Chapter 27. The Electric Field

Electric fields are responsible for the electric currents that flow through your computer and the nerves in your body. Electric fields also line up polymer molecules to form the images in a liquid crystal display (LCD).

Chapter Goal: To learn how to calculate and use the electric field.

Chapter 27. The Electric Field

Topics:

- Electric Field Models
- The Electric Field of Multiple Point Charges
- The Electric Field of a Continuous Charge Distribution
- The Electric Fields of Rings, Disks, Planes, and Spheres
- The Parallel-Plate Capacitor
- Motion of a Charged Particle in an Electric Field
- Motion of a Dipole in an Electric Field

Chapter 27. Reading Quizzes

What device provides a practical way to produce a uniform electric field?

A. A long thin resistor
B. A Faraday cage
C. A parallel plate capacitor
D. A toroidal inductor
E. An electric field uniformizer

What device provides a practical way to produce a uniform electric field?

A. A long thin resistor
B. A Faraday cage
C. A parallel plate capacitor ✔
D. A toroidal inductor
E. An electric field uniformizer

For charged particles, what is the quantity q/m called?

A. Linear charge density
B. Charge-to-mass ratio
C. Charged mass density
D. Massive electric dipole
E. Quadrupole moment

For charged particles, what is the quantity q/m called?

A. Linear charge density
✔ **B. Charge-to-mass ratio**
C. Charged mass density
D. Massive electric dipole
E. Quadrupole moment

Which of these charge distributions did *not* have its electric field determined in Chapter 27?

A. A line of charge
B. A parallel-plate capacitor
C. A ring of charge
D. A plane of charge
E. They were *all* determined

Which of these charge distributions did *not* have its electric field determined in Chapter 27?

A. A line of charge
B. A parallel-plate capacitor
C. A ring of charge
D. A plane of charge
✔ **E. They were *all* determined**

The worked examples of charged-particle motion are relevant to

A. a transistor.
B. a cathode ray tube.
C. magnetic resonance imaging.
D. cosmic rays.
E. lasers.

The worked examples of charged-particle motion are relevant to

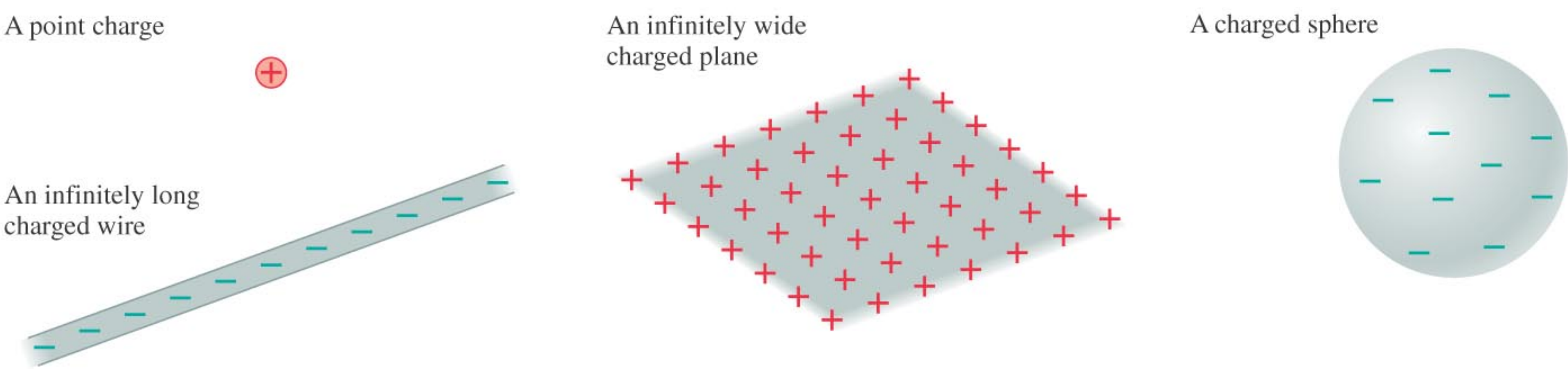
A. a transistor.
B. a cathode ray tube.
C. magnetic resonance imaging.
D. cosmic rays.
E. lasers.

Learning Objectives

- Here is what you should get out of this weeks lectures and Chapter 27
  - To use the principle of superposition to calculate the electric field of multiple charges and of continuous charge distributions
  - To learn the electric field of common charge distributions
  - To learn the electric field of a parallel-plate capacitor and some of its applications
  - To study the motion of charged particles and dipoles in simple electric fields

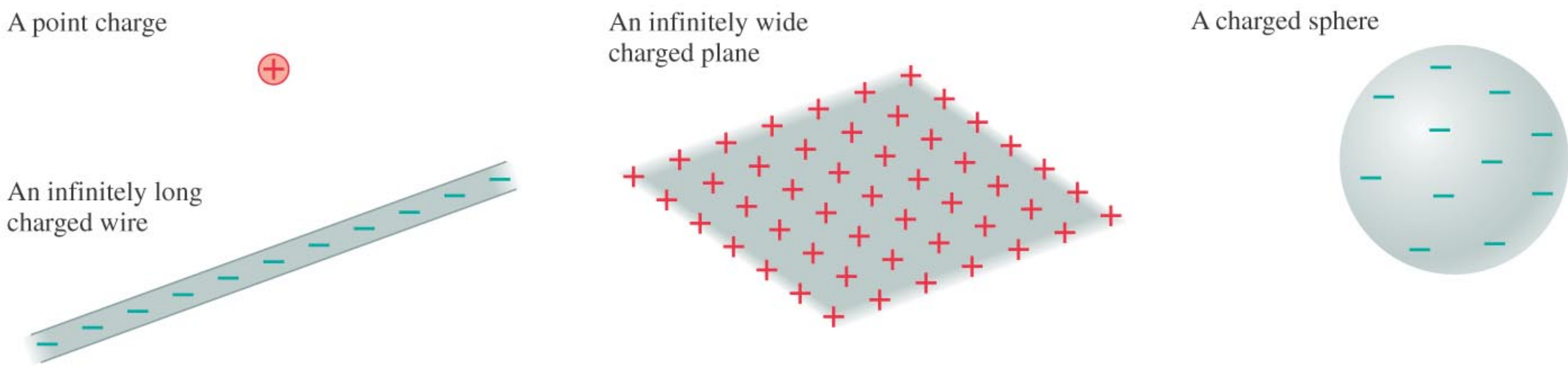
Physical Examples

- Last class we introduce the concept of electric fields to explain the interaction of charge at a distance
  - Electric Fields are responsible for the functioning of your LCD monitor, stereo, heart, brain
- Last class we introduced the electric field of a point charge
  - To make practical use of electric fields we need to calculate the electric field due to complicated distributions of charge - note that these are sources
- Primary goal today is to develop procedures for calculating the electric field for specific arrangements of charge
  - After we know how to calculate electric field due to various source charge distributions we can compute the force on a test charge placed at some position in the resulting electric field



Commonly Used Electric Field Models

- Electric Field of a point charge
- Electric Field of an infinite line of charge
- Electric Field of an infinite plan of charge
- Electric Field of a charged sphere

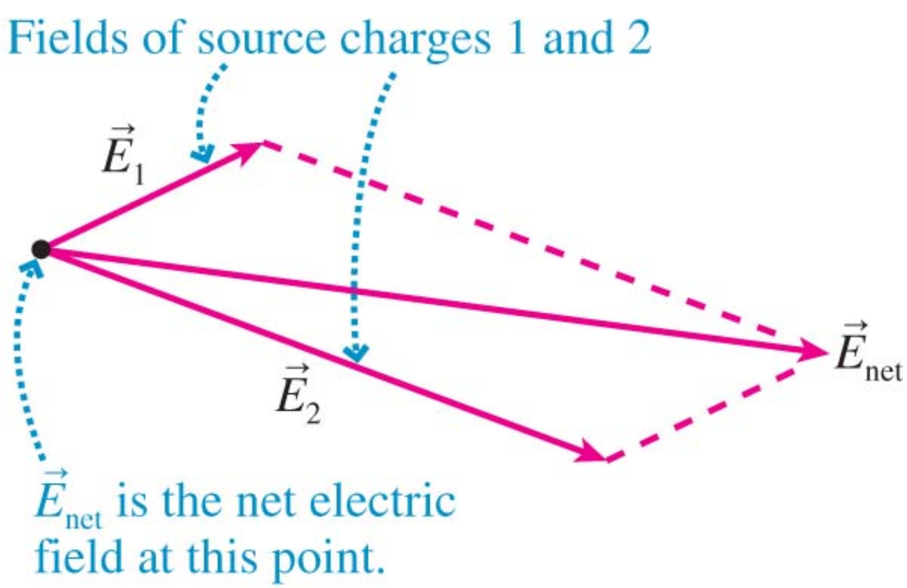


Electric Field Models

The electric field of a point charge q at the origin, $r = 0$, is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)}$$

where $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N\ m^2}$ is the permittivity constant.



