# Chapter 27. The Electric Field - Part 2

Electric fields are responsible for the electric currents that flow through your computer and the nerves in your body. Electric fields also line up polymer molecules to form the images in a liquid crystal display (LCD).

**Chapter Goal:** To learn how to calculate and use the electric field.

# Chapter 27. The Electric Field

**Topics:**

- Electric Field Models
- The Electric Field of Multiple Point Charges
- The Electric Field of a Continuous Charge Distribution
- The Electric Fields of Rings, Disks, Planes, and Spheres
- The Parallel-Plate Capacitor
- Motion of a Charged Particle in an Electric Field
- Motion of a Dipole in an Electric Field

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To be discused today

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- Motion of a Charged Particle in an E Field
- Motion of a Dipole in an Electric Field

# Learning Objectives

- Here is what you should get out of this weeks lectures and Chapter 27
  - To use the principle of superposition to calculate the electric field of multiple charges and of continuous charge distributions
  - To learn the electric field of common charge distributions
  - To learn the electric field of a parallel-plate capacitor and some of its applications
  - To study the motion of charged particles and dipoles in simple electric fields

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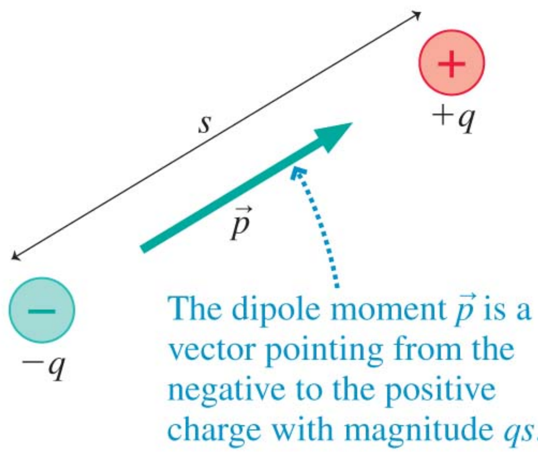
# Brief Review of the Electric Field of a Dipole

We can represent an electric dipole by two opposite charges $\pm q$ separated by the small distance $s$.
The dipole moment is defined as the vector

$$\vec{p} = (qs, \text{from the negative to the positive charge})$$

The dipole-moment magnitude $p = qs$ determines the electric field strength. The SI units of the dipole moment are C m.



# The Electric Field of a Dipole

The electric field at a point on the axis of a dipole is

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \qquad \text{(on the axis of an electric dipole)}$$

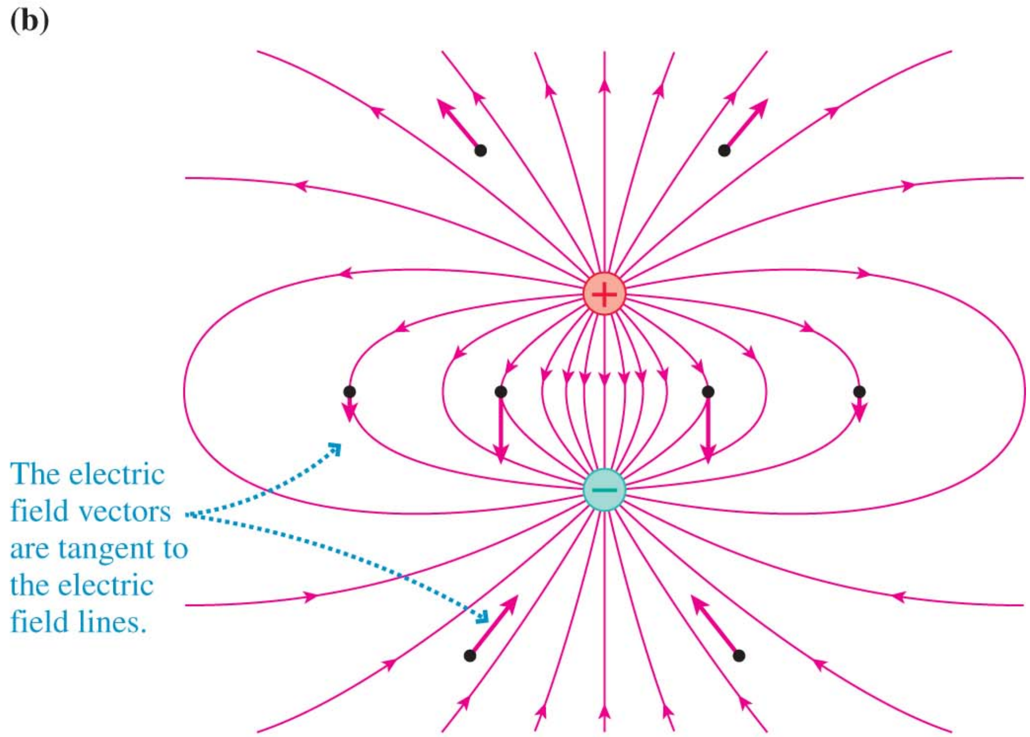
where $r$ is the distance measured from the *center* of the dipole.

The electric field in the plane that bisects and is perpendicular to the dipole is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \qquad \text{(perpendicular plane)}$$

This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

**FIGURE 27.9** The electric field of a dipole.

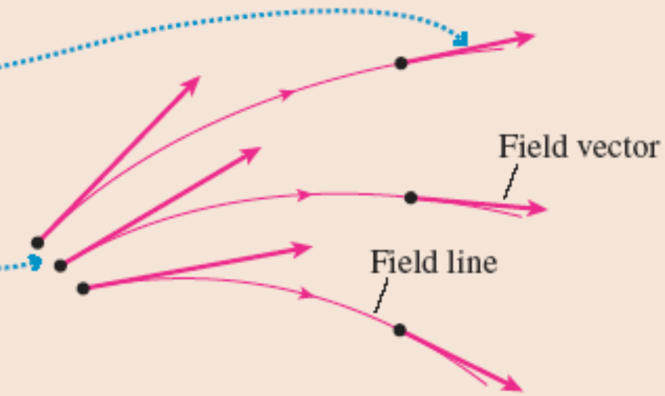


# Tactics: Drawing and using electric field lines

**TACTICS BOX 27.1** **Drawing and using electric field lines**

❶ Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point.

❷ Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.




❸ Electric field lines never cross.

❹ Electric field lines start from positive charges and end on negative charges.

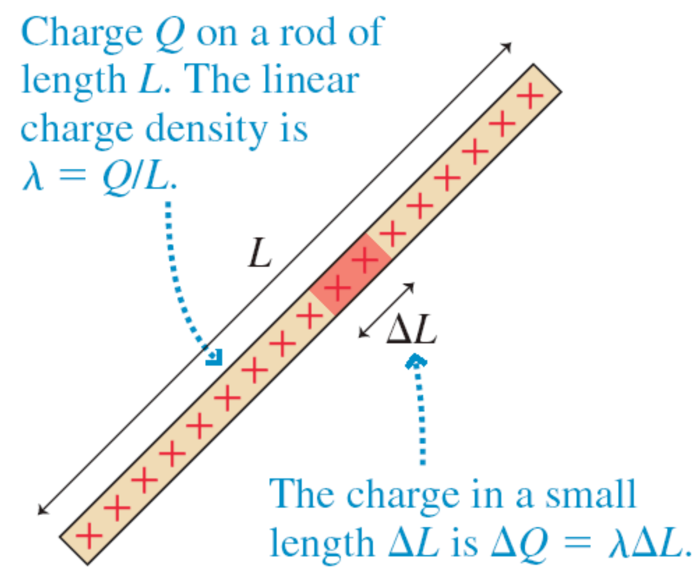
Exercises 2–4, 12, 13

# The Electric Field of a Continuous Charge Distribution

The linear charge density of an object of length $L$ and charge $Q$, is defined as

$$\lambda = \frac{Q}{L}$$

(a)




Linear charge density, which has units of C/m, is the amount of charge *per meter* of length.

