Chapter 29. The Electric Potential

At any time, millions of light bulbs are transforming electric energy into light and thermal energy. Just as electric fields allowed us to understand electric forces, *Electric Potential* allows us to understand electric energy.

Chapter Goal: To calculate and use the electric potential and electric potential energy.

Chapter 29. The Electric Potential

Topics:

- Electric Potential Energy
- The Potential Energy of Point Charges
- The Potential Energy of a Dipole
- The Electric Potential
- The Electric Potential Inside a Parallel-Plate Capacitor
- The Electric Potential of a Point Charge
- The Electric Potential of Many Charges

Learning Objectives

- To introduce electric potential energy and use it in conservation of energy problems.
- To define the electric potential.
- To find and use the electric potential of point charges, charged spheres, and parallel-plate capacitors.
- To find the electric potential of a continuous distribution of charge.
- To introduce and use potential graphs and equipotential surfaces.

Student Difficulties Understanding Electric Field, Electric Potential Energy and Electric Potential

- Have great difficulty using energy conservation in problems involving the motion of charged particles.
- Don’t acquire a conceptual model of potential or potential difference.
- Are unable to relate the electric potential to the electric field.
- Rarely invoke potential or potential difference when asked to explain a phenomenon.
- Are later unable to use the idea of potential difference in circuits.

Electric Potential Energy

- We need to understand how electric energy is related to electric charges, forces and fields

Conservation of Energy

$$\Delta K + \Delta U = \Delta E_{sys} = W_{ext}$$

- Recall $\Delta K = K_f - K_i$ (f = final, i = initial)
- K - Kinetic energy: Energy associated with motion
- U - Potential energy: Energy associated with position
  - Gravitational Potential Energy
  - Potential Energy of Spring
  - Also Electric Potential Energy

FIGURE 29.3 Potential energy is transformed into kinetic energy as a particle moves in a gravitational field.

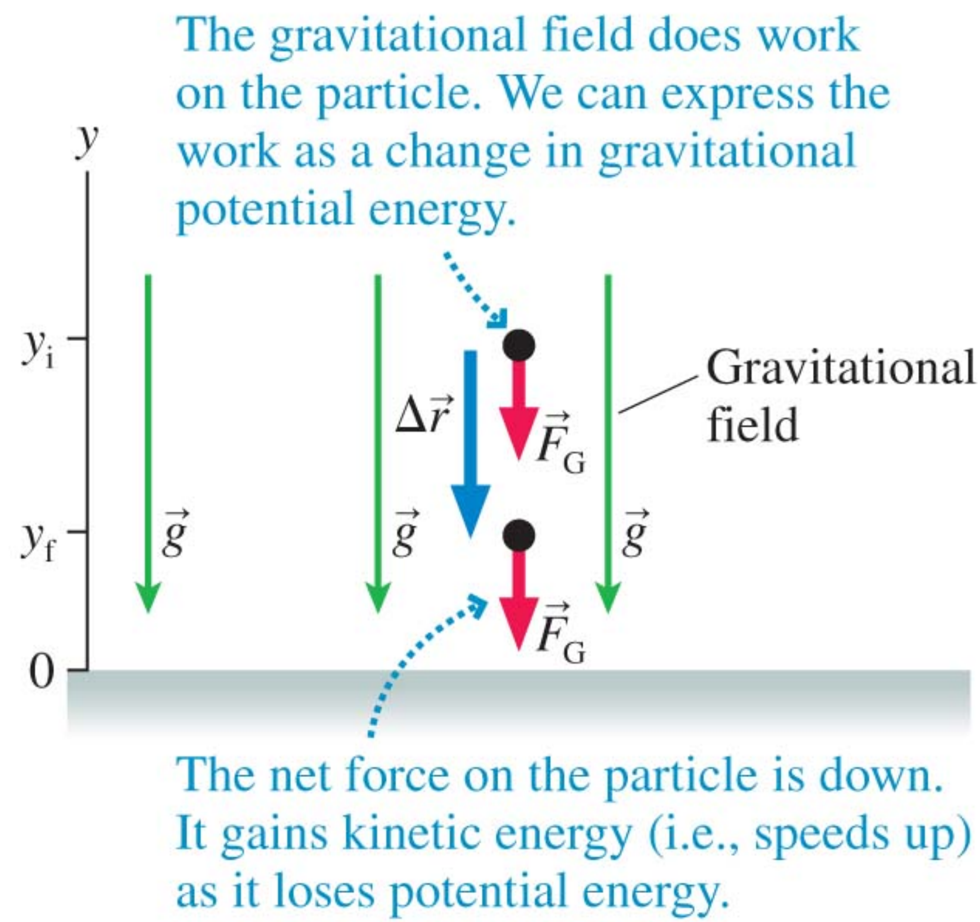
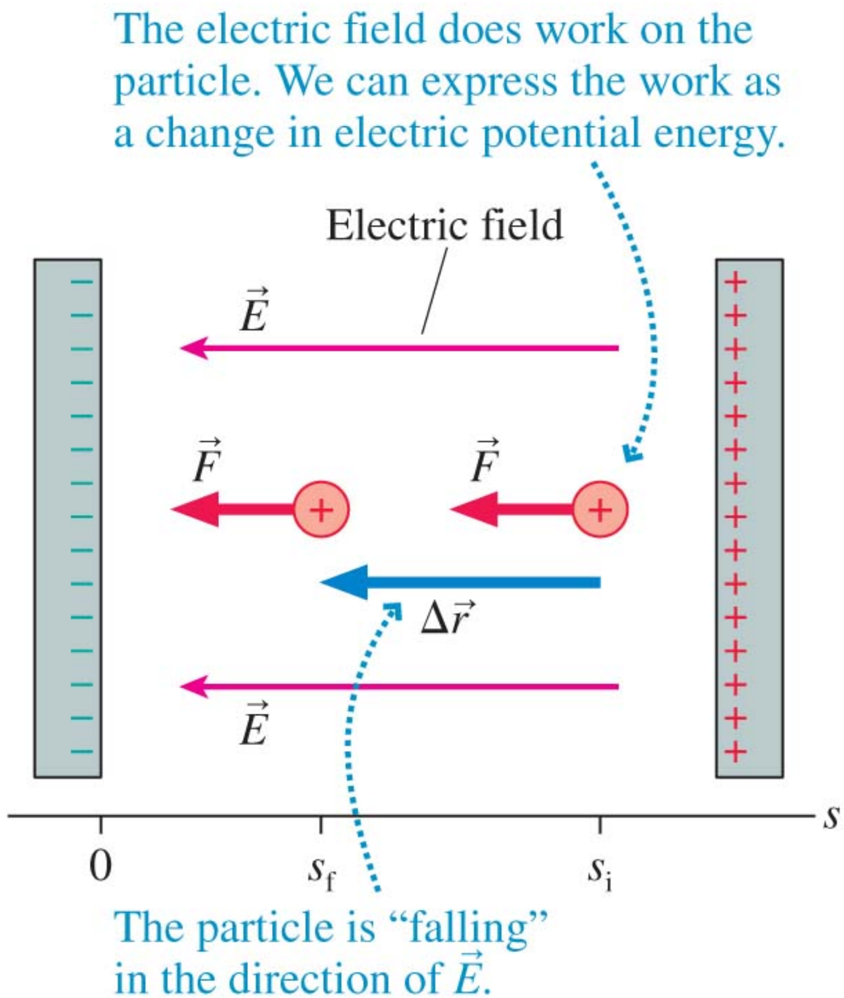


FIGURE 29.4 The electric field does work on the charged particle.