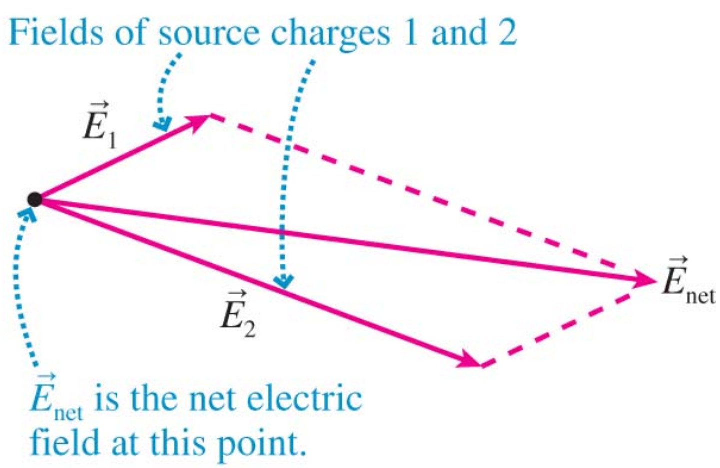
Electric Field Models

The net electric field due to a group of point charges is

$$\vec{E}_{\text{net}} = \frac{\vec{F}_{\text{on } q}}{q} = \frac{\vec{F}_{1 \text{ on } q}}{q} + \frac{\vec{F}_{2 \text{ on } q}}{q} + \cdots = \vec{E}_1 + \vec{E}_2 + \cdots = \sum_i \vec{E}_i$$

where E_i is the field from point charge i.

FIGURE 27.3 Electric fields obey the principle of superposition.



Electric Field of Common Charge Distributions

- The principle of superposition is used to obtain the electric field of
  - A dipole
  - A line of charge
  - A plane of charge
  - A parallel plate capacitor
  - A sphere of charge

PROBLEM-SOLVING STRATEGY 27.1

The electric field of multiple point charges

MODEL Model charged objects as point charges.

VISUALIZE For the pictorial representation:

- Establish a coordinate system and show the locations of the charges.
- Identify the point P at which you want to calculate the electric field.
- Draw the electric field of each charge at P.
- Use symmetry to determine if any components of $\vec{E}_{net}$ are zero.

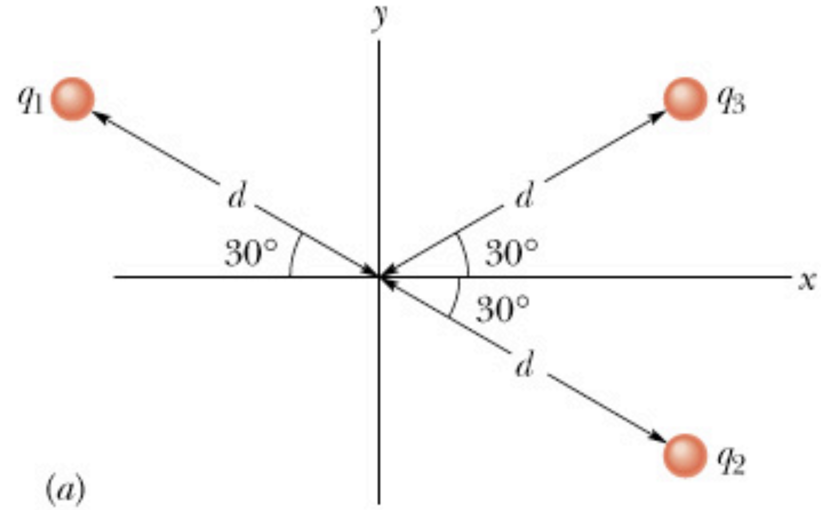
SOLVE The mathematical representation is $\vec{E}_{net} = \sum \vec{E}_i$.

- For each charge, determine its distance from P and the angle of $\vec{E}_i$ from the axes.
- Calculate the field strength of each charge's electric field.
- Write each vector $\vec{E}_i$ in component form.
- Sum the vector components to determine $\vec{E}_{net}$.
- If needed, determine the magnitude and direction of $\vec{E}_{net}$.

ASSESS Check that your result has the correct units, is reasonable, and agrees with any known limiting cases.

Example

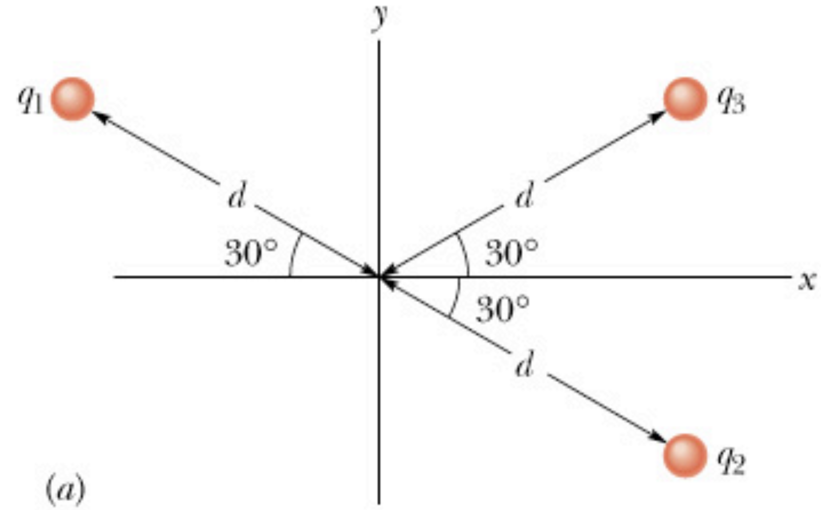
What is net $\vec{E}$ at origin when q_1=+2Q, q_2=-2Q and q_3=-4Q



–

Example

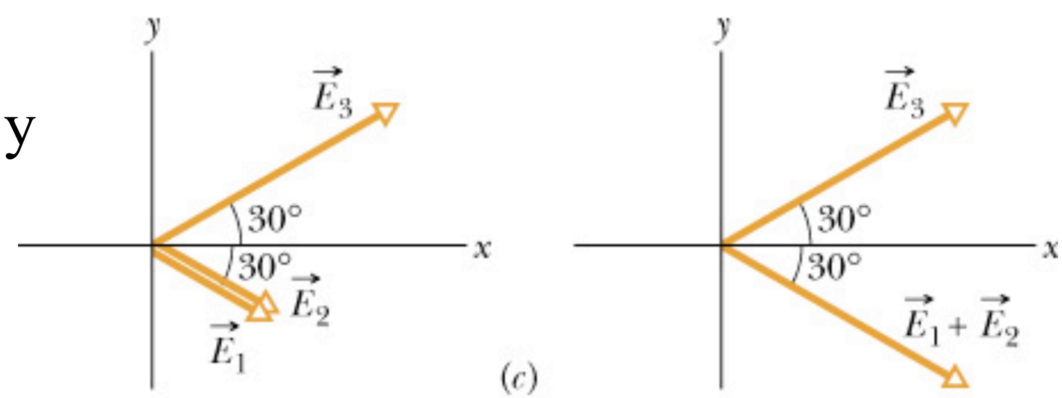
What is net $\vec{E}$ at origin when q_1=+2Q, q_2=-2Q and q_3=-4Q



Key idea: Use symmetry

E1, E2 exactly offset E3 in the y direction.

We need to add up the x-component of each of them.



Binomial Theorem: A useful mathematical tool

$$\boxed{(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \qquad (x^2 < 1)}$$

$$\frac{1}{(1+x)} = 1 - x + x^2 - x^3 + \cdots \qquad (x^2 < 1)$$

$$\frac{1}{(1-x)} = 1 + x + x^2 + x^3 + \cdots \qquad (x^2 < 1)$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots \qquad (x^2 < 1)$$

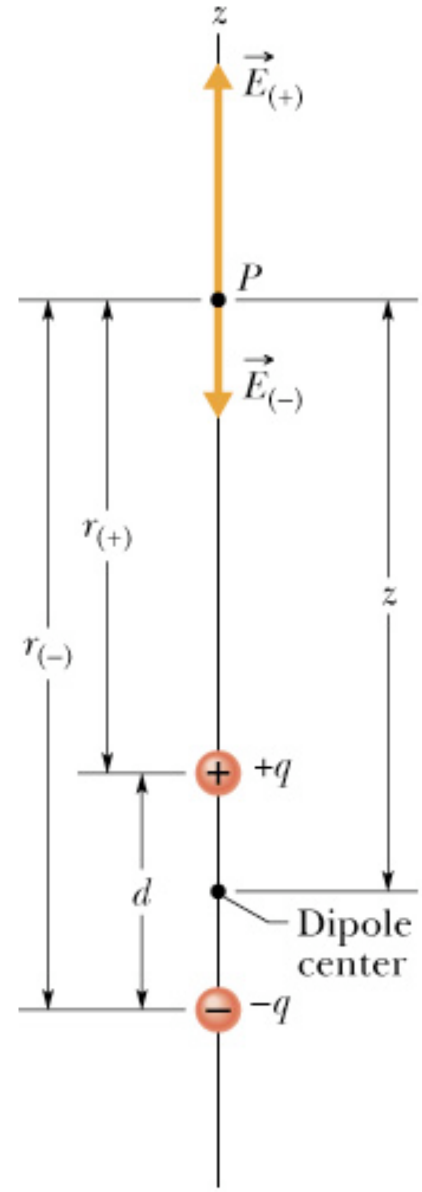
$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + \cdots \qquad (x^2 < 1)$$

Electric field due to an electric dipole

Dipole: 2 charged particles, magnitude q, distance d.