# Infinite Line of Charge

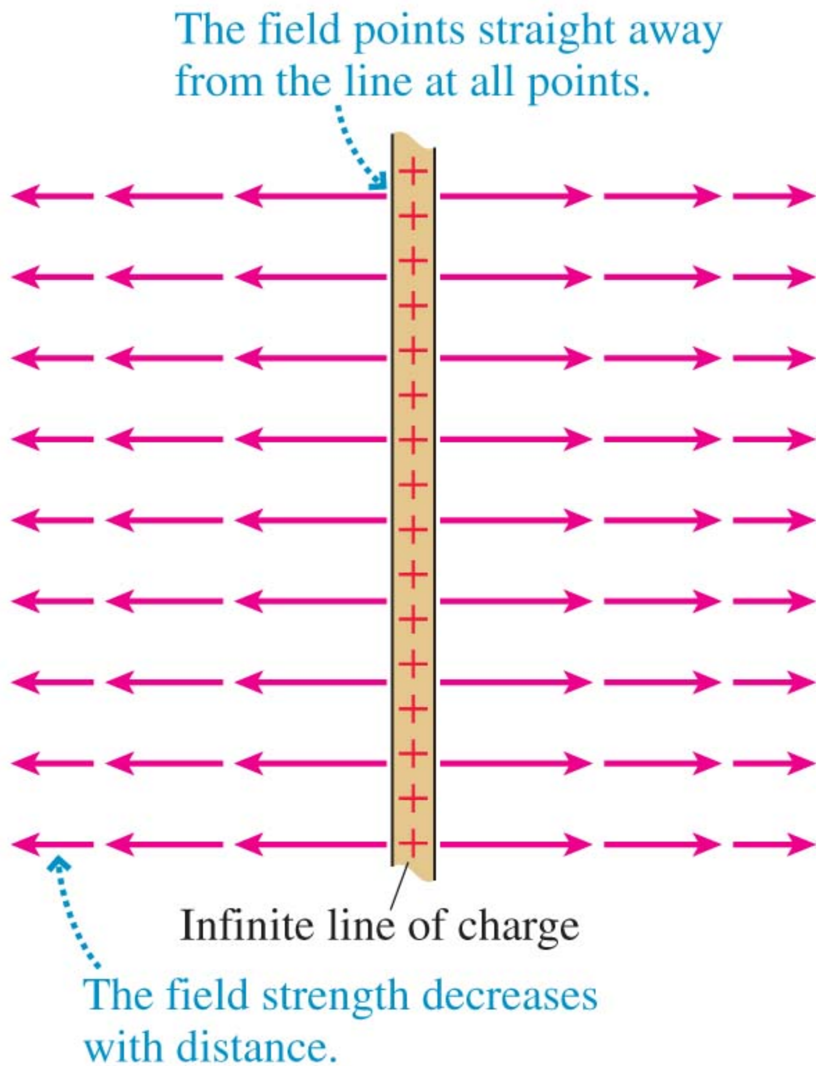
A very long, thin rod, with linear charge density λ=Q/L, has an electric field

$$E_{\text{line}} = \lim_{L\to\infty} \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi\epsilon_0} \frac{2|\lambda|}{r}$$

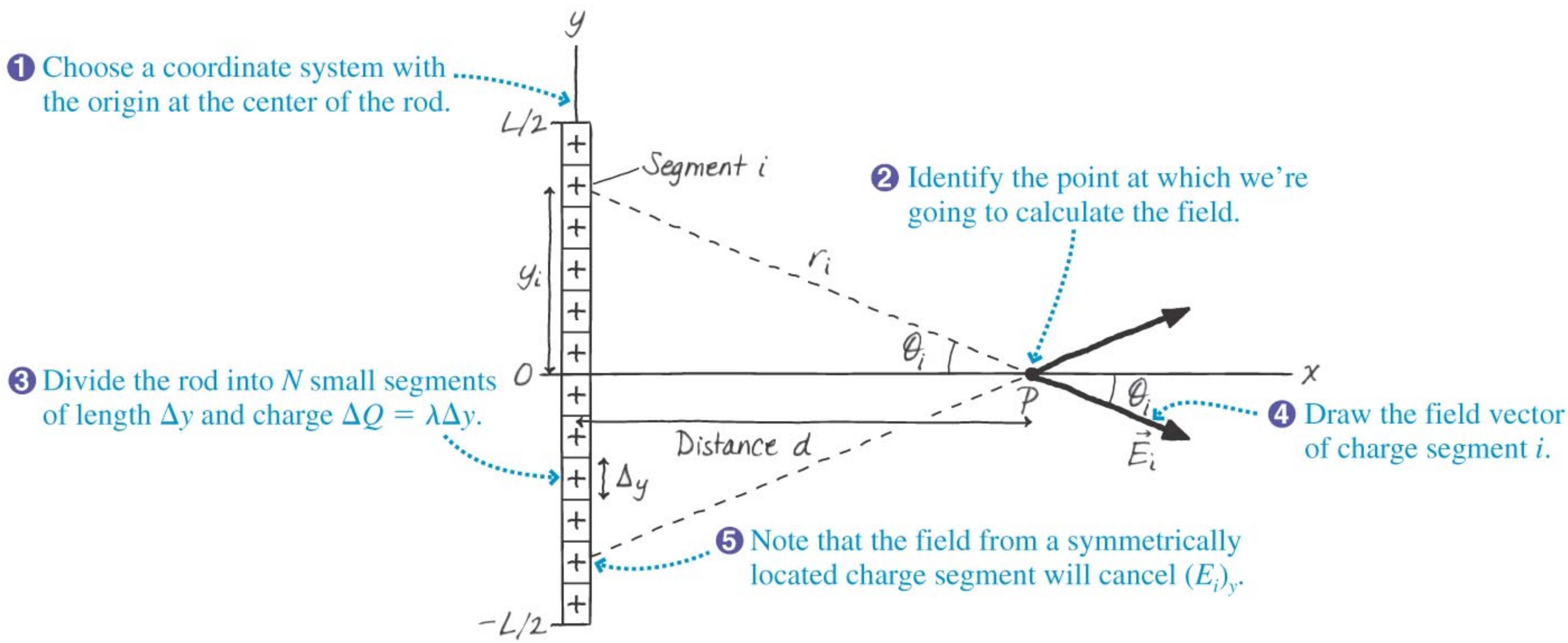
Where *r* is the radial distance away from the rod, Q is the total charge on the rod and L is the total length

**FIGURE 27.14** The electric field of an infinite line of charge.




Recall that we did the example of a rod of length L last lecture

# Electric Field of Line Charge

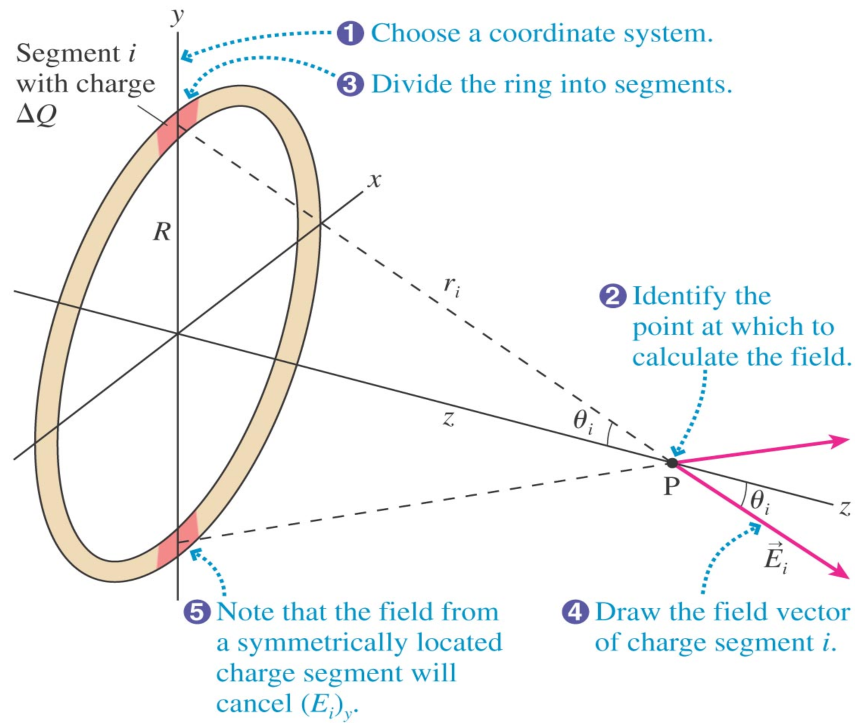


# “Setting up the Integral

1) determine length element ds (=r dθ or dx usually)
2) relate dq to ds (dq=λds usually)
3) relate dE to dq using $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$
4) consider symmetry and look for cancellations of components or entire parts of the field. Add all of the non-cancelling components by integration
5) take constants outside the integral and reduce the integrals to being over a single variable
6) use integration limits that give a +ve result, and relate λ to total charge if possible.

