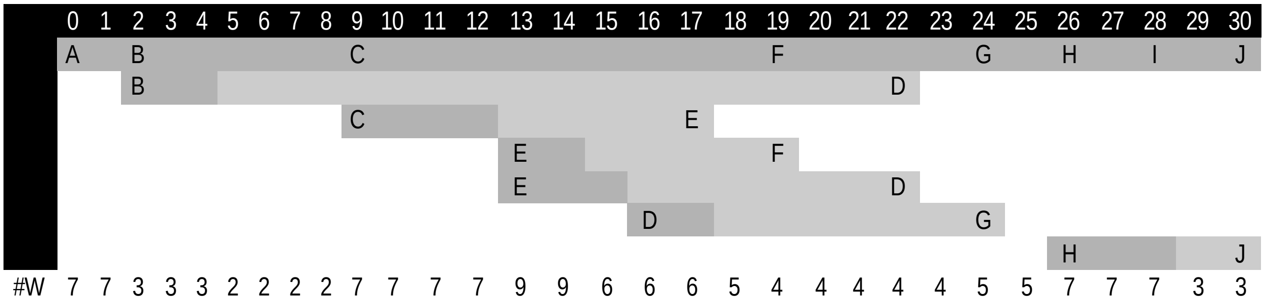
a) See the PERT chart below. Activities are labelled [activity time, total slack of activity]. Nodes are labelled [earliest time, latest time, slack of node].

The critical path is A-B-C-F-G-H-I-J.

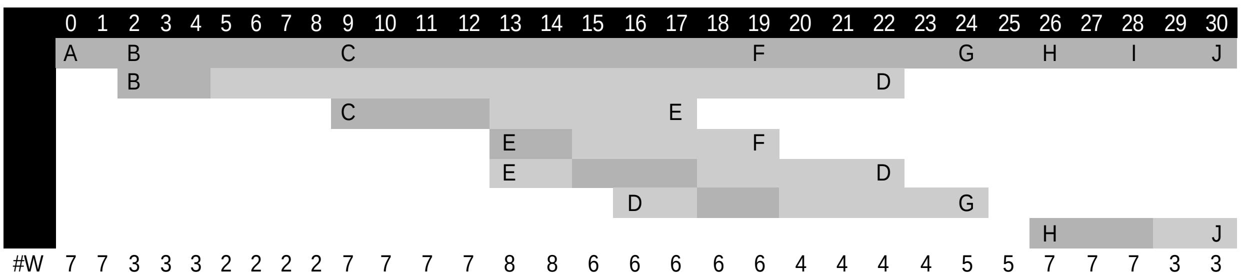
The earliest termination time is 30 days.


A
[0, 0, 0]
High-Level
Design
[2, 0]
B
[2, 2, 0]
Schedule
Creation
[7, 0]
Hardware
Purchase
[3, 17]
C
[9, 9, 0]
Test Case
Creation
[4, 4]
Programming
[10, 0]
[13, 17, 4]
E
Test Bed
Design
[3, 6]
D
[16, 22, 6]
Test Code
Creation
[2, 4]
[19, 19, 0]
F
Test
System
Creation
[2, 6]
Test Case
Execution
[5, 0]
[24, 24, 0]
G
System
Test
[2, 0]
H
[26, 26, 0]
User
Training
[2, 0]
System
Documentation
[3, 1]
[28, 28, 0]
I
Product
Production
[2, 0]
J
[30, 30, 0]

b) If every activity is started as early as possible, then the maximum number of workers needed on any day is 9 as shown in the Gantt chart below.



c) Yes the work can be completed using only 8 workers using the activity timings as shown in the Gantt chart below.