For a constant force

$$W = \vec{F} \cdot \Delta\vec{r}$$

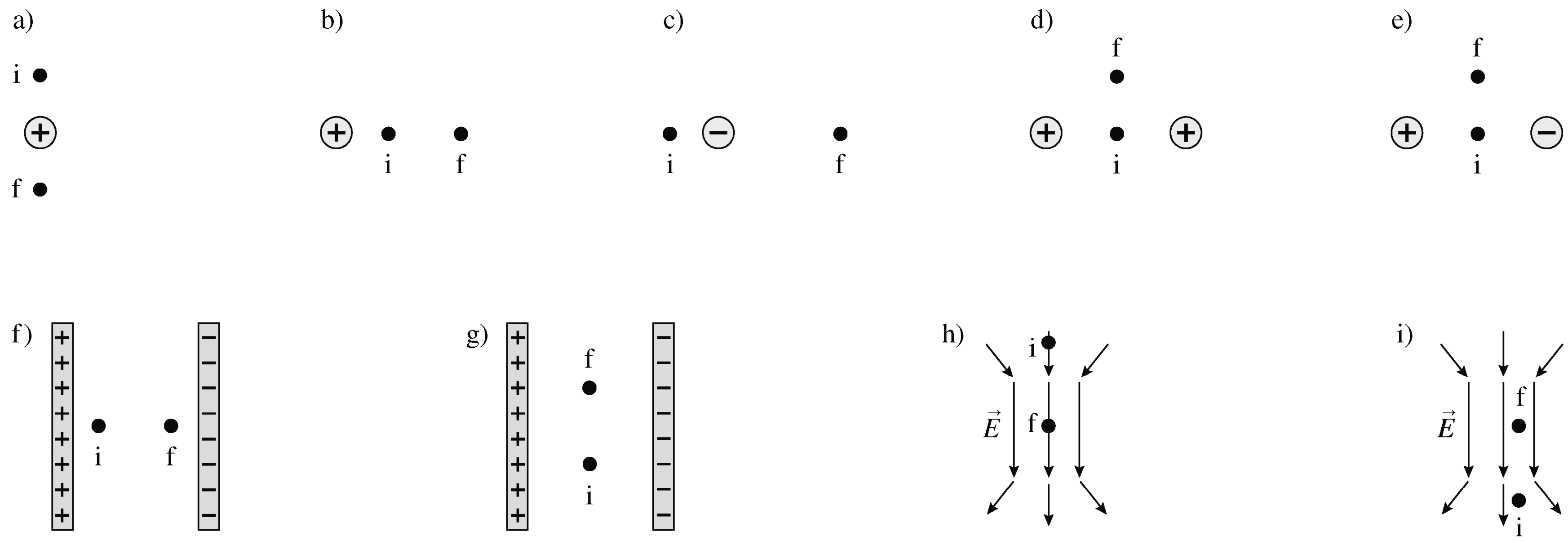
Force on charge q in electric field is

$$\vec{F} = \vec{E}q$$

Is ΔU of the a particle with charge +q positive, negative or zero as it moves from i to f?

Is ΔU of the a particle with charge -q positive, negative or zero as it moves from i to f?

Is ΔU of the a hydrogen atom positive, negative or zero as it moves from i to f?



Is ΔU of the particle positive, negative, or zero as the particle moves from i to f?

Electric Potential Energy

The **electric potential energy** of charge q in a uniform electric field is

$$U_{\text{elec}} = U_0 + qEs$$

where s is measured from the negative plate and U_0 is the potential energy at the negative plate ($s = 0$). It will often be convenient to choose $U_0 = 0$, but the choice has no physical consequences because it doesn't affect ΔU_{elec}, the *change* in the electric potential energy. Only the *change* is significant.

The Potential Energy of Point Charges

For a non-constant force $W = \int_{x_i}^{x_f} \vec{F} \cdot dx$

Consider two point charges, q_1 and q_2, separated by a distance r. The electric potential energy is work done to arrange charge distribution.

Here F is Coulomb's law, $x_i = \infty$ and $x_f = r$

$$U_{elec} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

This is explicitly the energy of *the system*, not the energy of just q_1 or q_2.

Note that the potential energy of two charged particles approaches zero as $r \to \infty$.

FIGURE 29.9 The potential-energy diagrams for two like charges and two opposite charges.