# Now do variation:Ring

- A thin ring of radius R is uniformly charged with total charge Q. Find the electric field at a point on the axis of the ring



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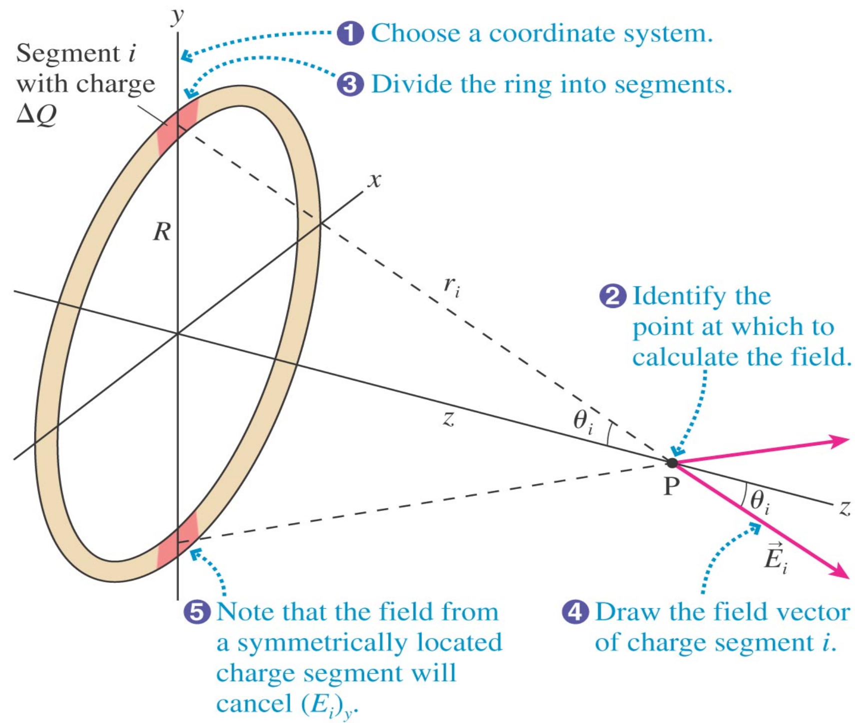


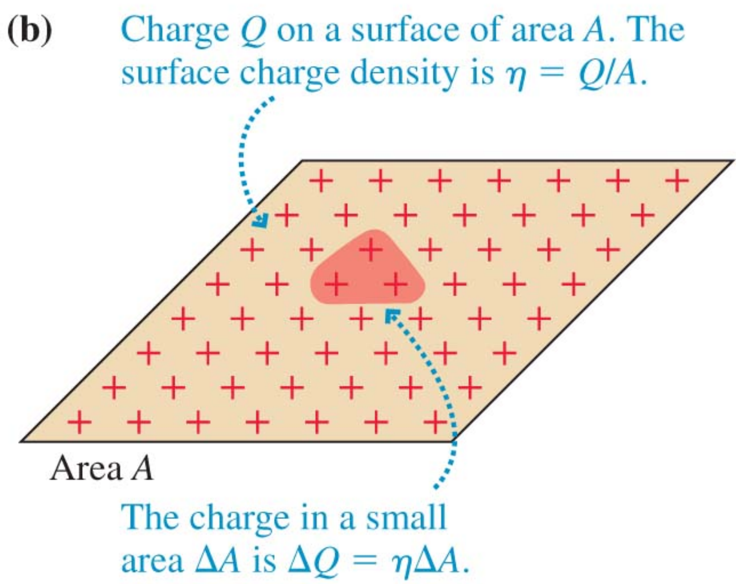

$$(E_{ring})_z = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+R^2)^{3/2}}$$

- Try problem 27.11

# The Electric Field of a Continuous Charge Distribution

The surface charge density of a two-dimensional distribution of charge across a surface of area *A* is defined as

$$\eta = \frac{Q}{A}$$




Surface charge density, with units $C/m^2$, is the amount of charge *per square meter*.

# A Plane of Charge

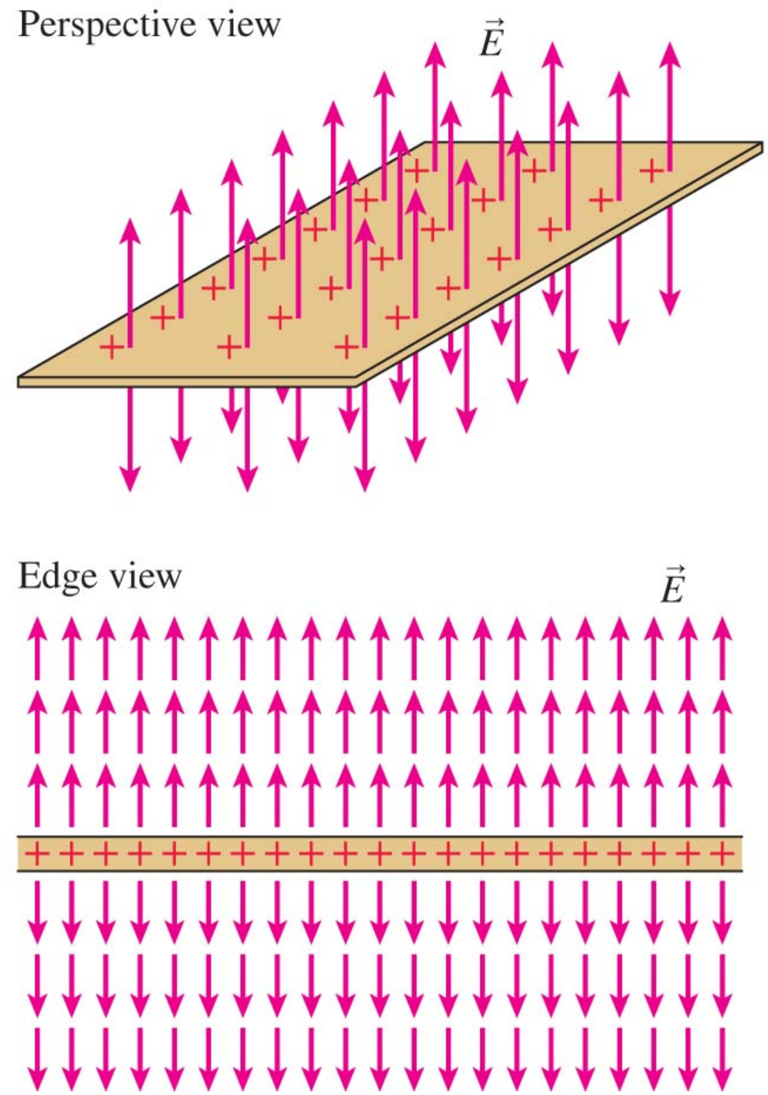
The electric field of an infinite plane of charge with surface charge density $\eta$ is:

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.

For a negatively charged plane, with $\eta < 0$, the electric field points *towards* the plane on both sides of the plane.