Find electric field at point P at a distance z from the mid-point on the dipole axis.

$$
\begin{aligned}
E &= E_{(+)} - E_{(-)} \\
&= \frac{1}{4\pi\epsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_{(-)}^2} \\
&= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(z-\frac{1}{2}d)^2} - \frac{1}{(z+\frac{1}{2}d)^2}\right] \\
&= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]
\end{aligned}
$$



Electric field due to an electric dipole(cont)

$$E = \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$

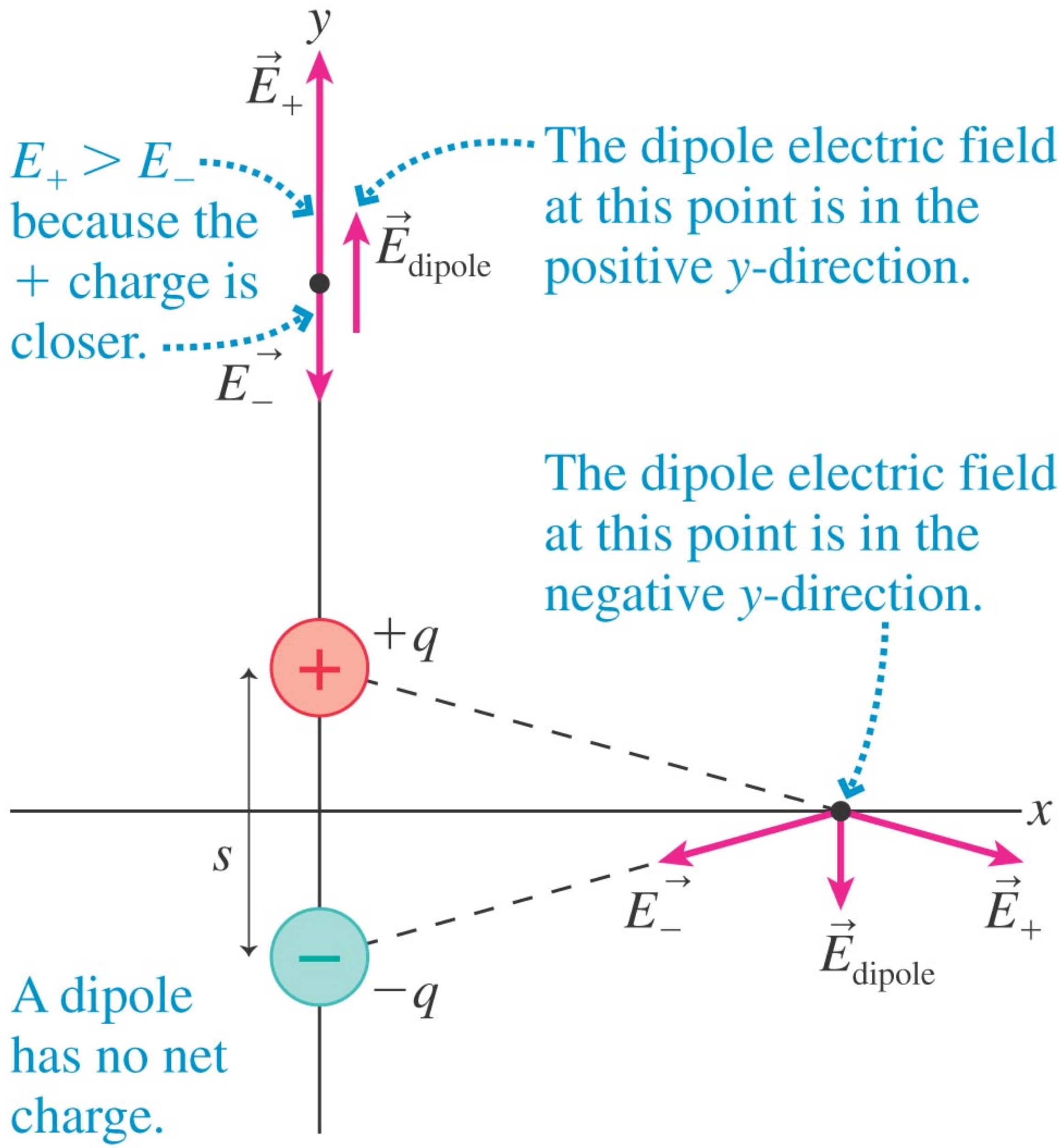
$$\frac{1}{(1-x)^2} = 1 + 2x$$

$\frac{d}{2z} \ll 1$ => use 2 terms of binomial theorem

$$= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+\frac{2d}{2z}+\cdots\right) - \left(1-\frac{2d}{2z}+\cdots\right)\right]$$

$$= \frac{q}{4\pi\epsilon_0 z^2}\frac{2d}{z}$$

$$= \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3} \qquad \left[\frac{N\cdot m^2}{C^2}\frac{C\cdot m}{m^3}\right] = \left[\frac{N}{C}\right]$$



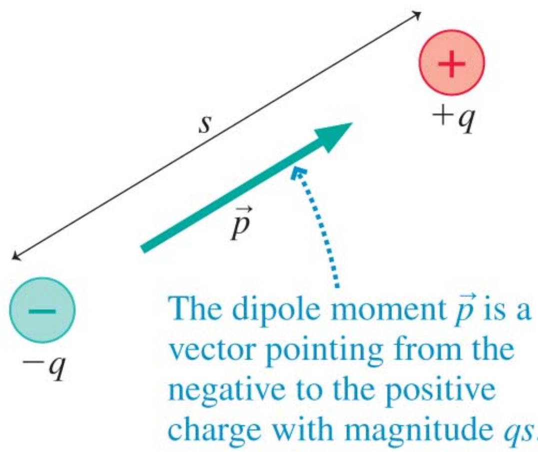
y
$\vec{E}_+$
$E_+ > E_-$ because the + charge is closer.
The dipole electric field at this point is in the positive y-direction.
$\vec{E}_{\text{dipole}}$
$\vec{E}_-$
The dipole electric field at this point is in the negative y-direction.
$+q$
s
x
$\vec{E}_-$
$\vec{E}_{\text{dipole}}$
$\vec{E}_+$
$-q$
A dipole has no net charge.

The Electric Field of a Dipole

We can represent an electric dipole by two opposite charges $\pm q$ separated by the small distance s.
The dipole moment is defined as the vector

$$\vec{p} = (qs, \text{from the negative to the positive charge})$$

The dipole-moment magnitude $p = qs$ determines the electric field strength. The SI units of the dipole moment are C m.



The Electric Field of a Dipole

The electric field at a point on the axis of a dipole is

$$\vec{E}_{\text{dipole}} \approx \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \qquad \text{(on the axis of an electric dipole)}$$

where r is the distance measured from the *center* of the dipole.

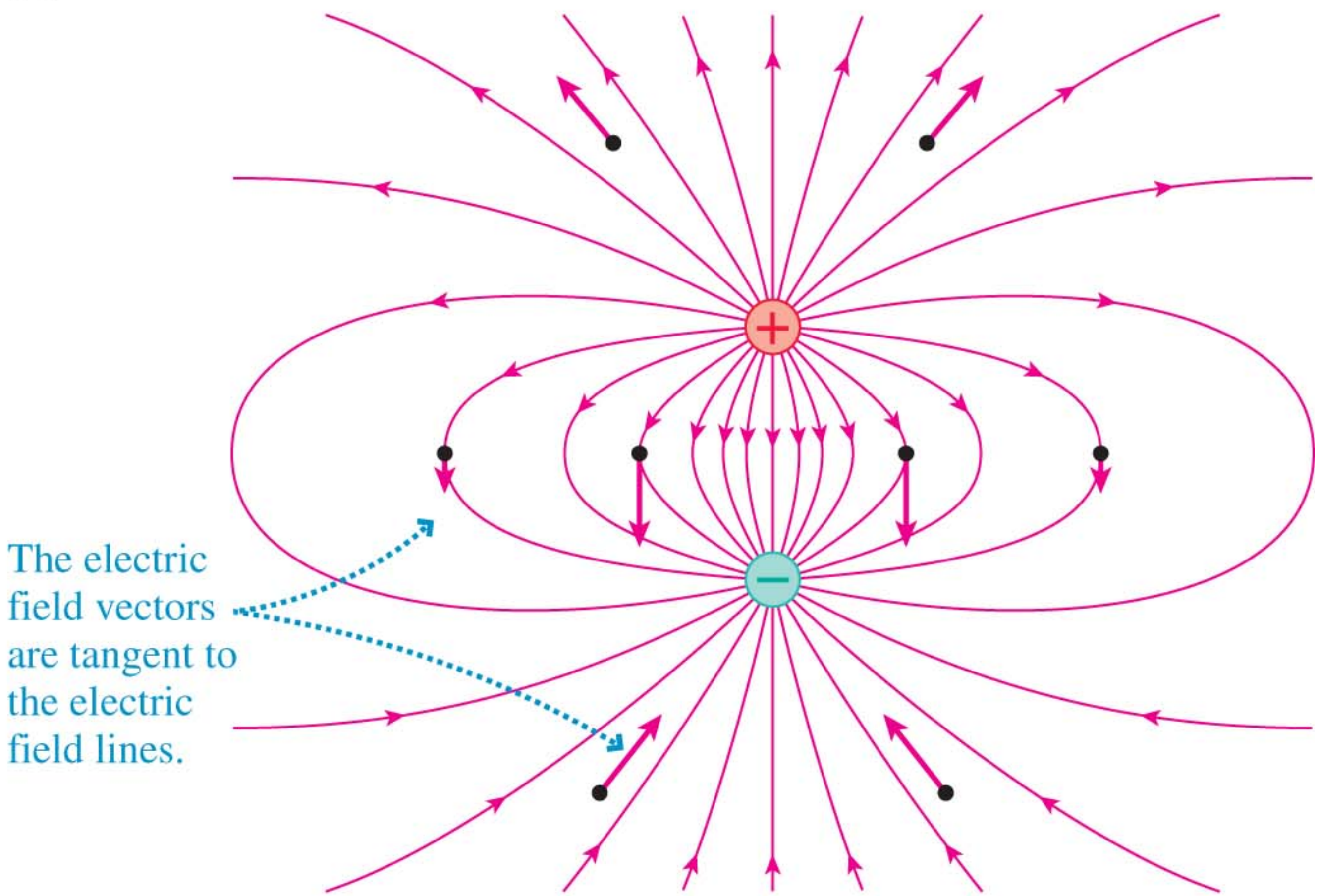
The electric field in the plane that bisects and is perpendicular to the dipole is