(a) Like charges

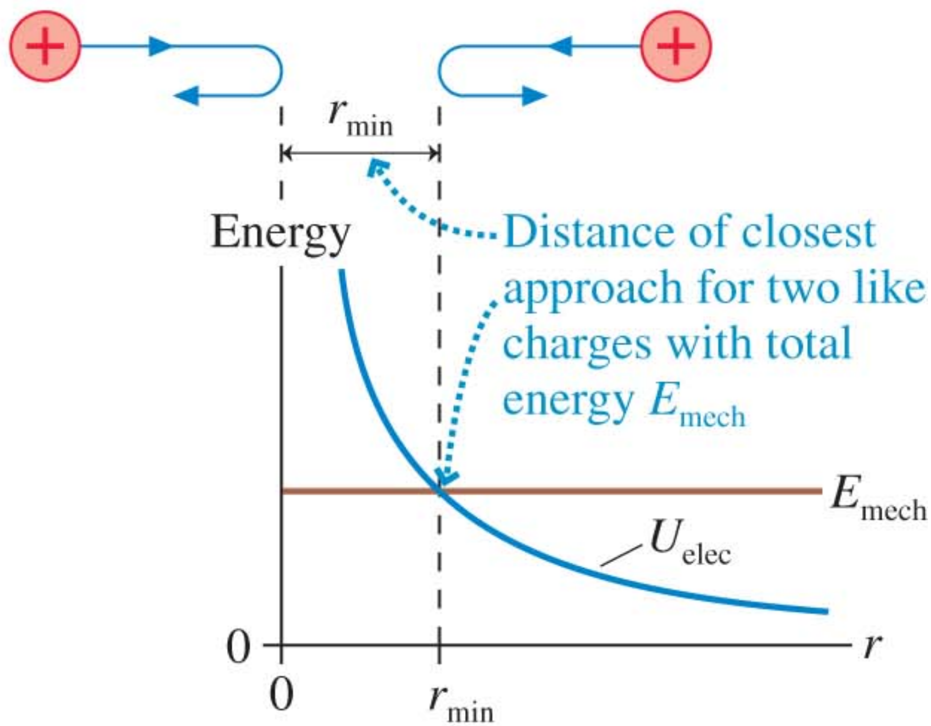
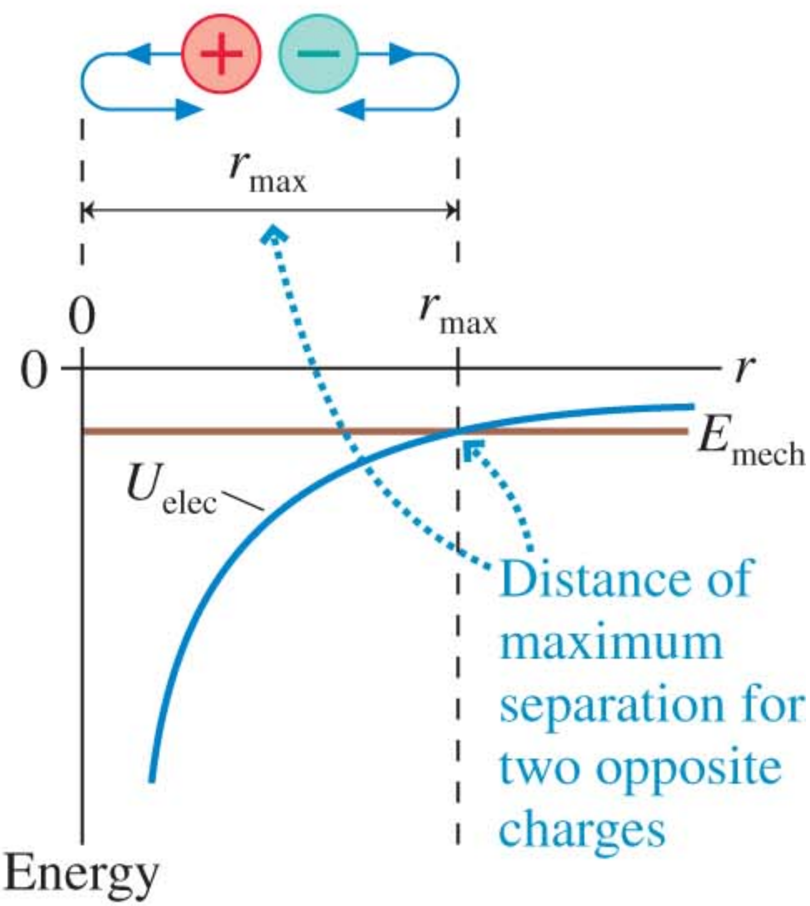


FIGURE 29.9 The potential-energy diagrams for two like charges and two opposite charges.

(b) Opposite charges



EXAMPLE 29.2 Approaching a charged sphere

QUESTION:

EXAMPLE 29.2 Approaching a charged sphere

A proton is fired from far away at a 1.0-mm-diameter glass sphere that has been charged to $+100$ nC. What initial speed must the proton have to just reach the surface of the glass?

Potential Energy of Multiple Point Charges

$$U_{elec} = \sum_{i<j} \frac{Kq_i q_j}{r_{ij}}$$

- Where r_{ij} is the distance between q_i and q_j
- The sum over i<j ensures each pair of charges is only considered once

Important Note

- Force and electric field are vectors

$$\vec{F} = \vec{E}q$$

- All potential energy are scalars

$$\Delta U = \int_{x_i}^{x_f} \vec{F} \cdot dx$$

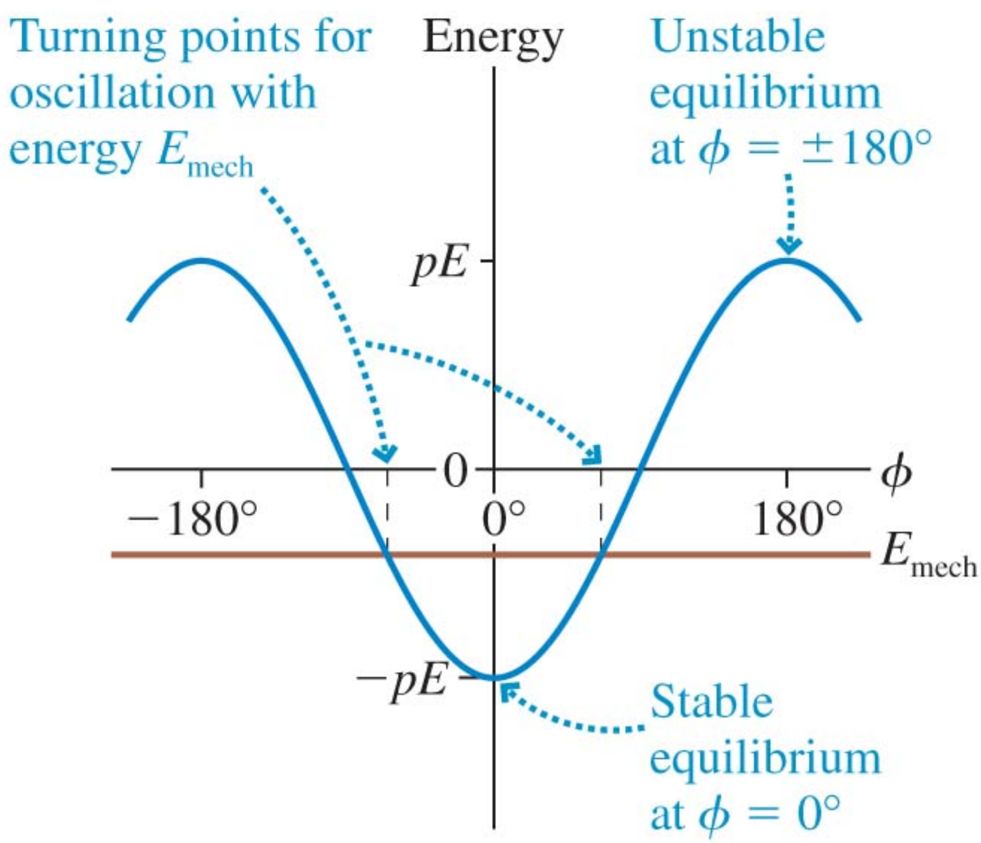
The Potential Energy of a Dipole

The potential energy of an electric dipole p in a uniform electric field E is

$$U_{\text{dipole}} = -pE\cos\phi = -\vec{p} \cdot E$$

The potential energy is minimum at $\phi = 0°$ where the dipole is aligned with the electric field.