**FIGURE 27.18** Two views of the electric field of a plane of charge.



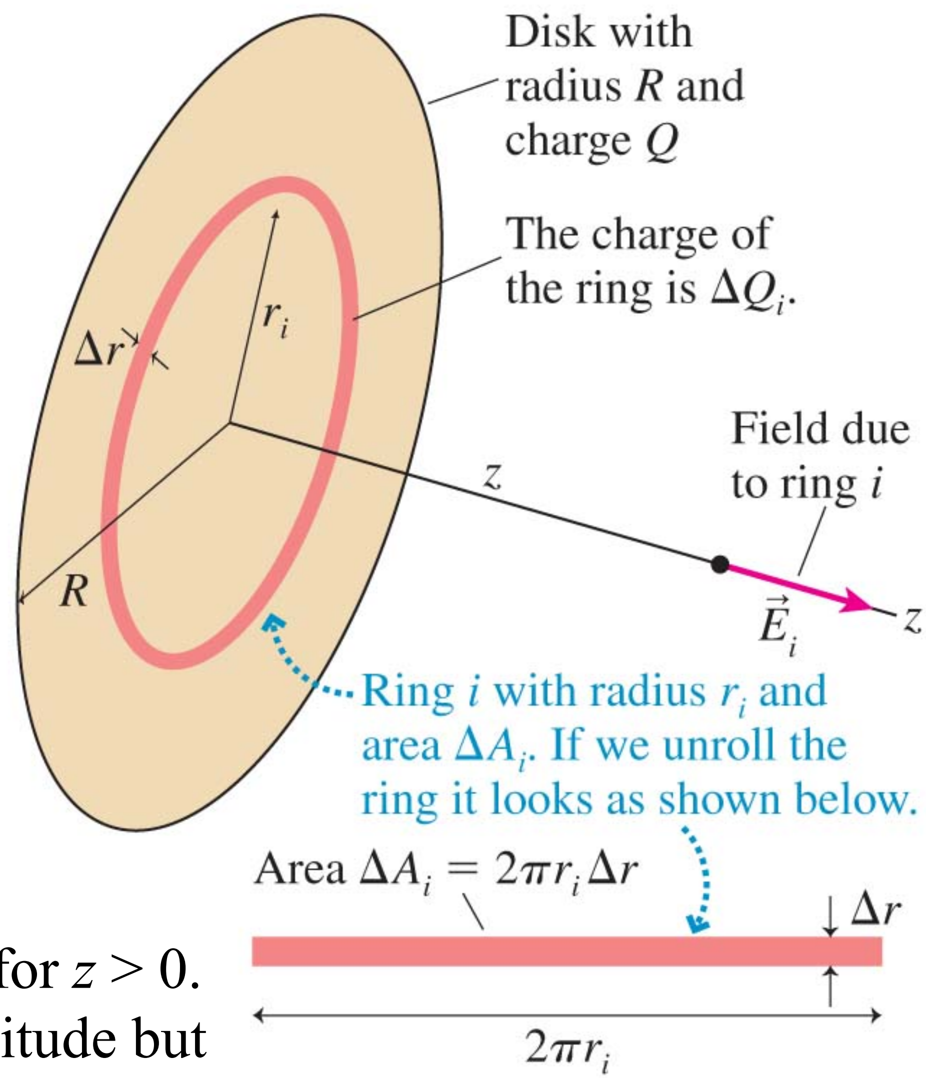
# A Disk of Charge

The on-axis electric field of a charged disk of radius $R$, centered on the origin with axis parallel to $z$, and surface charge density $\eta = Q/\pi R^2$ is

$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

NOTE: This expression is only valid for $z > 0$. The field for $z < 0$ has the same magnitude but points in the opposite direction.

**FIGURE 27.17** Calculating the on-axis field of a charged disk.



# A Sphere of Charge

A sphere of charge $Q$ and radius $R$, be it a uniformly charged sphere or just a spherical shell, has an electric field *outside* the sphere that is exactly the same as that of a point charge $Q$ located at the center of the sphere:

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad \text{for } r \geq R$$

# The Parallel-Plate Capacitor

- The figure shows two electrodes, one with charge $+Q$ and the other with $-Q$ placed face-to-face a distance $d$ apart.
- This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.**
- Capacitors play important roles in many electric circuits.

**FIGURE 27.20** A parallel-plate capacitor.

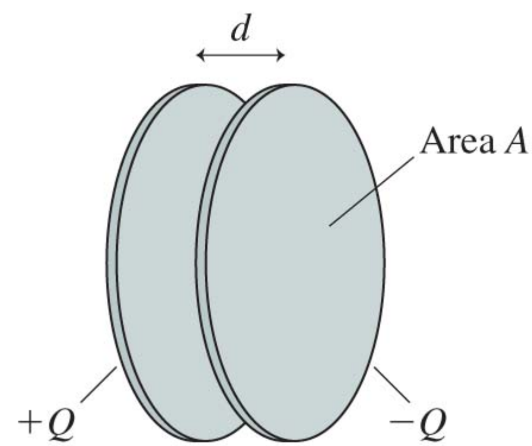
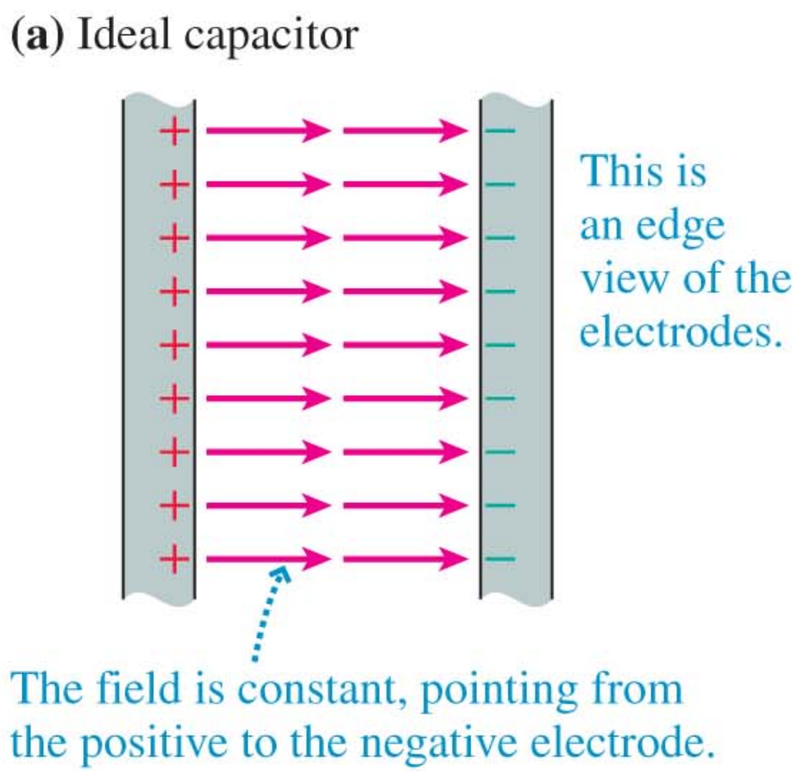


**FIGURE 27.22** The electric field of a capacitor.



# The Parallel-Plate Capacitor

The electric field inside a capacitor is

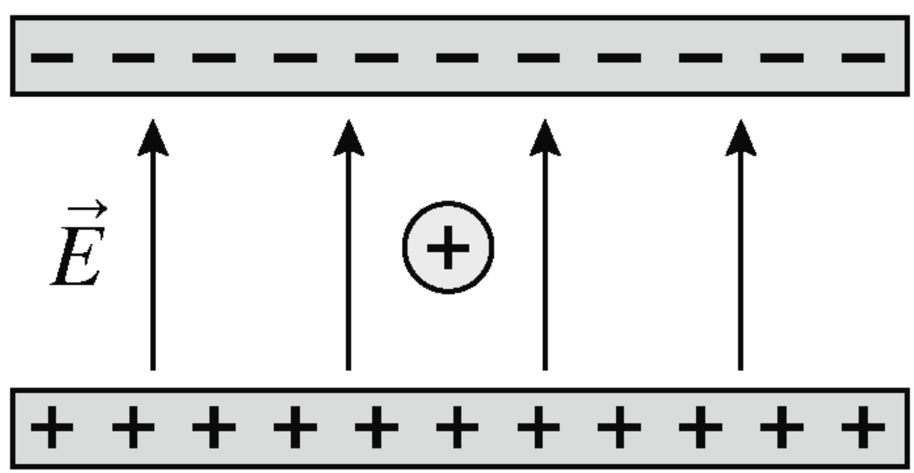
$$\vec{E}_{\text{capacitor}} = \vec{E}_+ + \vec{E}_- = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$

$$= \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

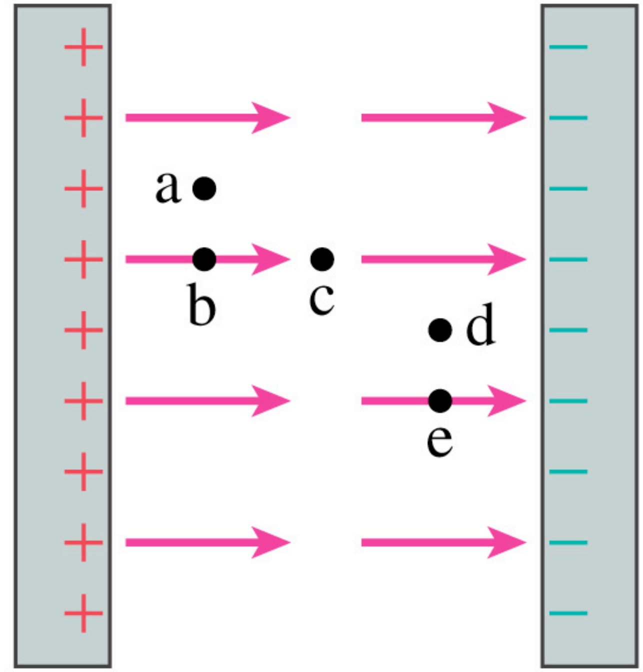
where $A$ is the surface area of each electrode. Outside the capacitor plates, where $E_+$ and $E_-$ have equal magnitudes but *opposite* directions, the electric field is zero.