$$\vec{E}_{\text{dipole}} \approx -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \qquad \text{(perpendicular plane)}$$

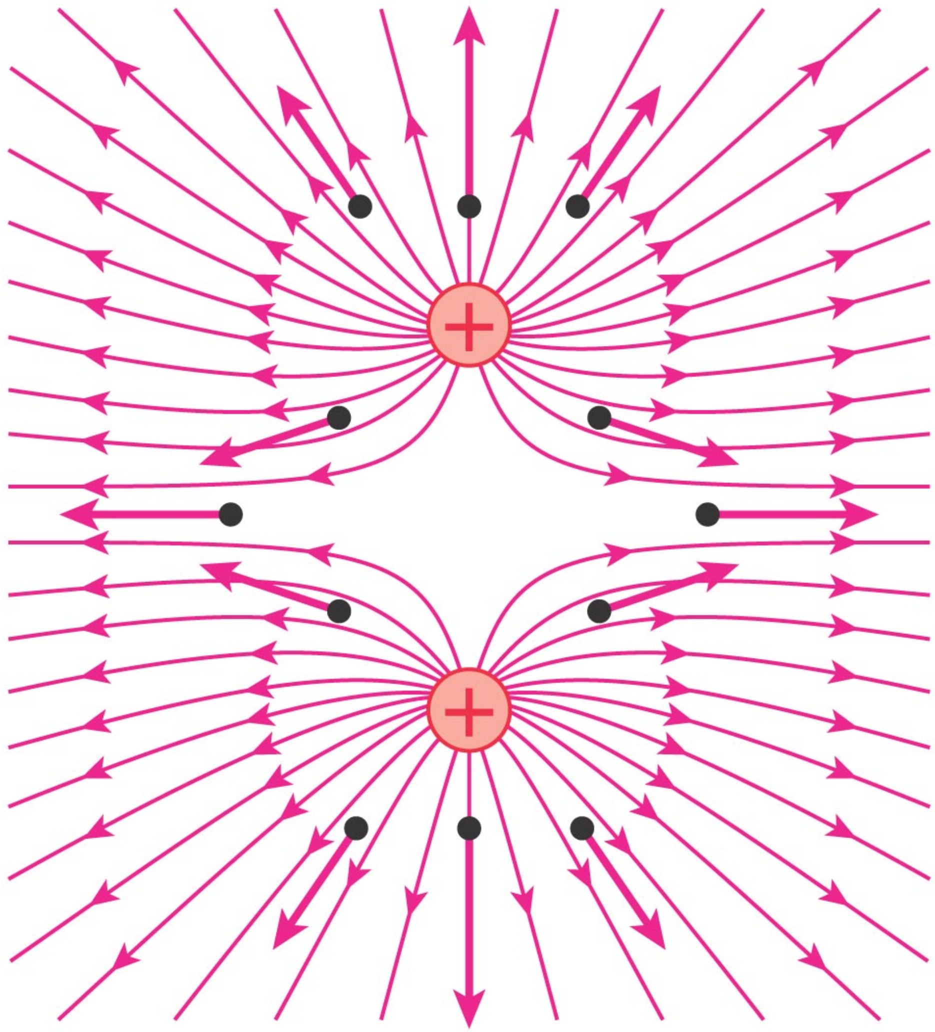
This field is opposite to the dipole direction, and it is only half the strength of the on-axis field at the same distance.

The electric field of a dipole.

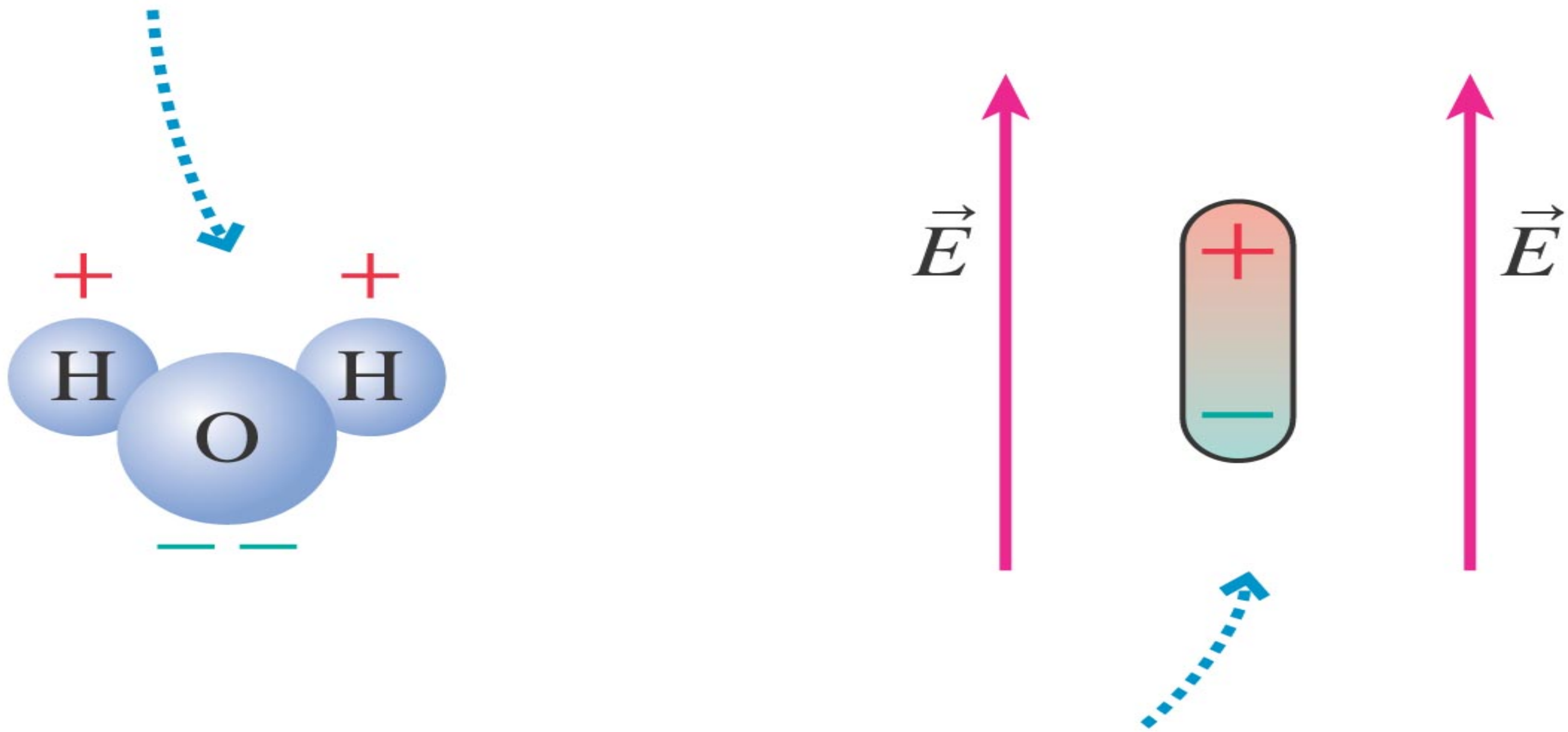
(b)



Electric Field of Two Positive Charges
Not a Dipole



A water molecule is a *permanent* dipole because the negative electrons spend more time with the oxygen atom.



This dipole is *induced*, or stretched, by the electric field acting on the + and − charges.

EXAMPLE 27.2 The electric field of a water molecule

QUESTION:

EXAMPLE 27.2 The electric field of a water molecule

The water molecule H_2O has a permanent dipole moment of magnitude 6.2×10^{-30} C m. What is the electric field strength 1.0 nm from a water molecule at a point on the dipole's axis?

EXAMPLE 27.2 The electric field of a water molecule

MODEL The size of a molecule is ≈ 0.1 nm. Thus $r \gg s$, and we can use Equation 27.11 for the on-axis electric field of the molecule's dipole moment.