FIGURE 29.16 The energy of a dipole in an electric field.



EXAMPLE 29.5 Rotating a molecule

QUESTION:

EXAMPLE 29.5 Rotating a molecule

The water molecule is a permanent electric dipole with dipole moment 6.2×10^{-30} C m. A water molecule is aligned in an electric field with field strength 1.0×10^{7} N/C. How much energy is needed to rotate the molecule 90°?

The Electric Potential

We define the electric potential *V* (or, for brevity, just the potential) as

$$V \equiv \frac{U_{q+\text{sources}}}{q}$$

Notice analogy with

$$\boxed{\vec{E} = \frac{\vec{F}}{q}}$$

Charge q is used as a probe to determine the electric potential, but the value of *V* is *independent of q*. **The electric potential, like the electric field, is a property of the source charges.**

The unit of electric potential is the joule per coulomb, which is called the volt V:

$$1 \text{ volt} = 1 \text{ V} \equiv 1 \text{ J/C}$$

More detail on Force/Potential Analogy

- Force on charge q = [charge q] x [alteration of space by the source charges]

$$\vec{F} = q\vec{E}$$

- Potential energy of q = [charge q] x [potential for interaction of the source charges]

$$U_{q+sources} = qV$$

What good is the Electric Potential *V* ?

- The electric potential depends only on the source charges and their geometry. The potential is the "ability" or potential of the source charges to have an interaction *if* a charge *q* shows up.
  - The potential is present throughout space whether or not charge *q* is there to experience it
- Given the electric potential V (due to the source charges), we know the interaction energy U=qV of any charge

PROBLEM-SOLVING STRATEGY 29.1

Conservation of energy in charge interactions

MP

MODEL Check whether there are any dissipative forces that would keep the mechanical energy from being conserved.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + qV_f = K_i + qV_i$$

- Is the electric potential given in the problem statement? If not, you'll need to use a known potential, such as that of a point charge, or calculate the potential using the procedure given later, in Problem-Solving Strategy 29.2.
- K_i and K_f are the sums of the kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

The Electric Potential of a Point Charge

Let q be the source charge, and let a second charge q', a distance r away, probe the electric potential of q. The potential energy of the two point charges is

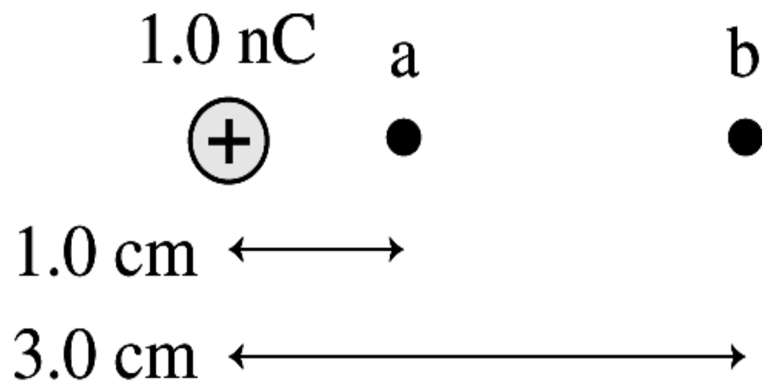
$$U_{q'+q} = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r}$$

By definition, the electric potential of charge q is

$$V = \frac{U_{q'+q}}{q'} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \text{(electric potential of a point charge)}$$

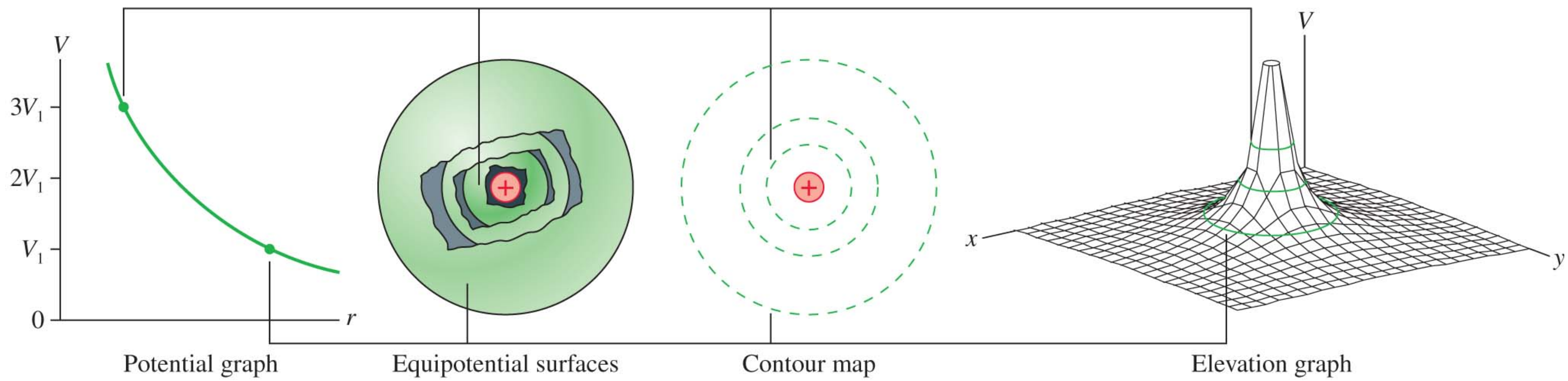
The potential extends through all of space, showing the influence of charge q, but it weakens with distance as $1/r$. This expression for V assumes that we have chosen $V = 0$ to be at $r = \infty$.

Example



- For the situation shown in the figure, find
  a) The potential at points a and b.
  b) The potential difference between a and b.
  c) The potential energy of a proton at a and b.
  d) The speed at point b of a proton that was moving to the right at point a with a speed of 4.0×10^5 m/s.
  e) The speed at point a of a proton that was moving to the left at point b with a speed of 4.0×10^5 m/s.

FIGURE 29.27 Four graphical representations of the electric potential of a point charge.



EXAMPLE 29.8 Calculating the potential of a point charge

QUESTIONS:

EXAMPLE 29.8 Calculating the potential of a point charge

What is the electric potential 1.0 cm from a +1.0 nC charge? What is the potential difference between a point 1.0 cm away and a second point 3.0 cm away?