# Example

- A positively charged particle is in the center of a capacitor. Sketch the particle's trajectory if the particle's initial velocity is (a) zero, (b) straight down, and (c) to the right.



**Rank in order, from largest to smallest, the forces $F_a$ to $F_e$ a proton would experience if placed at points a – e in this parallel-plate capacitor.**




A. $F_a = F_b = F_d = F_e > F_c$
B. $F_a = F_b > F_c > F_d = F_e$
C. $F_a = F_b = F_c = F_d = F_e$
D. $F_e = F_d > F_c > F_a = F_b$
E. $F_e > F_d > F_c > F_b > F_a$

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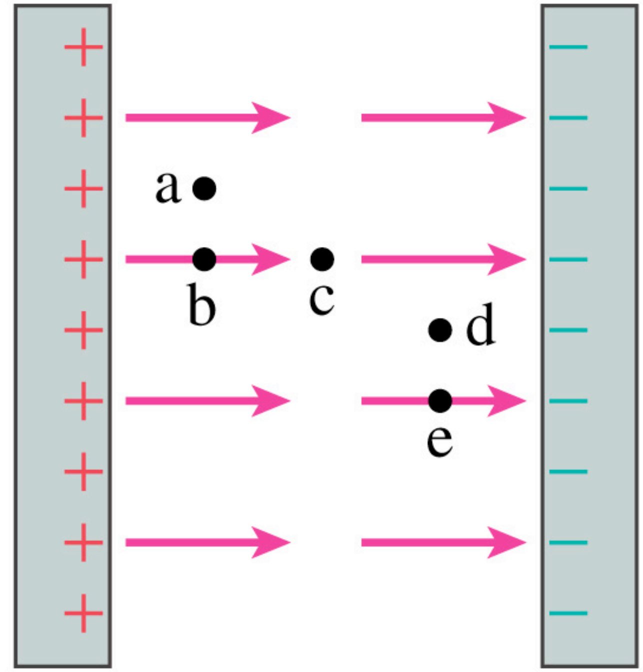


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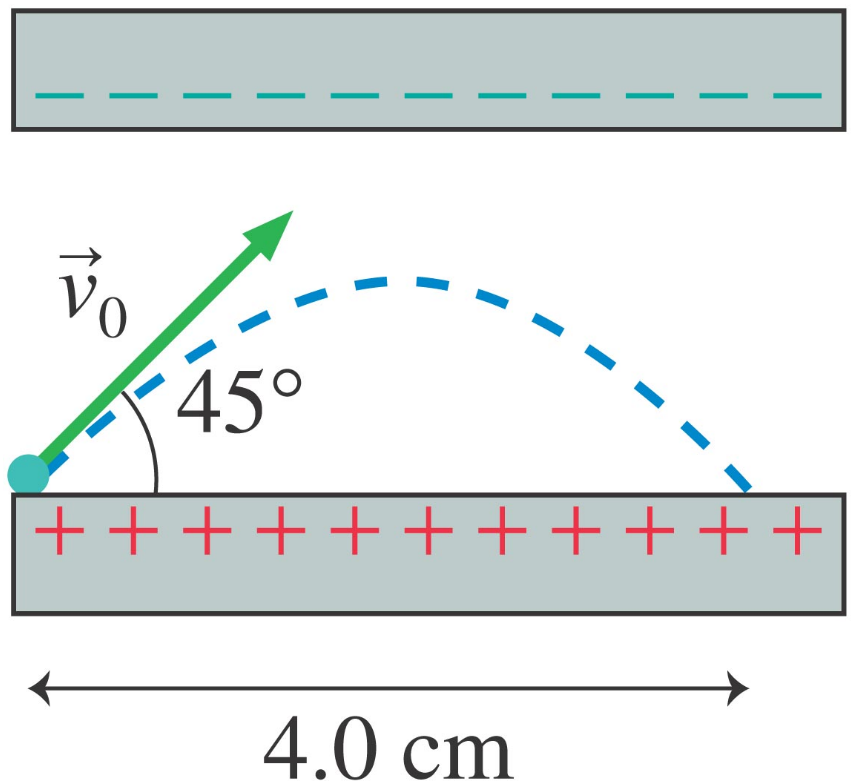
D. $F_e = F_d > F_c > F_a = F_b$
E. $F_e > F_d > F_c > F_b > F_a$

# Example: The electric field inside a capacitor

- Two 1.0 cm x 2.0 cm rectangular electrodes are 1.0 mm apart. What charge must be placed on each electrode to create a uniform electric field of strength 2.0 x $10^6$ N/C?
- How many electrons must be moved from one electrode to the other to accomplish this?

# Example

- An electron is launched at a 45 degree angle and a speed of $5.0 \times 10^6$ m/s. from the positive plate of a parallel plate capacitor as shown. The electron lands 4.0 cm away
  a) What is the electric field strength inside the capacitor?
  b) What is the smallest possible spacing between the plates?



# Motion of a Charged Particle in an Electric Field

The electric field exerts a force

$$\vec{F}_{\text{on } q} = q\vec{E}$$

on a charged particle. If this is the only force acting on $q$, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

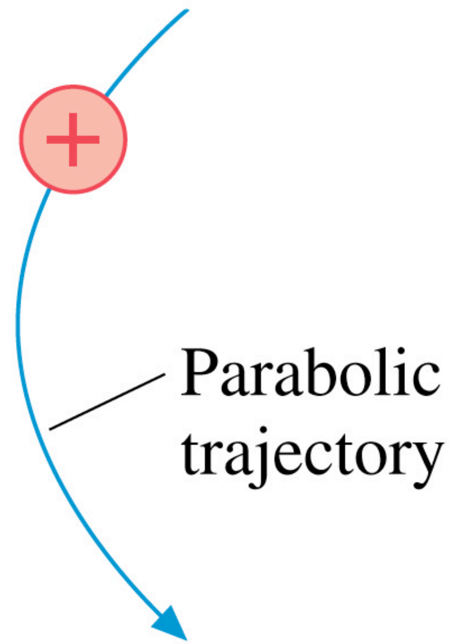
In a uniform field, the acceleration is constant:

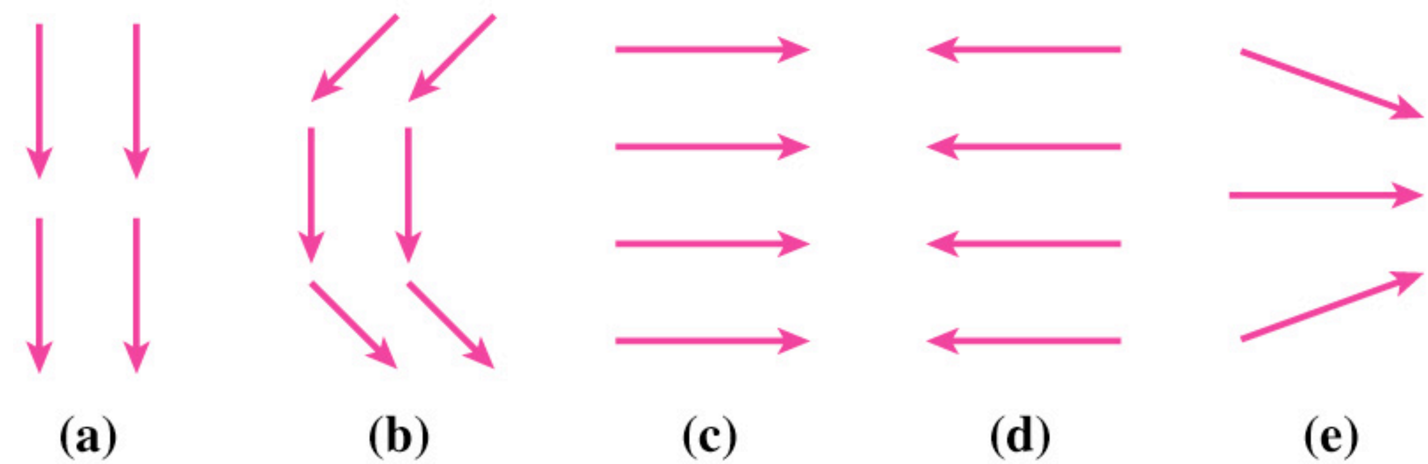
$$\boxed{a = \frac{qE}{m} = \text{constant}}$$

## Example

- An electron is released from rest 2.0 cm from an infinite charged plane. It accelerates toward the plane and collides with a speed of $1.0 \times 10^7$ m/s. What are
  a) The surface charge density of the plane?
  b) The time required for the electron to travel the 2.0 cm?