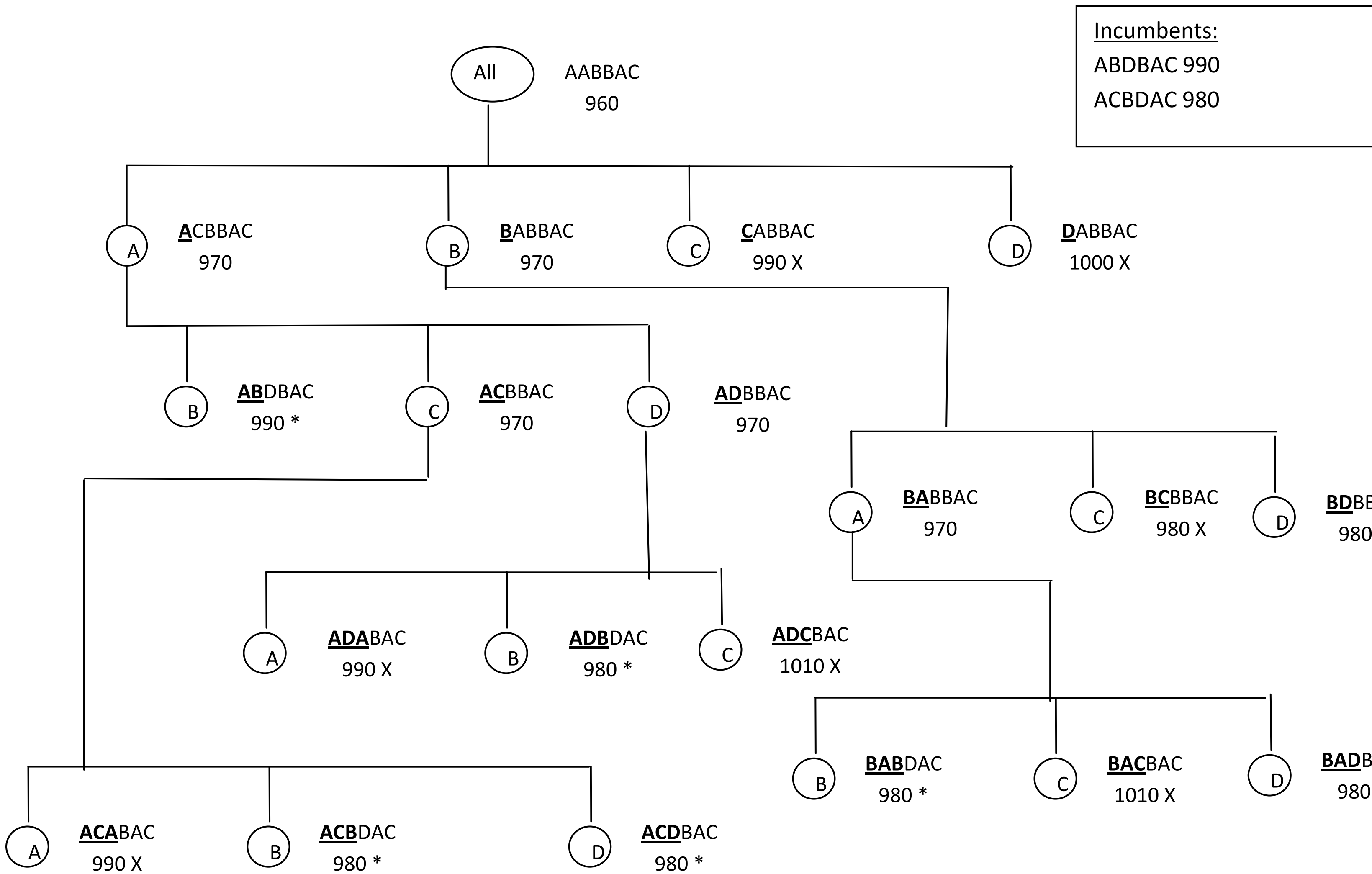
Question 3 [10 marks]

Formulation -> 3 marks
Branch & Bound Tree -> 4 marks
Correct Incumbents -> 1 mark
Correct Assignments -> 1 mark
Correct Minimum Repair Time -> 1 mark

Formulation:

- **Node in decision tree** – a set of mechanic-car assignments.
- **Node selection policy** – global best.
- **Bounding formula** – for unassigned cars choose the best/fastest mechanic. First unassigned car must use a different mechanic than the last assigned car.
- **Terminating rule** – when the incumbent solution has an objective function value that is lower than or equal to all the bounding function values for all the bud nodes.
- **Feasibility test** – a solution is feasible if no mechanic is used for consecutive car repairs.