SOLVE The on-axis electric field strength at $r = 1.0$ nm is

$$E \approx \frac{1}{4\pi\epsilon_0}\frac{2p}{r^3} = (9.0 \times 10^9\ \mathrm{Nm^2/C^2})\frac{2(6.2 \times 10^{-30}\ \mathrm{Cm})}{(1.0 \times 10^{-9}\ \mathrm{m})^3}$$

$$= 1.1 \times 10^8\ \mathrm{N/C}$$

ASSESS By referring to Table 27.1 you can see that the field strength is "strong" compared to our everyday experience with charged objects but "weak" compared to the electric field inside the atoms themselves. This seems reasonable.

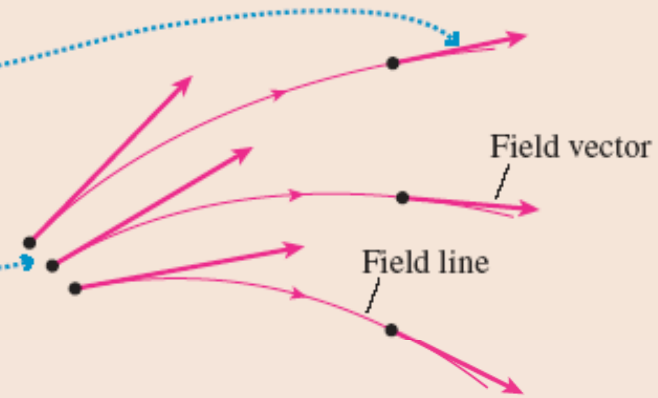
Tactics: Drawing and using electric field lines

TACTICS BOX 27.1 **Drawing and using electric field lines**

MP

1. Electric field lines are continuous curves drawn tangent to the electric field vectors. Conversely, the electric field vector at any point is tangent to the field line at that point.

2. Closely spaced field lines represent a larger field strength, with longer field vectors. Widely spaced lines indicate a smaller field strength.

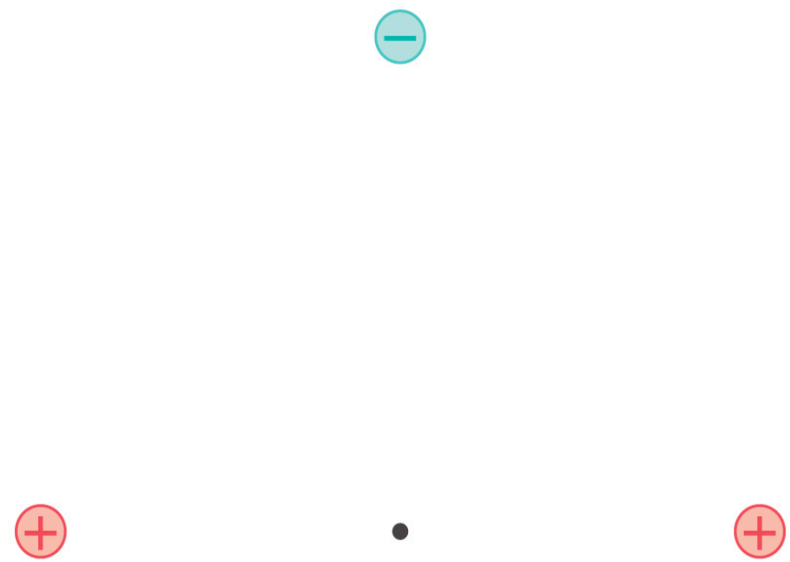


3. Electric field lines never cross.

4. Electric field lines start from positive charges and end on negative charges.

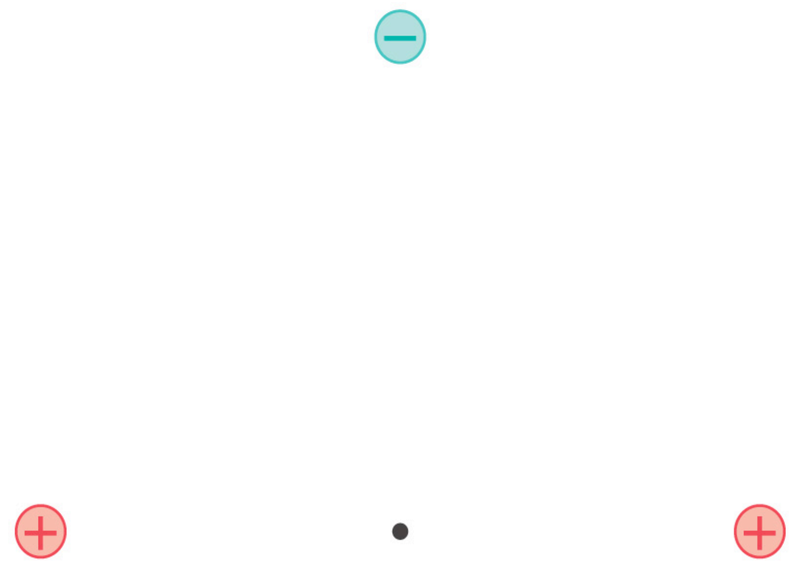
Exercises 2–4, 12, 13

At the position of the dot, the electric field points



A. Up.
B. Down.
C. Left.
D. Right.
E. The electric field is zero.

At the position of the dot, the electric field points



A. Up.

B. Down.

C. Left.

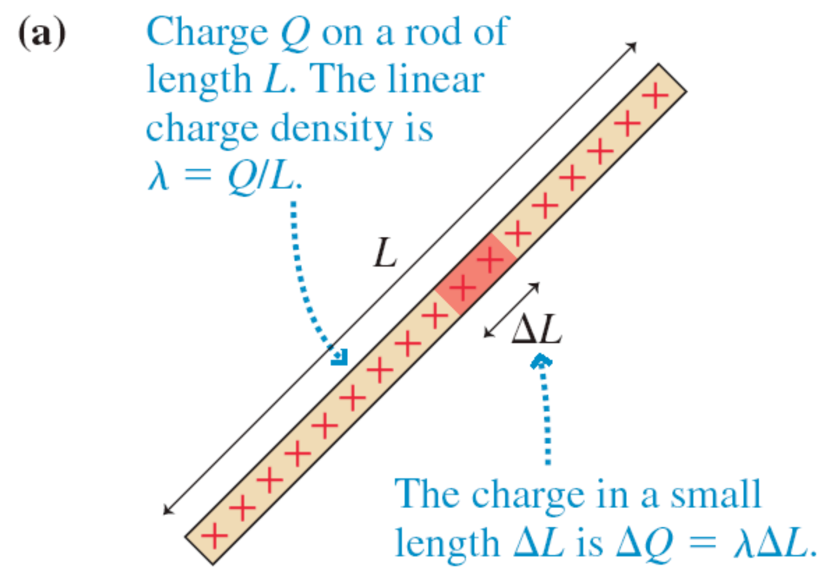
D. Right.

E. The electric field is zero.

The Electric Field of a Continuous Charge Distribution

The linear charge density of an object of length L and charge Q, is defined as

$$\lambda = \frac{Q}{L}$$



Linear charge density, which has units of C/m, is the amount of charge *per meter* of length.

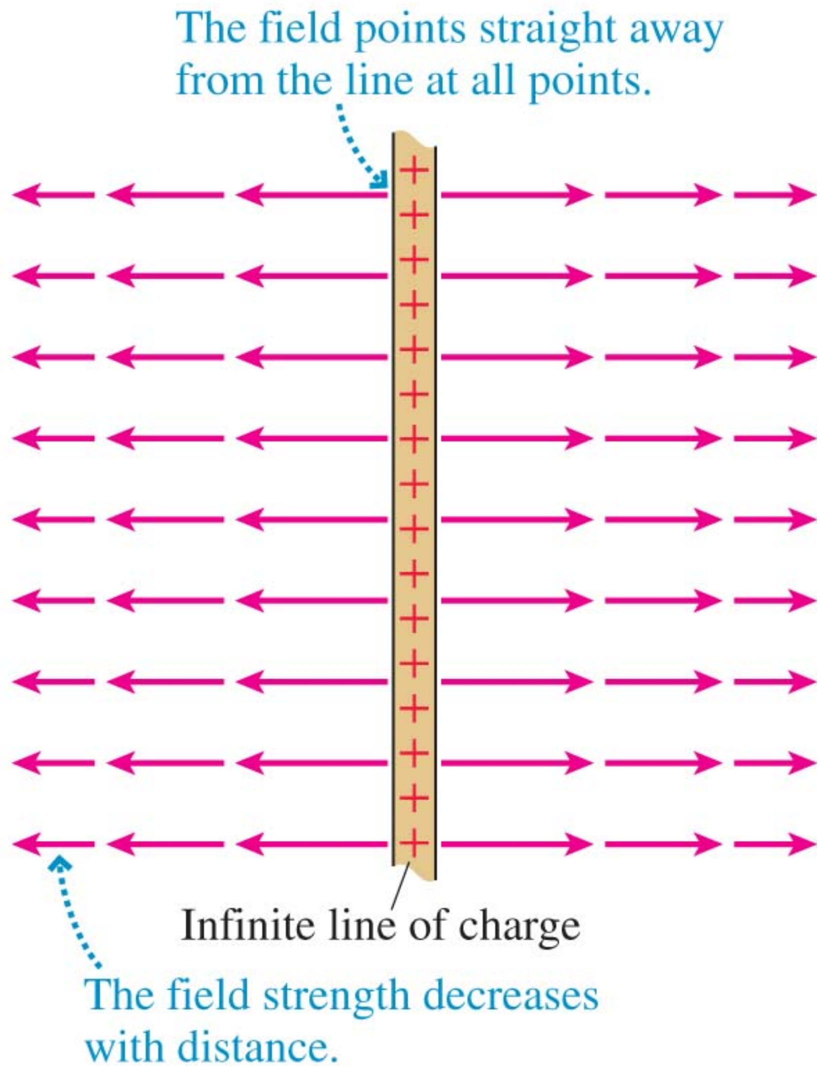
An Infinite Line of Charge

A very long, thin rod, with linear charge density λ, has an electric field

$$E_{\text{line}} = \lim_{L\to\infty} \frac{1}{4\pi\epsilon_0} \frac{|Q|}{r\sqrt{r^2 + (L/2)^2}} = \frac{1}{4\pi\epsilon_0} \frac{|Q|}{rL/2} = \frac{1}{4\pi\epsilon_0} \frac{2|\lambda|}{r}$$

Where r is the radial distance away from the rod.