## Which electric field is responsible for the trajectory of the proton?



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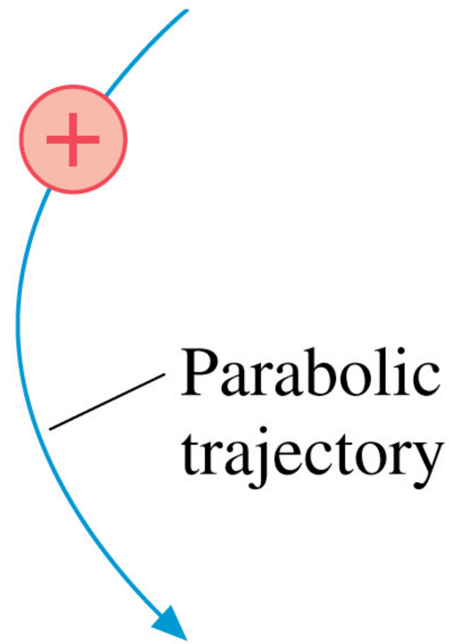


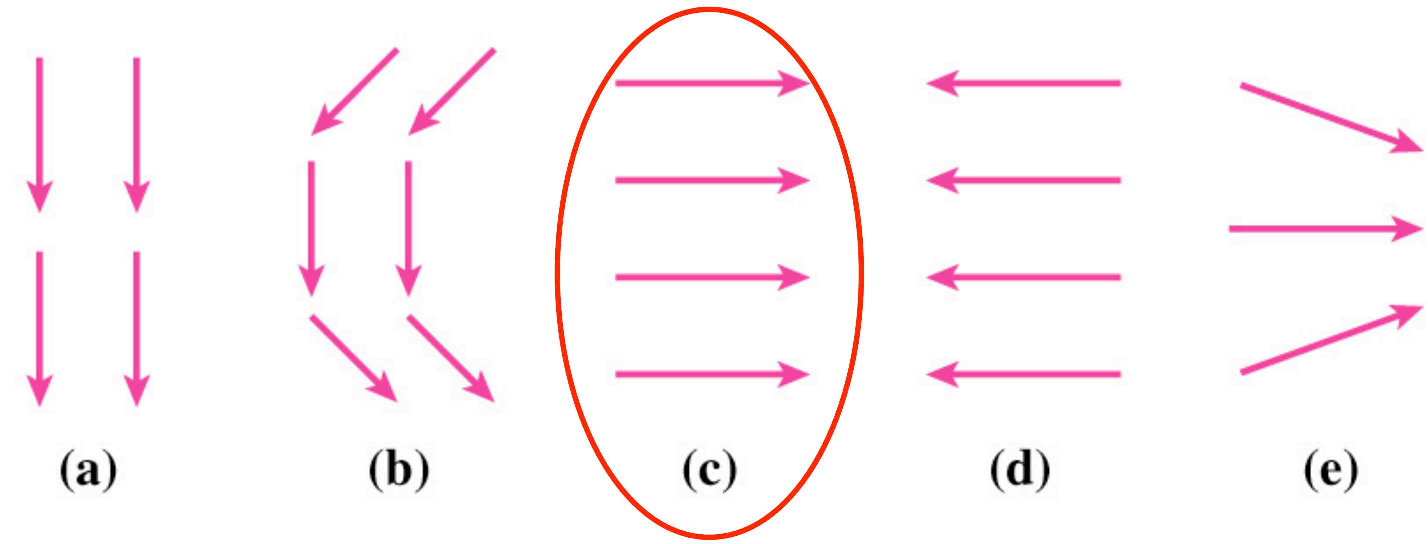

# Brief Review of the Electric Field of a Dipole

We can represent an electric dipole by two opposite charges $\pm q$ separated by the small distance $s$.
The dipole moment is defined as the vector

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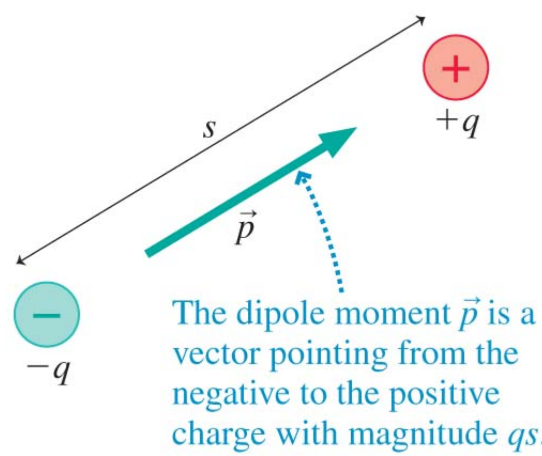
The dipole-moment magnitude $p = qs$ determines the electric field strength. The SI units of the dipole moment are C m.



# The Electric Field of a Dipole

The electric field at a point on the axis of a dipole is

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \qquad \text{(on the axis of an electric dipole)}$$

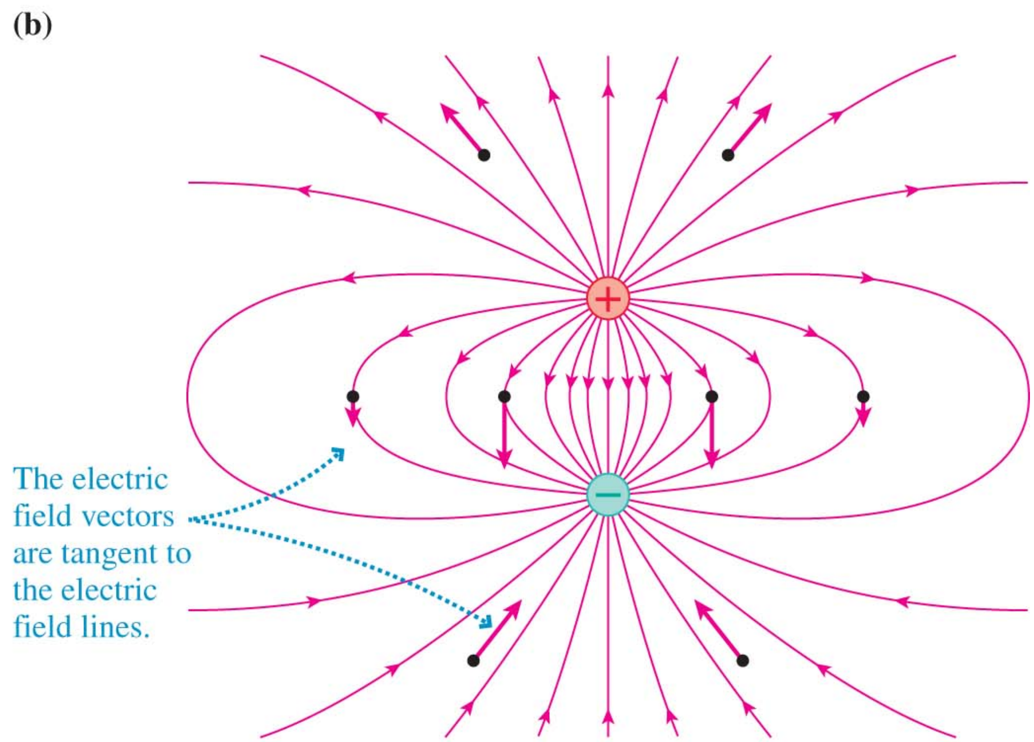
where $r$ is the distance measured from the *center* of the dipole.