The Electric Potential of a Charged Sphere

In practice, you are more likely to work with a charged sphere, of radius R and total charge Q, than with a point charge. Outside a uniformly charged sphere, the electric potential is identical to that of a point charge Q at the center. That is,

$$V = \frac{1}{4\pi\epsilon}\frac{Q}{r} \qquad \text{(sphere of charge, } r \geq R\text{)}$$

Or, in a more useful form, the potential outside a sphere that is charged to potential V_0 is

$$V = \frac{R}{r}V_0 \qquad \text{(sphere charged to potential } V_0\text{)}$$

The Electric Potential of Many Charges

The electric potential V at a point in space is the sum of the potentials due to each charge:

$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

where r_i is the distance from charge q_i to the point in space where the potential is being calculated.
In other words, **the electric potential, like the electric field, obeys the principle of superposition.**

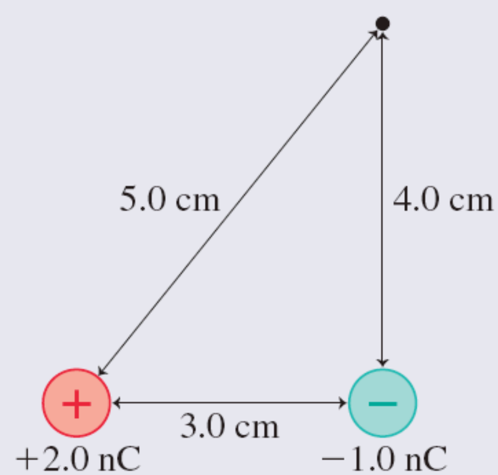
EXAMPLE 29.10 The potential of two charges

QUESTION:

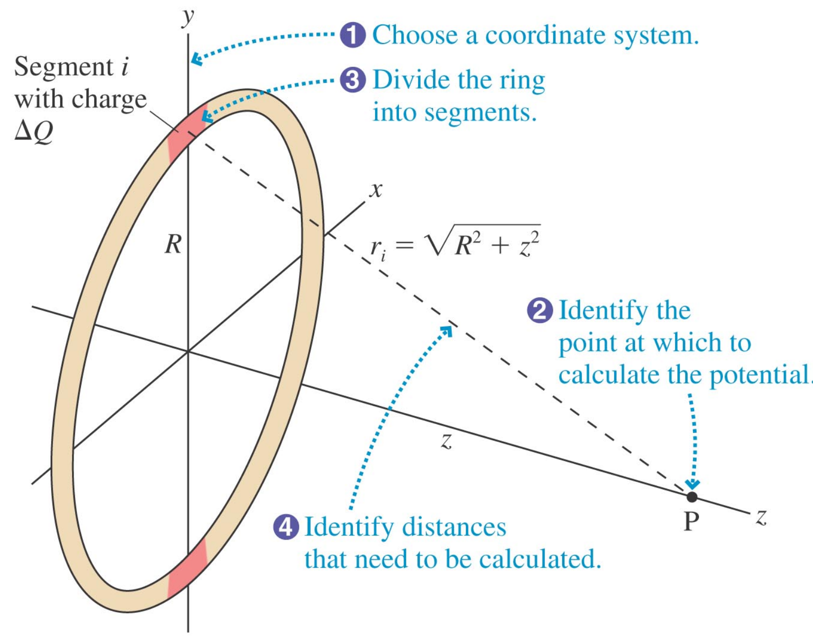
EXAMPLE 29.10 The potential of two charges

What is the electric potential at the point indicated in **FIGURE 29.30**?

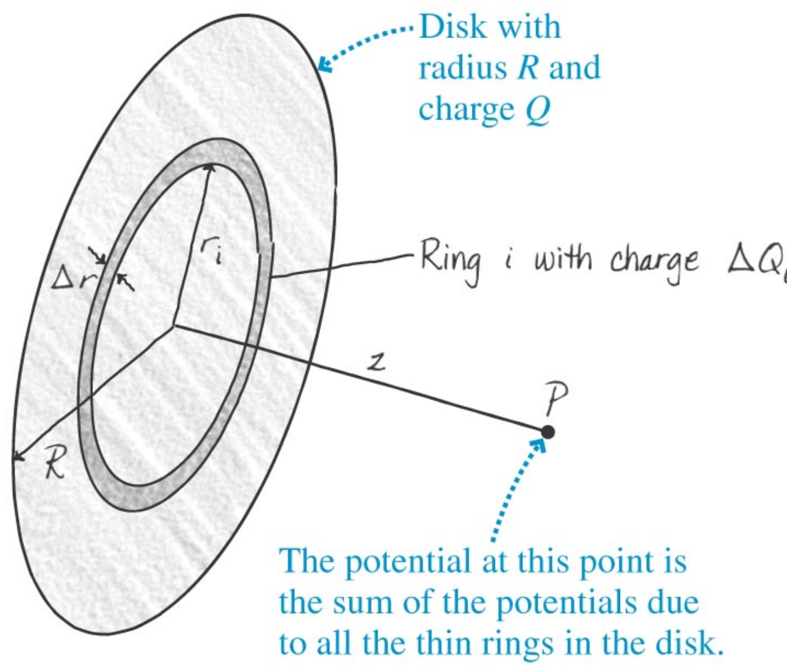
FIGURE 29.30 Finding the potential of two charges.



- Compute the potential of a ring of charge



- Compute the potential of a disk of charge



The Electric Potential Inside a Parallel-Plate Capacitor

The electric potential inside a parallel-plate capacitor is

$$V = Es \qquad \text{(electric potential inside a parallel-plate capacitor)}$$

where ***s*** **is the distance from the *negative* electrode.**
The electric potential, like the electric field, exists at *all points* inside the capacitor.
The electric potential is created by the source charges on the capacitor plates and exists whether or not charge q is inside the capacitor.

EXAMPLE 29.7 A proton in a capacitor

QUESTIONS:

EXAMPLE 29.7 A proton in a capacitor

A parallel-plate capacitor is constructed of two 2.0-cm-diameter disks spaced 2.0 mm apart. It is charged to a potential difference of 500 V.

a. What is the electric field strength inside?
b. How much charge is on each plate?
c. A proton is shot through a small hole in the negative plate with a speed of 2.0×10^5 m/s. Does it reach the other side? If not, where is the turning point?

PROBLEM-SOLVING STRATEGY 29.2

The electric potential of a continuous distribution of charge

MP

MODEL Model the charges as a simple shape, such as a line or a disk. Assume the charge is uniformly distributed.

VISUALIZE For the pictorial representation:

1. Draw a picture and establish a coordinate system.
2. Identify the point P at which you want to calculate the electric potential.
3. Divide the total charge Q into small pieces of charge ΔQ, using shapes for which you *already know* how to determine V. This division is often, but not always, into point charges.
4. Identify distances that need to be calculated.

SOLVE The mathematical representation is $V = \sum V_i$.

- Use superposition to form an algebraic expression for the potential at P.
- Let the (x, y, z) coordinates of the point remain as variables.
- Replace the small charge ΔQ with an equivalent expression involving a *charge density* and a *coordinate,* such as dx, that describes the shape of charge ΔQ. **This is the critical step in making the transition from a sum to an integral** because you need a coordinate to serve as the integration variable.
- All distances must be expressed in terms of the coordinates.
- Let the sum become an integral. The integration will be over the coordinate variable that is related to ΔQ. The integration limits for this variable will depend on the coordinate system you have chosen. Carry out the integration and simplify the result.