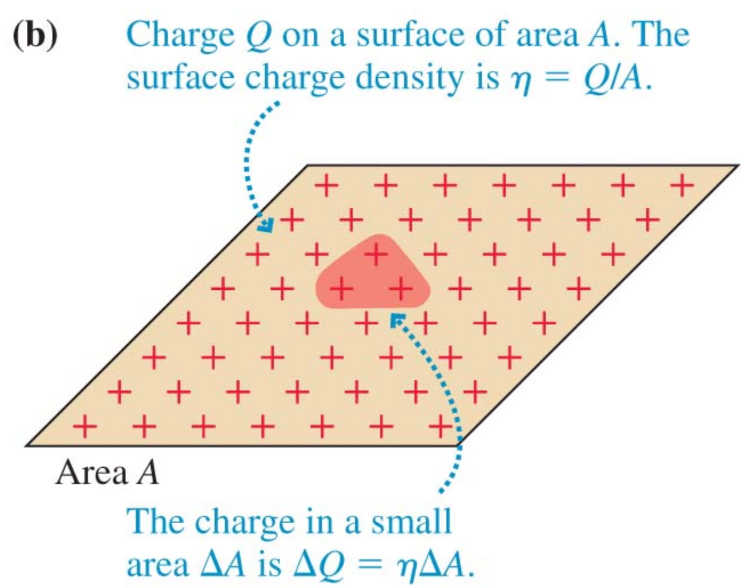
FIGURE 27.14 The electric field of an infinite line of charge.



The Electric Field of a Continuous Charge Distribution

The surface charge density of a two-dimensional distribution of charge across a surface of area *A* is defined as

$$\eta = \frac{Q}{A}$$



Surface charge density, with units C/m^2, is the amount of charge *per square meter*.

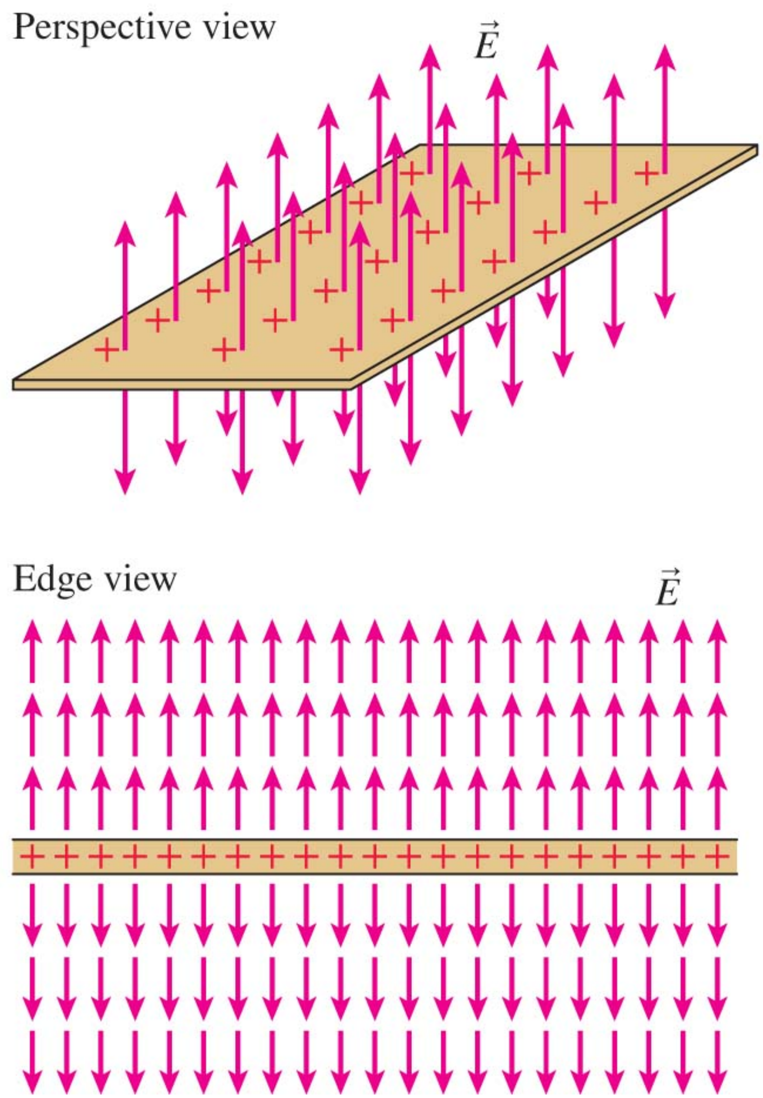
A Plane of Charge

The electric field of an infinite plane of charge with surface charge density η is:

$$E_{\text{plane}} = \frac{\eta}{2\epsilon_0} = \text{constant}$$

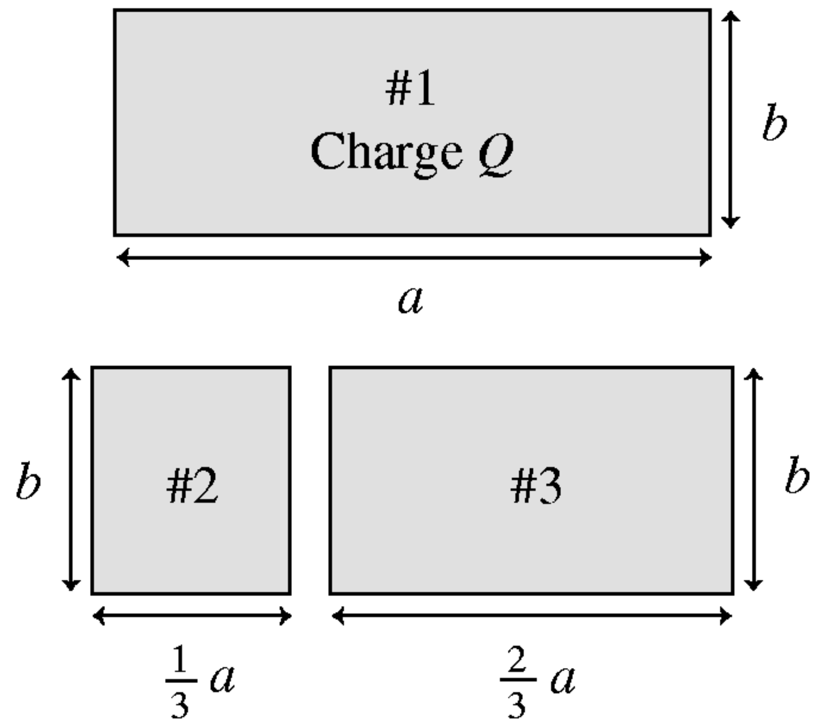
For a positively charged plane, with $\eta > 0$, the electric field points *away from* the plane on both sides of the plane.
For a negatively charged plane, with $\eta < 0$, the electric field points *towards* the plane on both sides of the plane.

FIGURE 27.18 Two views of the electric field of a plane of charge.



Example: Surface Charge Density

- Charge Q is spread uniformly on a rectangle of sides a & b.
  a) What is the surface charge density?
  b) The original rectangle, identified as number 1, is then broken into the smaller rectangles 2 & 3. Compare the surface charge densities η_1, η_2, & η_3.