The electric field in the plane that bisects and is perpendicular to the dipole is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \qquad \text{(perpendicular plane)}$$

This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

**FIGURE 27.9** The electric field of a dipole.



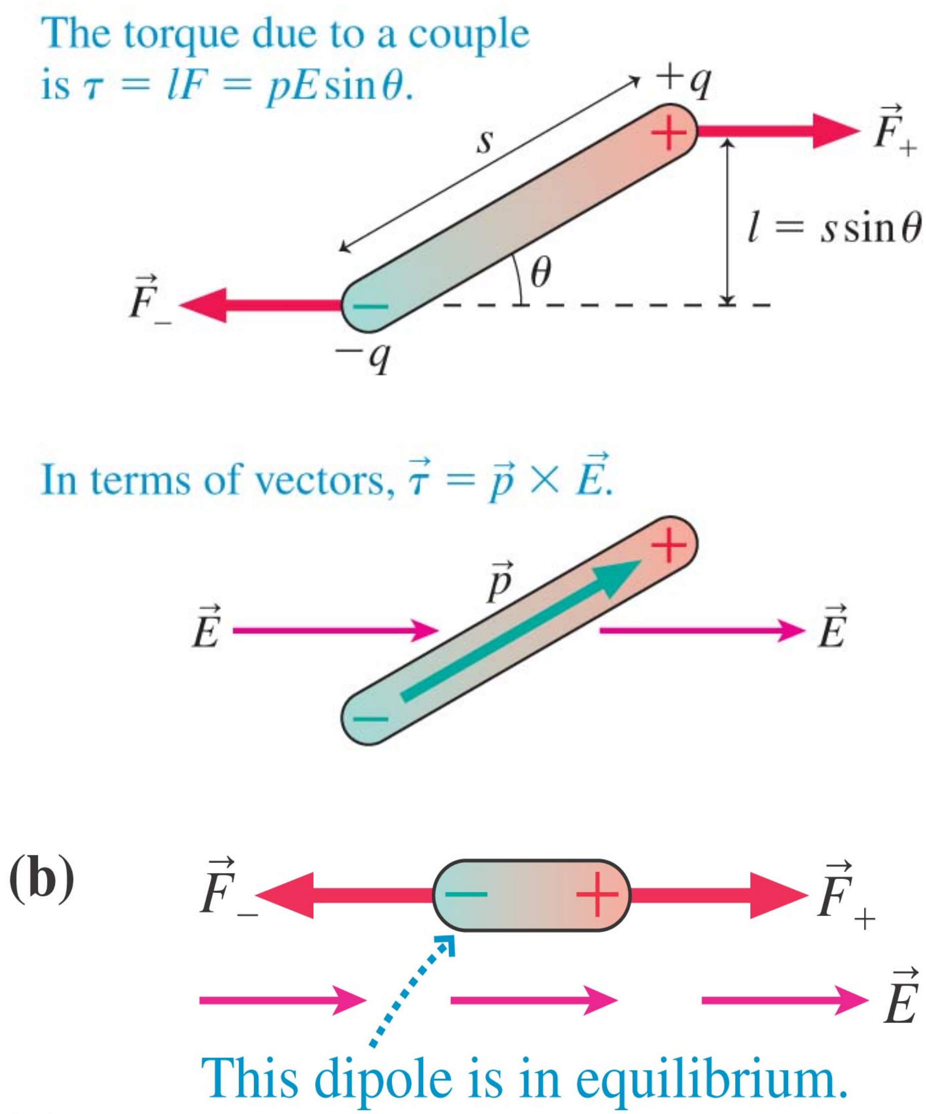
# Dipoles in an Electric Field

The torque on a dipole in an electric field is

$$\tau = lF = (s\sin\theta)(qE) = pE\sin\theta$$

where $\theta$ is the angle the dipole makes with the electric field.

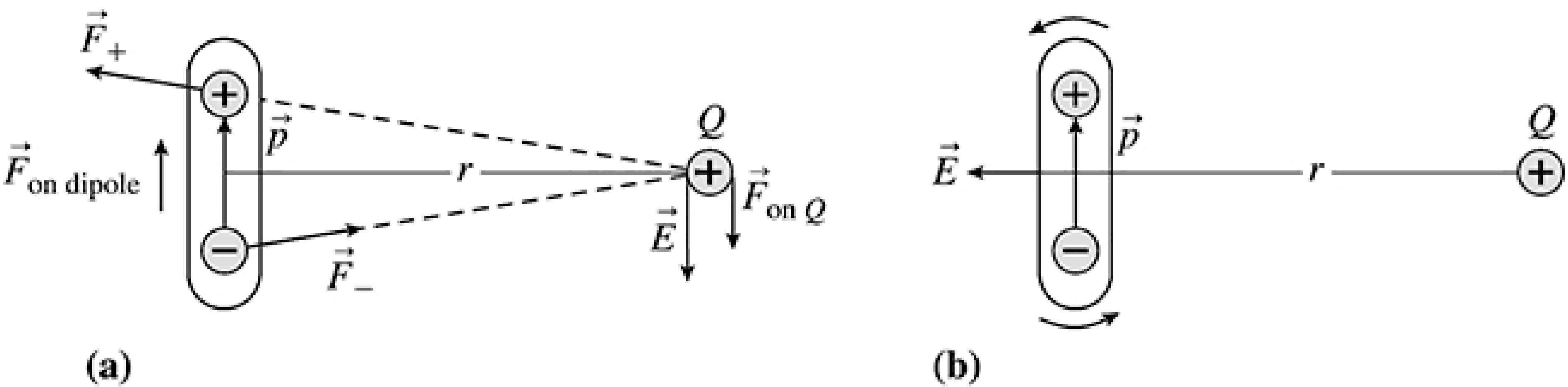
**FIGURE 27.30** The torque on a dipole.



# Example

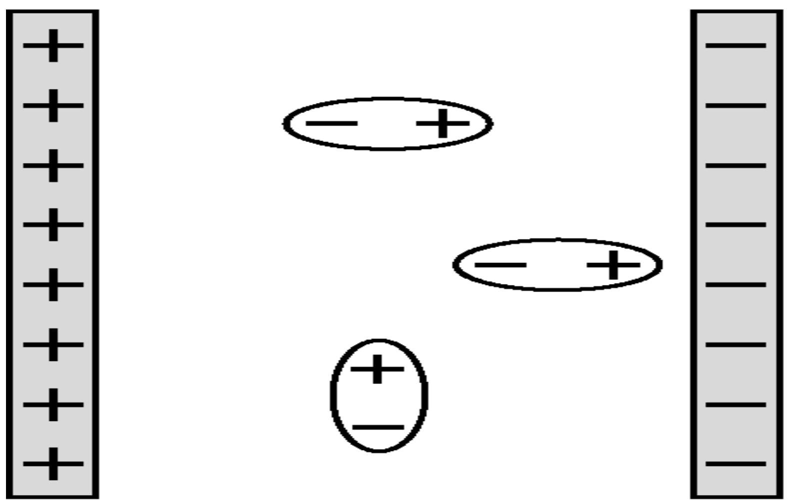
- A point charge $Q$ is a distance $r$ from the center of a dipole consisting of charges $\pm q$ separated by a distance $s$. The charge is located in the plane that bisects the dipole. At this instant, what are
  a) The force (magnitude and directions) on the dipole
  b) The torque on the dipole

  Assuming that $r \gg s$



# Example

- Three dipoles are between the plates of a capacitor. For each dipole, does it
  - a. not move,
  - b. move to the right,
  - c. move to the left,
  - d. rotate clockwise,
  - e. rotate counterclockwise
  - f. some combination of these?



# Brief start to Chapter 28: Gauss's Law

## Symmetry

Some charge distributions have translational, rotational, or reflective symmetry. If this is the case, we can determine something about the field it produces:
**The symmetry of an electric field must match the symmetry of the charge distribution.**

**Symmetry can tell us the shape of the electric field**

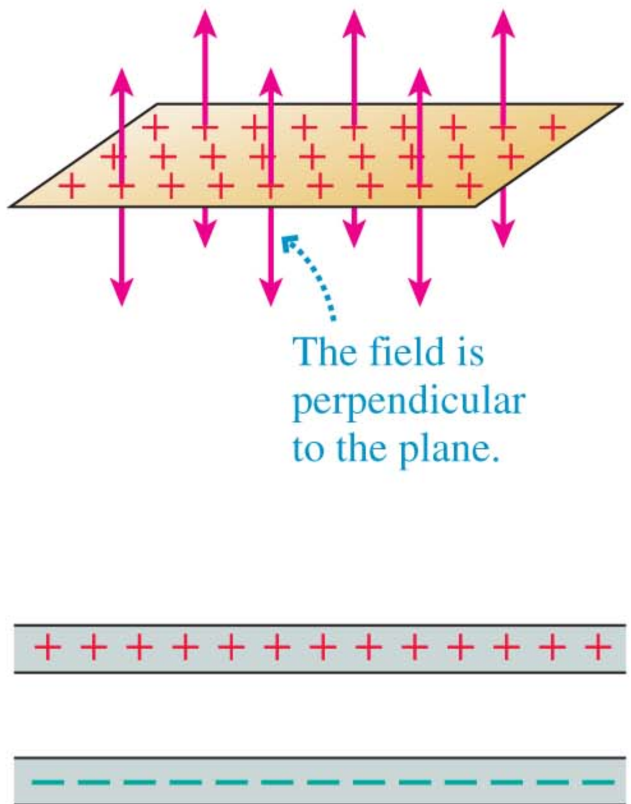
For example, the electric field of a cylindrically symmetric charge distribution
a) cannot have a component parallel to the cylinder axis.
b) cannot have a component tangent to the circular cross section.