ASSESS Check that your result is consistent with any limits for which you know what the potential should be.

Summary - General Principles

Sources of V

The electric potential, like the electric field, is created by charges.

Two major tools for calculating V are

- The potential of a point charge $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r}$
- The principle of superposition

Multiple point charges

Use superposition: $V = V_1 + V_2 + V_3 + \cdots$

Continuous distribution of charge

- Divide the charge into point-like ΔQ.
- Find the potential of each ΔQ.
- Find V by summing the potentials of all ΔQ.

The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a charge density and an integration coordinate. Calculating V is usually easier than calculating $\vec{E}$ because the potential is a scalar.

Summary - General Principles

Consequences of *V*

A charged particle has potential energy

$$U = qV$$

at a point where source charges have created an electric potential V.

The electric force is a conservative force, so the mechanical energy is conserved for a charged particle in an electric potential:

$$K_f + U_f = K_i + U_i$$

The potential energy of **two point charges** separated by distance r is

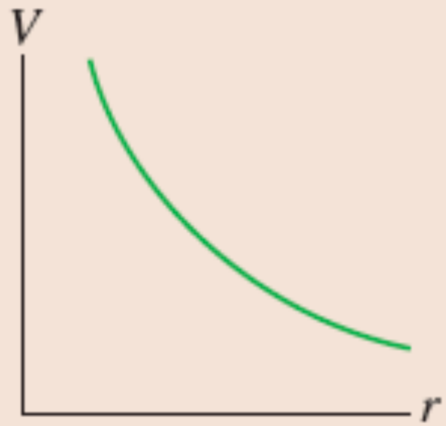
$$U_{q_1+q_2} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

The **zero point** of potential and potential energy is chosen to be convenient. For point charges, we let $U = 0$ when $r \to \infty$.

The potential energy in an electric field of an **electric dipole** with dipole moment $\vec{p}$ is

$$U_{\text{dipole}} = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$

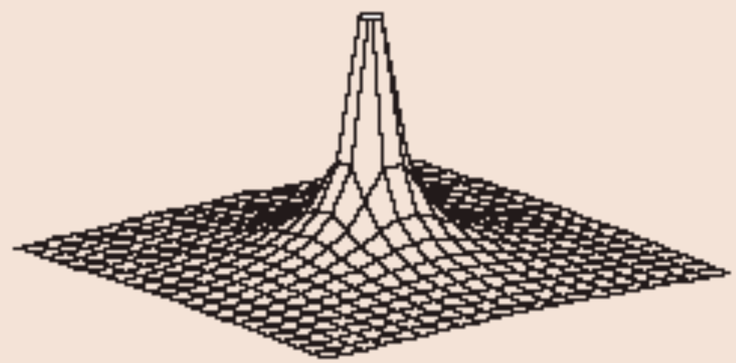
Graphical representations of the potential:



Potential graph

Equipotential surfaces

Contour map



Elevation graph

Summary - Applications

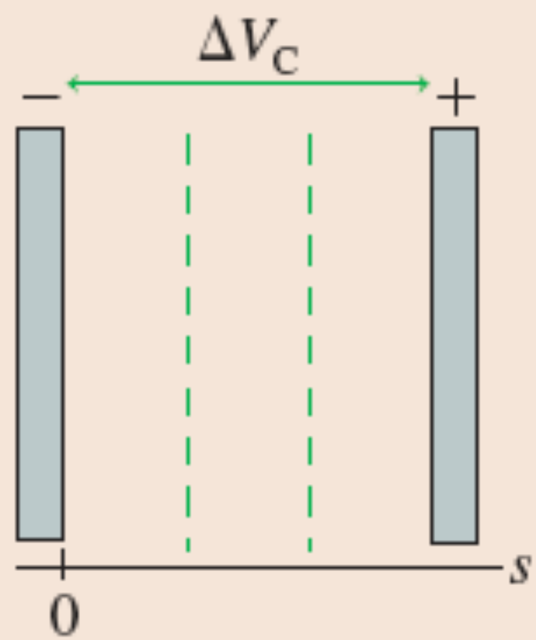
Sphere of charge Q

Same as a point charge if $r \geq R$

Parallel-plate capacitor

$V = Es$, where s is measured from the negative plate. The electric field inside is

$$E = \frac{\Delta V_C}{d}$$



Applications

Units

Electric potential: $1\ \text{V} = 1\ \text{J/C}$

Electric field: $1\ \text{V/m} = 1\ \text{N/C}$

Chapter 29. Reading Quizzes

What are the units of *potential difference*?

A. Amperes
B. Potentiometers
C. Farads
D. Volts
E. Henrys

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A. Amperes
B. Potentiometers
C. Farads
✔ **D. Volts**
E. Henrys

New units of the electric field were introduced in this chapter. They are:

A. V/C.
B. N/C.
C. V/m.
D. J/m^2.
E. Ω/m.

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A. V/C.
B. N/C.
✔ **C. V/m.**
D. J/m^2.
E. Ω/m.

The electric potential inside a capacitor

A. is constant.
B. increases linearly from the negative to the positive plate.
C. decreases linearly from the negative to the positive plate.
D. decreases inversely with distance from the negative plate.
E. decreases inversely with the square of the distance from the negative plate.

The electric potential inside a capacitor

A. is constant.

B. increases linearly from the negative to the positive plate.

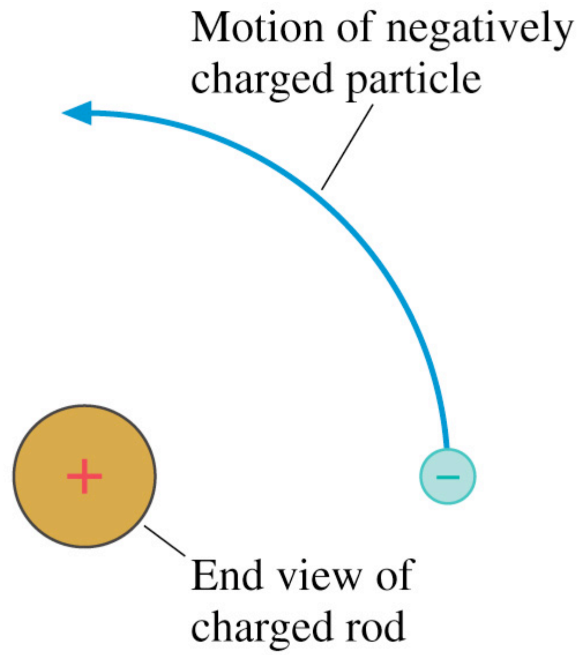
C. decreases linearly from the negative to the positive plate.