The branch and bound tree is shown on the next page. An X indicates a pruned solution; an * indicates a feasible solution. The total repair time is 980 minutes. There are several possible mechanic assignments that lead to this solution. One such solution is that mechanic A repairs car 1, mechanic C repairs car 2, mechanic B repairs car 3, mechanic D repairs car 4, A repairs car 5 and mechanic C repairs car 6.


Incumbents:
ABDBAC 990
ACBDAC 980
All
AABBAC
960
A
ACBBAC
970
B
BABBAC
970
C
CABBAC
990 X
D
DABBAC
1000 X
B
ABDBAC
990 *
C
ACBBAC
970
D
ADBBAC
970
A
BABBAC
970
C
BCBBAC
980 X
D
BDBBAC
980 X
A
ADABAC
990 X
B
ADBDAC
980 *
C
ADCBAC
1010 X
B
BABDAC
980 *
C
BACBAC
1010 X
D
BADBAC
980*
A
ACABAC
990 X
B
ACBDAC
980 *
D
ACDBAC
980 *

Question 4 [10 marks]

[marking: 4 formulation, 4 part a, 2 part b]

(a) This is binary integer programming problem which will be solved by Balas additive algorithm.

Variables: x_{ij} where i = plant 1, 2, 3 or 4; j = mode 1, 2, or 3.

$x_{ij} = 0$ means that the plant and node is not used, $x_{ij} = 1$ means that the plant and node is used.

The variables should be rearranged in the order shown in the objective function.

Objective:

Min Cost = $35x_{41} + 50x_{11} + 55x_{42} + 65x_{21} + 70x_{31} + 75x_{43} + 80x_{12} + 90x_{22} + 90x_{32} + 100x_{13} + 110x_{33} + 120x_{23}$

Constraints:

Only 1 mode in plant 1: $x_{11} + x_{12} + x_{13} \leq 1$ $\quad -x_{11} - x_{12} - x_{13} \geq -1$

Only 1 mode in plant 2: $x_{21} + x_{22} + x_{23} \leq 1 \rightarrow$ $\quad -x_{21} - x_{22} - x_{23} \geq -1$

Only 1 mode in plant 3: $x_{31} + x_{32} + x_{33} \leq 1$ $\quad -x_{31} - x_{32} - x_{33} \geq -1$

Only 1 mode in plant 3: $x_{41} + x_{42} + x_{43} \leq 1$ $\quad -x_{41} - x_{42} - x_{43} \geq -1$

Gasoline needed (thousands of barrels): $80x_{11} + 100x_{21} + 112x_{31} + 140x_{12} + 140x_{22} + 153x_{32} + 170x_{13} + 195x_{33} + 215x_{23} + 70x_{41} + 85x_{42} + 110x_{43} \geq 350$