**FIGURE 28.6** Three fundamental symmetries.

**Planar symmetry**

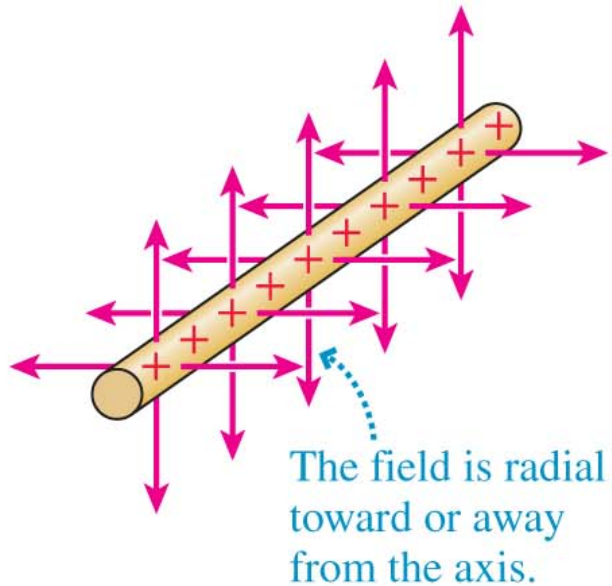
Basic symmetry:

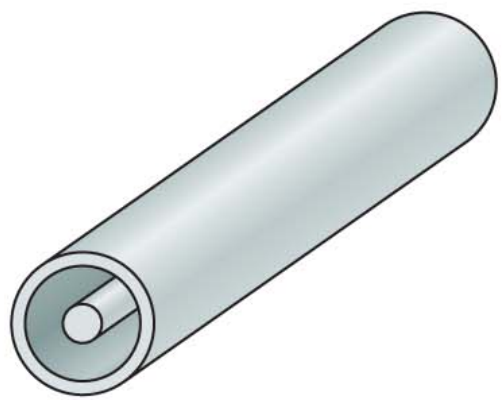
More complex example:




Infinite parallel-plate capacitor

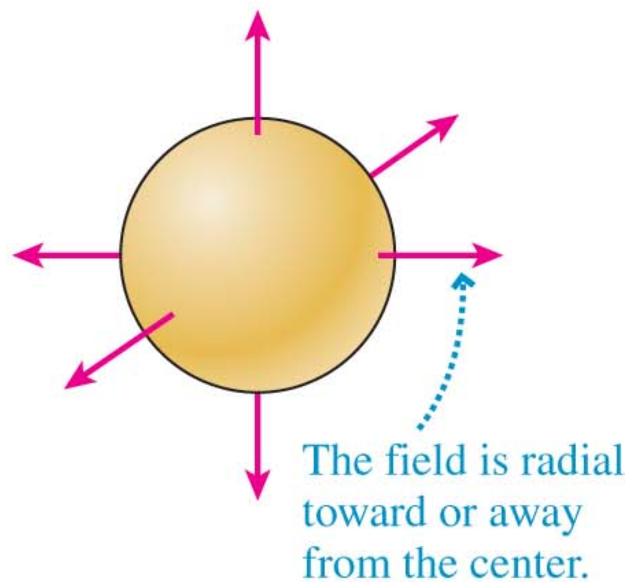
**Cylindrical symmetry**

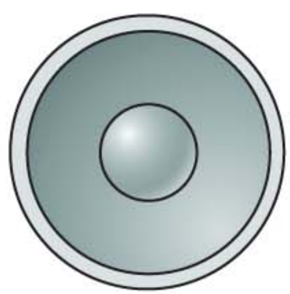


Coaxial cylinders

**Spherical symmetry**



Concentric spheres

# Chapter 27. Summary Slides

# General Principles

## Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

- The principle of superposition

### Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$

### Continuous distribution of charge

- Divide the charge into segments $\Delta Q$ for which you already know the field.
- Find the field of each $\Delta Q$.
- Find $\vec{E}$ by summing the fields of all $\Delta Q$.

The summation usually becomes an integral. A critical step is replacing $\Delta Q$ with an expression involving a **charge density** ($\lambda$ or $\eta$) and an integration coordinate.

# General Principles

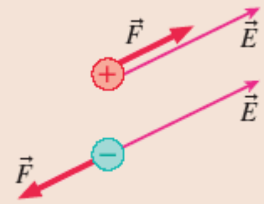
## Consequences of $\vec{E}$

The electric field exerts a force on a charged particle:

$$\vec{F} = q\vec{E}$$

The force causes acceleration:

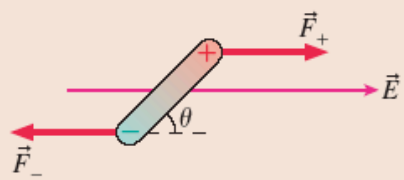
$$\vec{a} = (q/m)\vec{E}$$




Trajectories of charged particles are calculated with kinematics.

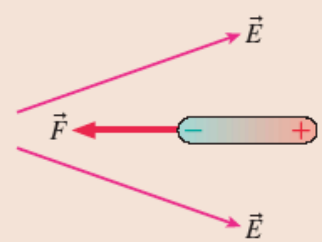
The electric field exerts a torque on a dipole:

$$\tau = pE\sin\theta$$



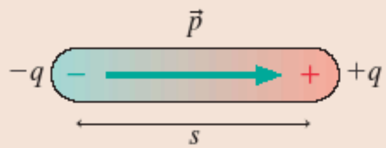

The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.



# Applications

**Electric dipole**



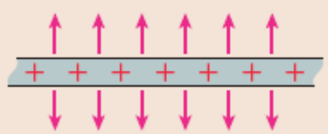

The electric dipole moment is

$\vec{p} = (qs$, from negative to positive)

Field on axis: $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{2\vec{p}}{r^3}$

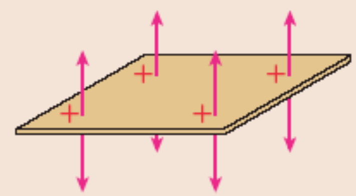
Field in bisecting plane: $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{\vec{p}}{r^3}$

**Infinite line of charge** with linear charge density $\lambda$



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}, \text{ perpendicular to line}\right)$$

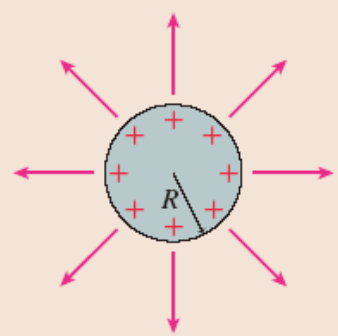
**Infinite plane of charge** with surface charge density $\eta$



$$\vec{E} = \left(\frac{\eta}{2\epsilon_0}, \text{ perpendicular to plane}\right)$$

**Sphere of charge**

Same as a point charge $Q$ for $r > R$

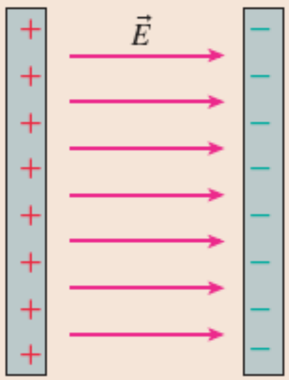


# Applications

## Parallel-plate capacitor

The electric field inside an ideal capacitor is a **uniform electric field:**

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$




A real capacitor has a weak **fringe field** around it.