D. decreases inversely with distance from the negative plate.

E. decreases inversely with the square of the distance from the negative plate.

Chapter 29. Clicker Questions

The positive charge is the end view of a positively charged glass rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod's electric field positive, negative or zero?



A. Positive
B. Negative
C. Zero

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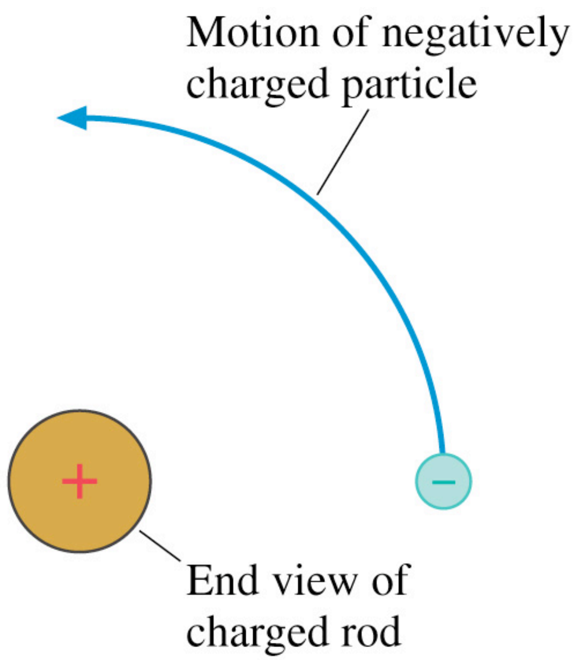


A. Positive
B. Negative
✔ **C. Zero**

Rank in order, from largest to smallest, the potential energies U_a to U_d of these four pairs of charges. Each + symbol represents the same amount of charge.

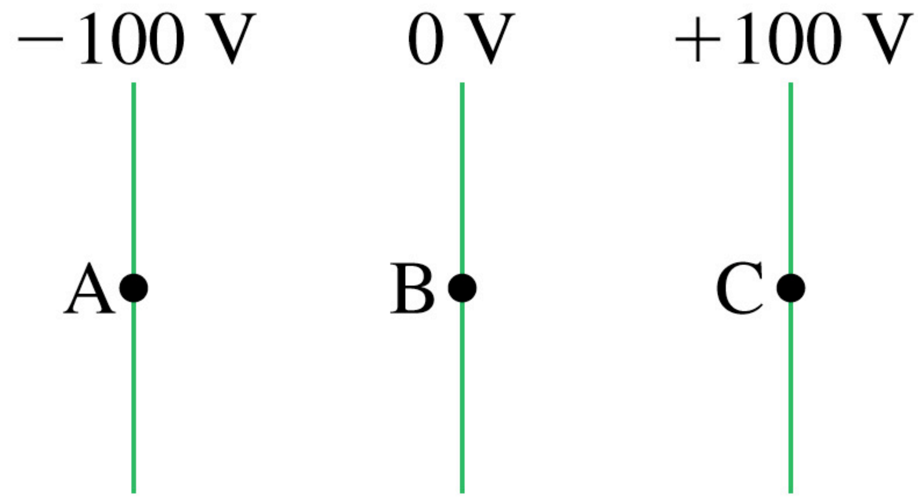
A. $U_a = U_b > U_c = U_d$
B. $U_b = U_d > U_a = U_c$
C. $U_a = U_c > U_b = U_d$
D. $U_d > U_c > U_b > U_a$
E. $U_d > U_b = U_c > U_a$

Rank in order, from largest to smallest, the potential energies U_a to U_d of these four pairs of charges. Each + symbol represents the same amount of charge.

(a) (b) (c) (d)

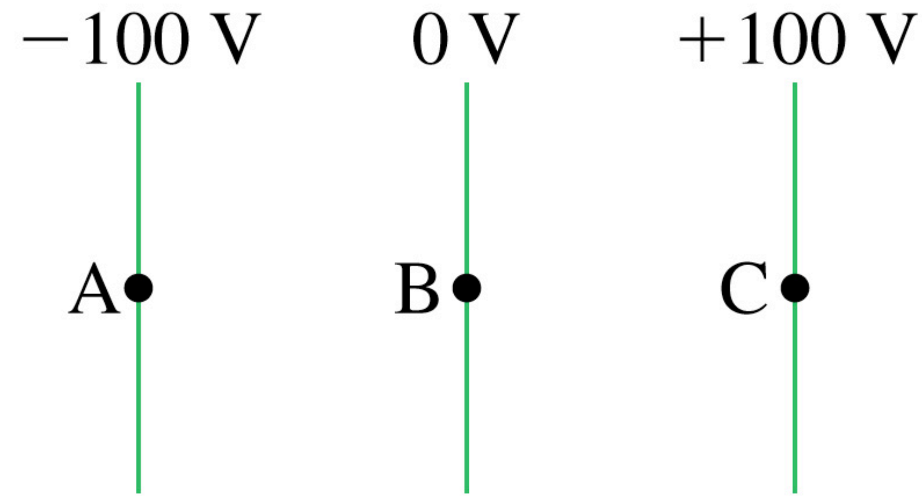
A. $U_a = U_b > U_c = U_d$
B. $U_b = U_d > U_a = U_c$
C. $U_a = U_c > U_b = U_d$
D. $U_d > U_c > U_b > U_a$
E. $U_d > U_b = U_c > U_a$

A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton



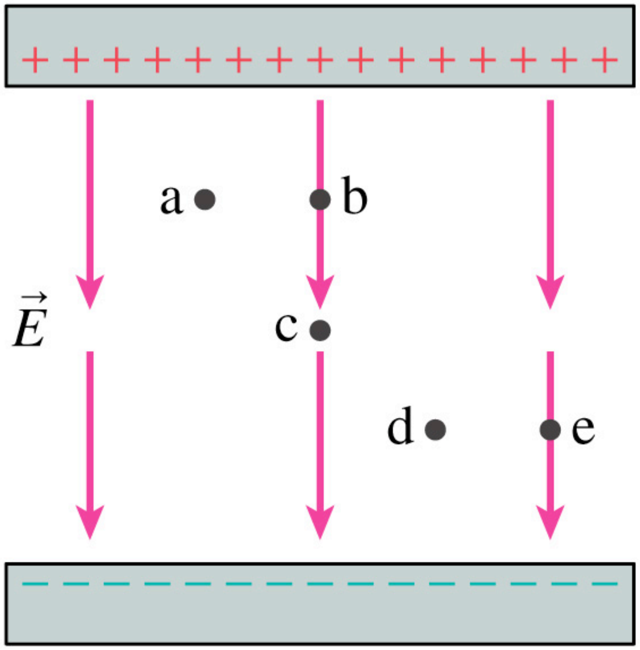
A. moves toward A with a steady speed.
B. moves toward A with an increasing speed.
C. moves toward C with a steady speed.
D. moves toward C with an increasing speed.
E. remains at rest at B.