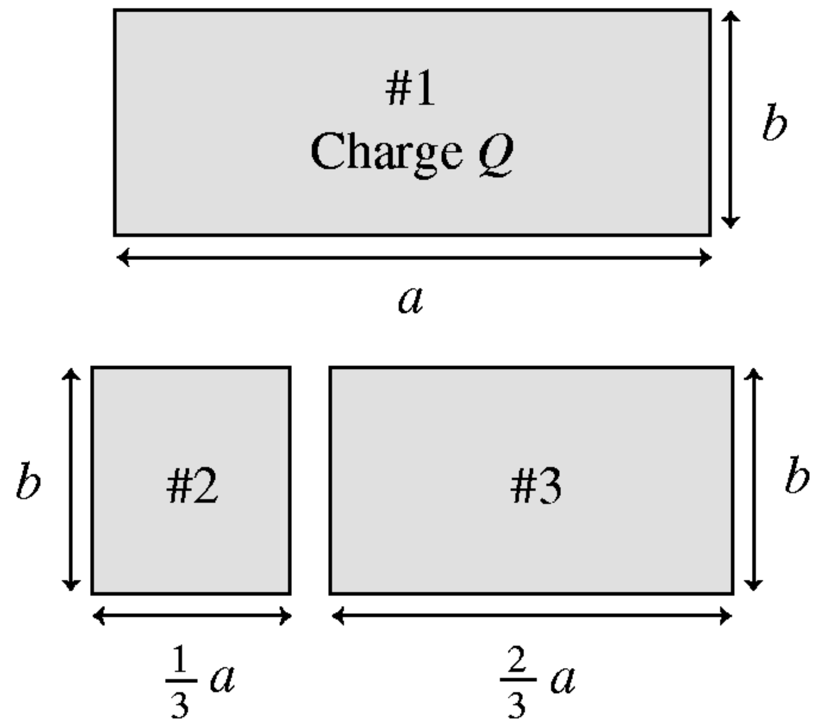


Example: Surface Charge Density

- Charge Q is spread uniformly on a rectangle of sides a & b.

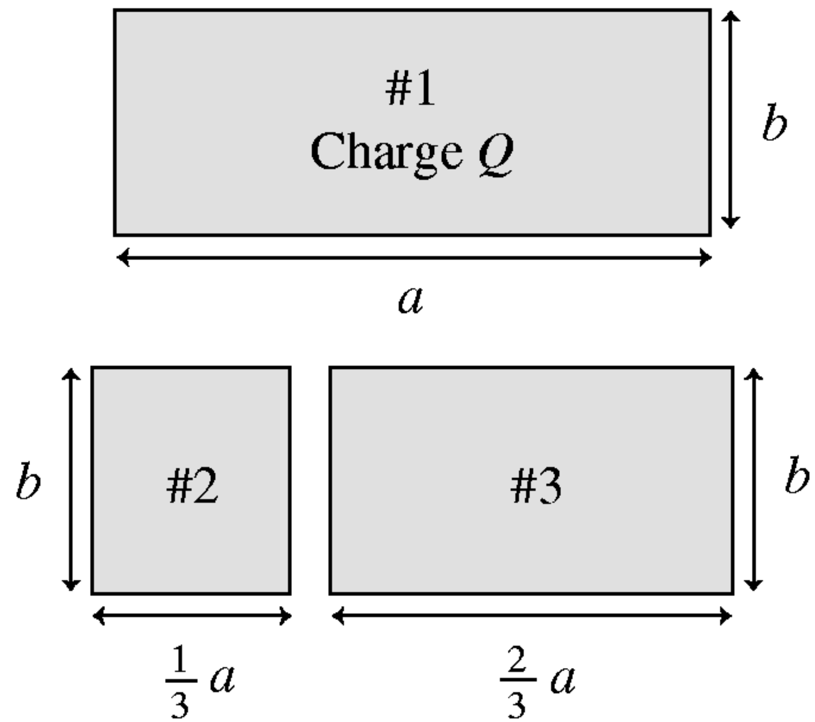
  a) What is the surface charge density? $\eta = Q/(ab)$

  b) The original rectangle, identified as number 1, is then broken into the smaller rectangles 2 & 3. Compare the surface charge densities η_1, η_2, & η_3.

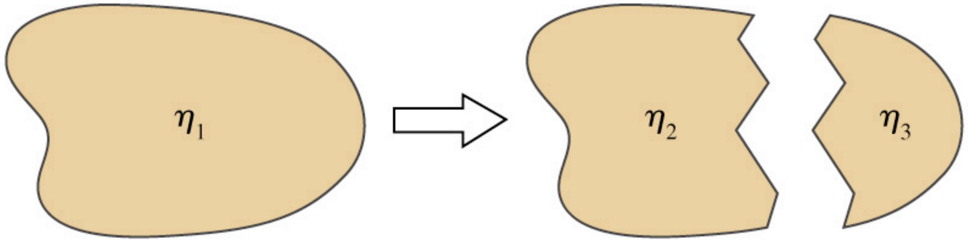


Example: Surface Charge Density

- Charge Q is spread uniformly on a rectangle of sides a & b.
  a) What is the surface charge density? $\eta = Q/(ab)$
  b) The original rectangle, identified as number 1, is then broken into the smaller rectangles 2 & 3. Compare the surface charge densities η_1, η_2, & η_3.

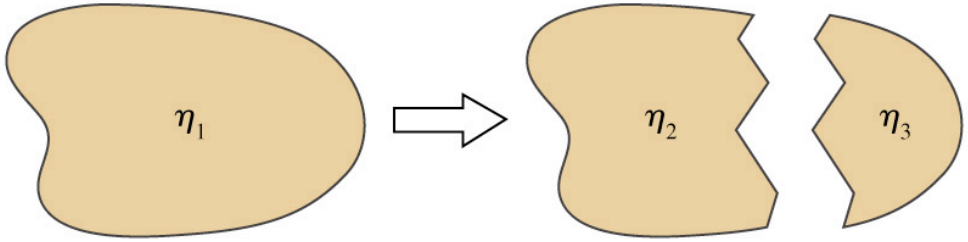


Answer: $\eta_1 = \eta_2 = \eta_3$.



A piece of plastic is uniformly charged with surface charge density η_1. The plastic is then broken into a large piece with surface charge density η_2 and a small piece with surface charge density η_3. Rank in order, from largest to smallest, the surface charge densities η_1 to η_3.

A. $\eta_2 = \eta_3 > \eta_1$
B. $\eta_1 > \eta_2 > \eta_3$
C. $\eta_1 > \eta_2 = \eta_3$
D. $\eta_3 > \eta_2 > \eta_1$
E. $\eta_1 = \eta_2 = \eta_3$



A piece of plastic is uniformly charged with surface charge density η_1. The plastic is then broken into a large piece with surface charge density η_2 and a small piece with surface charge density η_3. Rank in order, from largest to smallest, the surface charge densities η_1 to η_3.

A. $\eta_2 = \eta_3 > \eta_1$
B. $\eta_1 > \eta_2 > \eta_3$
C. $\eta_1 > \eta_2 = \eta_3$
D. $\eta_3 > \eta_2 > \eta_1$
✔ **E. $\boldsymbol{\eta_1 = \eta_2 = \eta_3}$**

Problem-Solving Strategy: The electric field of a continuous distribution of charge

PROBLEM-SOLVING STRATEGY 27.2 **The electric field of a continuous distribution of charge**

MODEL Model the distribution as a simple shape, such as a line of charge or a disk of charge. Assume the charge is uniformly distributed.

Problem-Solving Strategy: The electric field of a continuous distribution of charge

VISUALIZE For the pictorial representation:

1. Draw a picture and establish a coordinate system.
2. Identify the point P at which you want to calculate the electric field.
3. Divide the total charge Q into small pieces of charge ΔQ, using shapes for which you *already know* how to determine $\vec{E}$. This is often, but not always, a division into point charges.
4. Draw the electric field vector at P for one or two small pieces of charge. This will help you identify distances and angles that need to be calculated.
5. Look for symmetries of the charge distribution that simplify the field. You may conclude that some components of $\vec{E}$ are zero.

Problem-Solving Strategy: The electric field of a continuous distribution of charge

SOLVE The mathematical representation is $\vec{E}_{net} = \sum \vec{E}_i$.

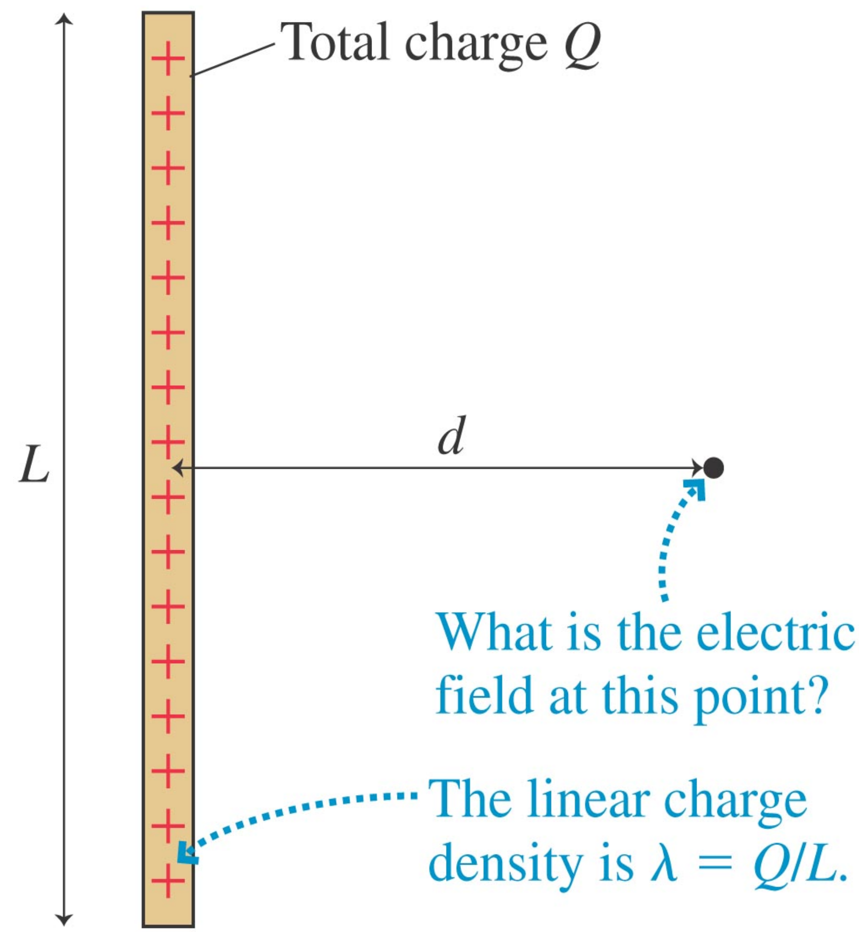
- Use superposition to form an algebraic expression for *each* of the three components of $\vec{E}$ (unless you are sure one or more is zero) at point P.
- Let the (x, y, z) coordinates of the point remain variables.
- Replace the small charge ΔQ with an equivalent expression involving a charge density and a coordinate, such as dx, that describes the shape of charge ΔQ. **This is the critical step in making the transition from a sum to an integral** because you need a coordinate to serve as the integration variable.
- Express all angles and distances in terms of the coordinates.
- Let the sum become an integral. The integration will be over the *one* coordinate variable that is related to ΔQ. The integration limits for this variable must "cover" the entire charged object.

