Section 3.4

1. From $m^2 + 3m + 2 = 0$ we find $m_1 = -1$ and $m_2 = -2$. Then $y_c = c_1e^{-x} + c_2e^{-2x}$ and we assume $y_p = A$. Substituting into the differential equation we obtain $2A = 6$. Then $A = 3$, $y_p = 3$ and

$$y = c_1e^{-x} + c_2e^{-2x} + 3.$$

2. From $4m^2 + 9 = 0$ we find $m_1 = -\frac{3}{2}\,i$ and $m_2 = \frac{3}{2}\,i$. Then $y_c = c_1 \cos \frac{3}{2}x + c_2 \sin \frac{3}{2}x$ and we assume $y_p = A$. Substituting into the differential equation we obtain $9A = 15$. Then $A = \frac{5}{3}$, $y_p = \frac{5}{3}$ and

$$y = c_1 \cos \frac{3}{2}x + c_2 \sin \frac{3}{2}x + \frac{5}{3}.$$

29. We have $y_c = c_1e^{-x/5} + c_2$ and we assume $y_p = Ax^2 + Bx$. Substituting into the differential equation we find $A = -3$ and $B = 30$. Thus $y = c_1e^{-x/5} + c_2 - 3x^2 + 30x$. From the initial conditions we obtain $c_1 = 200$ and $c_2 = -200$, so

$$y = 200e^{-x/5} - 200 - 3x^2 + 30x.$$

31. We have $y_c = e^{-2x}(c_1 \cos x + c_2 \sin x)$ and we assume $y_p = Ae^{-4x}$. Substituting into the differential equation we find $A = 7$. Thus $y = e^{-2x}(c_1 \cos x + c_2 \sin x) + 7e^{-4x}$. From the initial conditions we obtain $c_1 = -10$ and $c_2 = 9$, so

$$y = e^{-2x}(-10 \cos x + 9 \sin x) + 7e^{-4x}.$$

Section 3.5

1. The auxiliary equation is $m^2 + 1 = 0$, so $y_c = c_1 \cos x + c_2 \sin x$ and

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1.$$

Identifying $f(x) = \sec x$ we obtain

$$u_1' = -\frac{\sin x \sec x}{1} = -\tan x$$

$$u_2' = \frac{\cos x \sec x}{1} = 1.$$

Then $u_1 = \ln|\cos x|$, $u_2 = x$, and

$$y = c_1 \cos x + c_2 \sin x + \cos x \ln|\cos x| + x \sin x.$$

4. The auxiliary equation is $m^2+1=0$, so $y_c = c_1\cos x + c_2\sin x$ and

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1.$$

Identifying $f(x) = \sec x \tan x$ we obtain

$$u_1' = -\sin x(\sec x\tan x) = -\tan^2 x = 1 - \sec^2 x$$
$$u_2' = \cos x(\sec x \tan x) = \tan x.$$

Then $u_1 = x - \tan x$, $u_2 = -\ln|\cos x|$, and

$$\begin{aligned} y &= c_1\cos x + c_2\sin x + x\cos x - \sin x - \sin x\ln|\cos x| \\ &= c_1\cos x + c_3\sin x + x\cos x - \sin x\ln|\cos x|. \end{aligned}$$

22. The auxiliary equation is $m^2-4m+4=(m-2)^2=0$, so $y_c = c_1e^{2x}+c_2xe^{2x}$ and

$$W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x}+e^{2x} \end{vmatrix} = e^{4x}.$$

Identifying $f(x) = \left(12x^2-6x\right)e^{2x}$ we obtain

$$u_1' = 6x^2 - 12x^3$$
$$u_2' = 12x^2 - 6x.$$

Then

$$u_1 = 2x^3 - 3x^4$$
$$u_2 = 4x^3 - 3x^2.$$

Thus

$$\begin{aligned} y &= c_1e^{2x} + c_2xe^{2x} + \left(2x^3-3x^4\right)e^{2x} + \left(4x^3-3x^2\right)xe^{2x} \\ &= c_1e^{2x} + c_2xe^{2x} + e^{2x}\left(x^4-x^3\right) \end{aligned}$$

and

$$y' = 2c_1e^{2x} + c_2\left(2xe^{2x}+e^{2x}\right) + e^{2x}\left(4x^3-3x^2\right) + 2e^{2x}\left(x^4-x^3\right).$$

The initial conditions imply

$$\begin{aligned} c_1 \qquad &= 1 \\ 2c_1 + c_2 &= 0. \end{aligned}$$

Thus $c_1 = 1$ and $c_2 = -2$, and

$$y = e^{2x} - 2xe^{2x} + e^{2x}\left(x^4-x^3\right) = e^{2x}\left(x^4-x^3-2x+1\right).$$

Section 3.6

1. The auxiliary equation is $m^2 - m - 2 = (m+1)(m-2) = 0$ so that $y = c_1 x^{-1} + c_2 x^2$.

2. The auxiliary equation is $4m^2 - 4m + 1 = (2m-1)^2 = 0$ so that $y = c_1 x^{1/2} + c_2 x^{1/2} \ln x$.

4. The auxiliary equation is $m^2 - 4m = m(m-4) = 0$ so that $y = c_1 + c_2 x^4$.

5. The auxiliary equation is $m^2 + 4 = 0$ so that $y = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x)$.