A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton



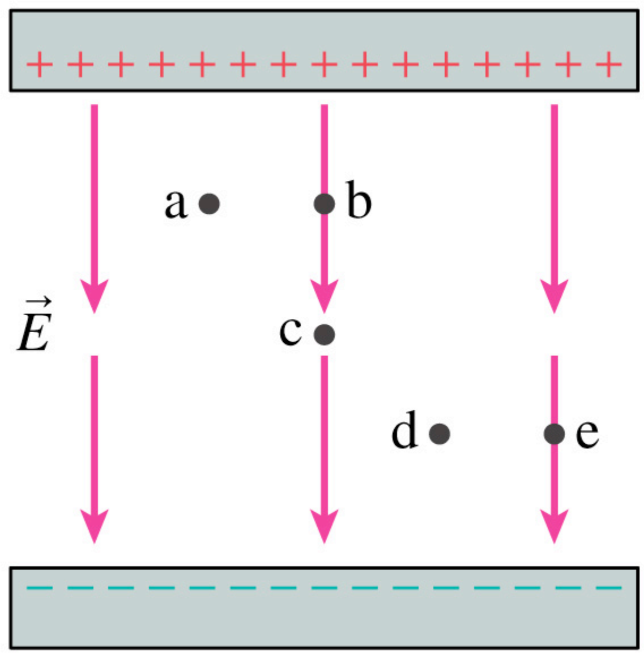
A. moves toward A with a steady speed.
✔ **B. moves toward A with an increasing speed.**
C. moves toward C with a steady speed.
D. moves toward C with an increasing speed.
E. remains at rest at B.

Rank in order, from largest to smallest, the potentials V_a to V_e at the points a to e.



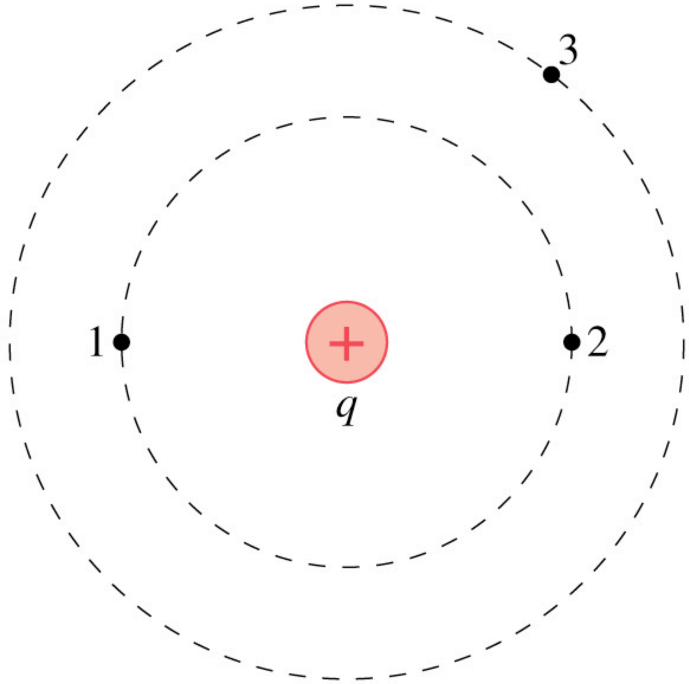
A. $V_d = V_e > V_c > V_a = V_b$
B. $V_b = V_c = V_e > V_a = V_d$
C. $V_a = V_b = V_c = V_d = V_e$
D. $V_a = V_b > V_c > V_d = V_e$
E. $V_a = V_b = V_d = V_e > V_c$

Rank in order, from largest to smallest, the potentials V_a to V_e at the points a to e.



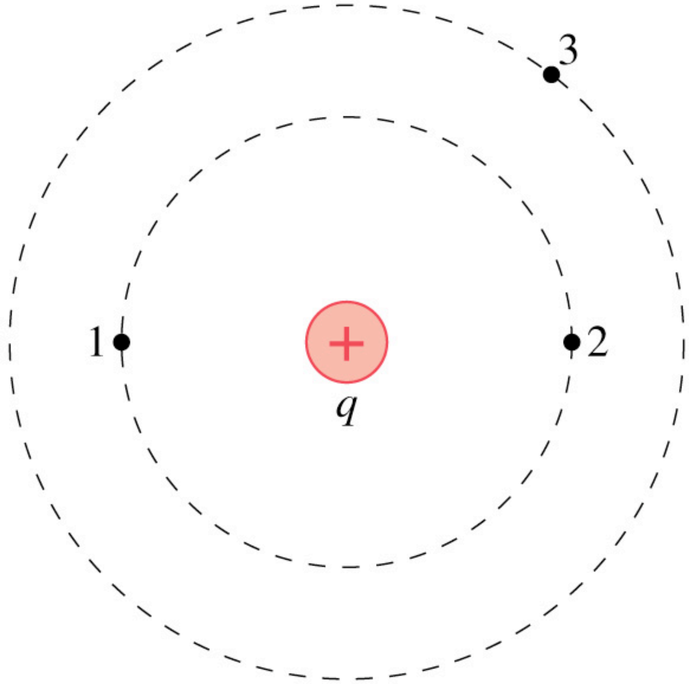
A. $V_d = V_e > V_c > V_a = V_b$
B. $V_b = V_c = V_e > V_a = V_d$
C. $V_a = V_b = V_c = V_d = V_e$
✔ **D.** $\boldsymbol{V_a = V_b > V_c > V_d = V_e}$
E. $V_a = V_b = V_d = V_e > V_c$

Rank in order, from largest to smallest, the potential differences ΔV_{12}, ΔV_{13}, and ΔV_{23} between points 1 and 2, points 1 and 3, and points 2 and 3.



A. $\Delta V_{13} > \Delta V_{12} > \Delta V_{23}$
B. $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$
C. $\Delta V_{13} > \Delta V_{23} > \Delta V_{12}$
D. $\Delta V_{12} > \Delta V_{13} = \Delta V_{23}$
E. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$

Rank in order, from largest to smallest, the potential differences ΔV_{12}, ΔV_{13}, and ΔV_{23} between points 1 and 2, points 1 and 3, and points 2 and 3.



A. $\Delta V_{13} > \Delta V_{12} > \Delta V_{23}$

✔ **B.** $\boldsymbol{\Delta V_{13} = \Delta V_{23} > \Delta V_{12}}$

C. $\Delta V_{13} > \Delta V_{23} > \Delta V_{12}$

D. $\Delta V_{12} > \Delta V_{13} = \Delta V_{23}$

E. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$