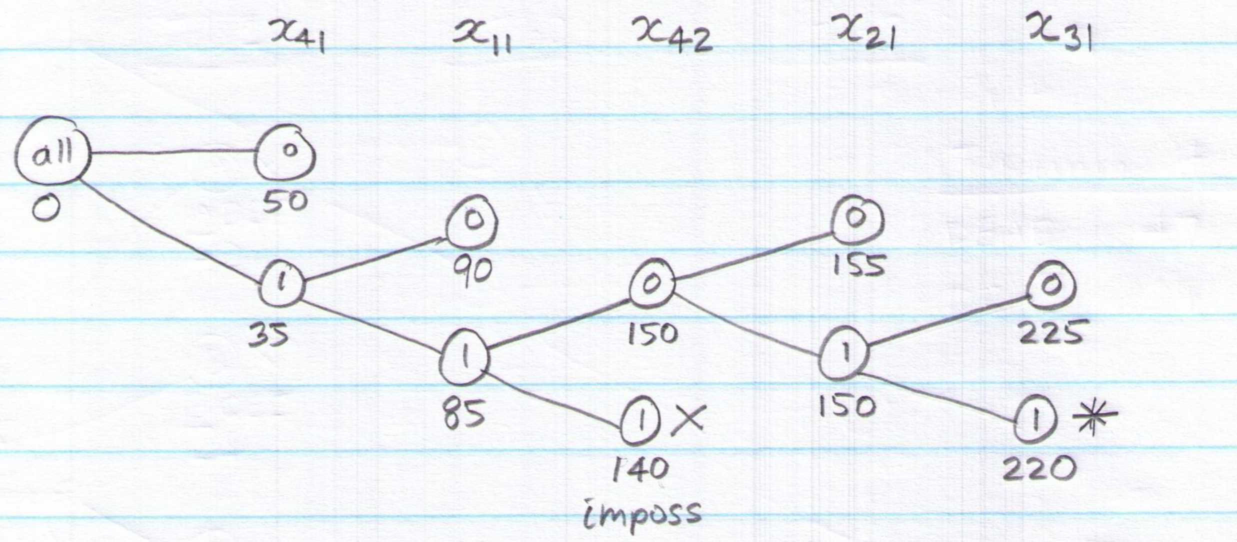
Bounds: all variables are binary.

Branch and Bound Formulation: use Balas algorithm because all variables are binary.

- **Meaning of node**: a selection of plants and their modes (partial or complete).
- **Node selection policy**: depth-first (as specified in Balas algorithm).
- **Bounding formula**: as described in Balas algorithm.
- **Terminating rule**: when first feasible solution is found.
- **Feasibility test**: as described in Balas algorithm.

The branch and bound tree appears in the diagram below.

Applying Balas' algorithm only until the first feasible solution:



The first feasible solution has a cost of $220,000 and specifies that all 4 plants be operated in mode 1.

In the next section we will see whether there is a cheaper solution.

(b) LINDO model and solution:

```
MIN     50 X11 + 80 X12 + 100 X13 + 65 X21 + 90 X22 + 120 X23
      + 70 X31 + 90 X32 + 110 X33 + 35 X41 + 55 X42 + 75 X43
 SUBJECT TO
   PLANT1)   X11 + X12 + X13 <=   1
   PLANT2)   X21 + X22 + X23 <=   1
   PLANT3)   X31 + X32 + X33 <=   1
   PLANT4)   X41 + X42 + X43 <=   1
 GASOLINE)   80 X11 + 140 X12 + 170 X13 + 100 X21 + 140 X22 + 215 X23
      + 112 X31 + 153 X32 + 195 X33 + 70 X41 + 85 X42 + 110 X43
      >=   350
 END
 INTE    12

       OBJECTIVE FUNCTION VALUE
       1)     200.00000

 VARIABLE        VALUE          REDUCED COST
      X11          .000000         50.000000
      X12         1.000000         80.000000
      X13          .000000        100.000000
      X21          .000000         65.000000
      X22          .000000         90.000000
      X23         1.000000        120.000000
      X31          .000000         70.000000
      X32          .000000         90.000000
      X33          .000000        110.000000
```

```
X41            .000000           35.000000
X42            .000000           55.000000
X43            .000000           75.000000
```

The company should operate plant 1 in mode 2, and plant 2 in mode 3 to produce 355 thousand barrels of gasoline at a minimum cost of $200 thousand dollars.