Problem-Solving Strategy: The electric field of a continuous distribution of charge

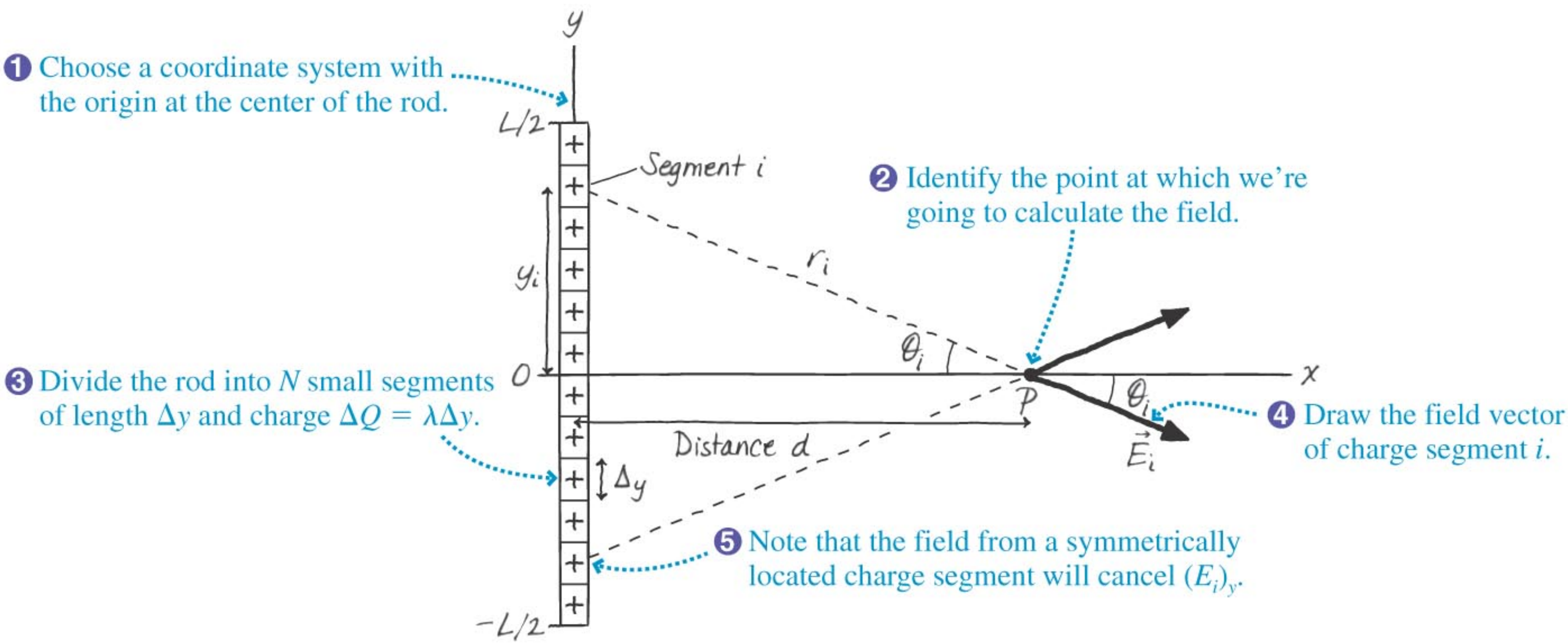
ASSESS Check that your result is consistent with any limits for which you know what the field should be.

Derive Field of a line charge

- Figure show a uniformly charged rod of length L with total charge Q that can be either positive or negative. Find the electric field stength E at a point that bisects the rod.

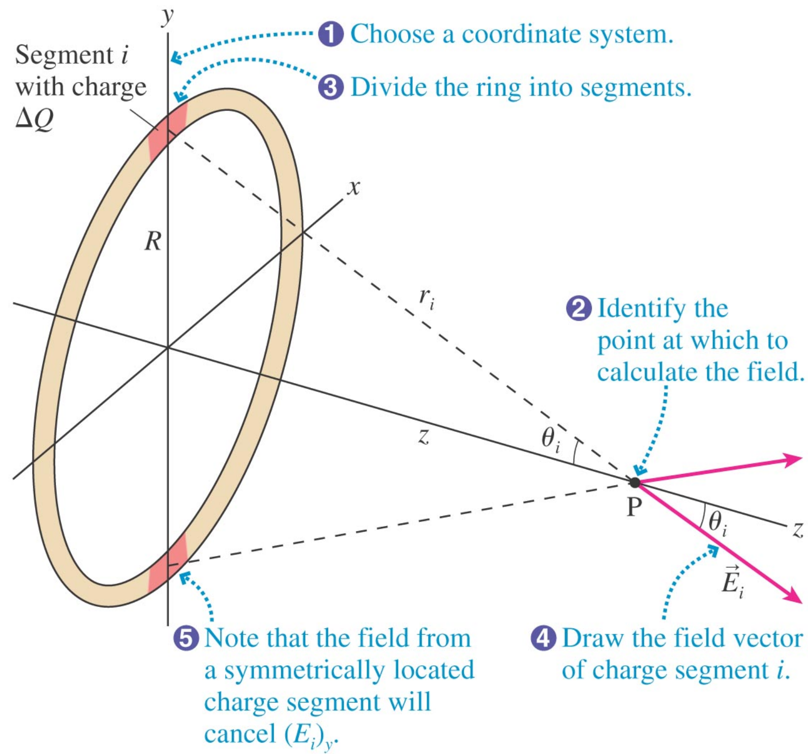


Electric Field of Line Charge

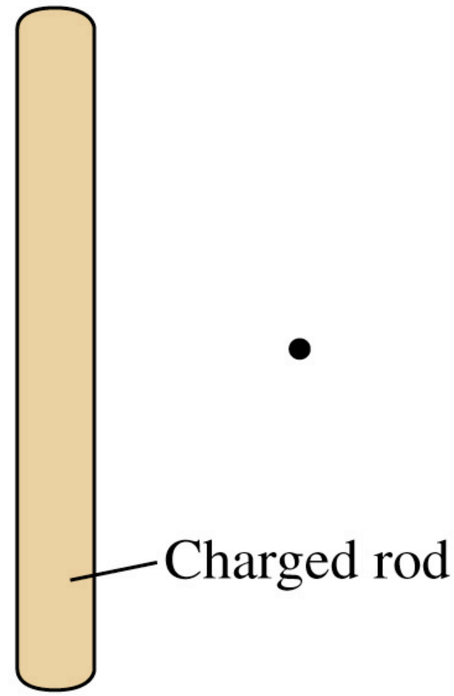


Now do variation:Ring

- A thin ring of radius R is uniformly charged with total charge Q. Find the electric field at a point on the axis of the ring

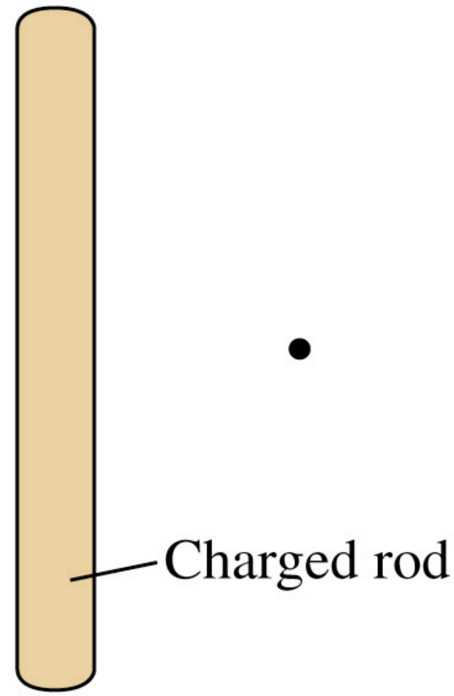


Which of the following actions will increase the electric field strength at the position of the dot?



A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
C. Make the rod shorter without changing the charge.
D. Remove charge from the rod.
E. Make the rod narrower without changing the charge.

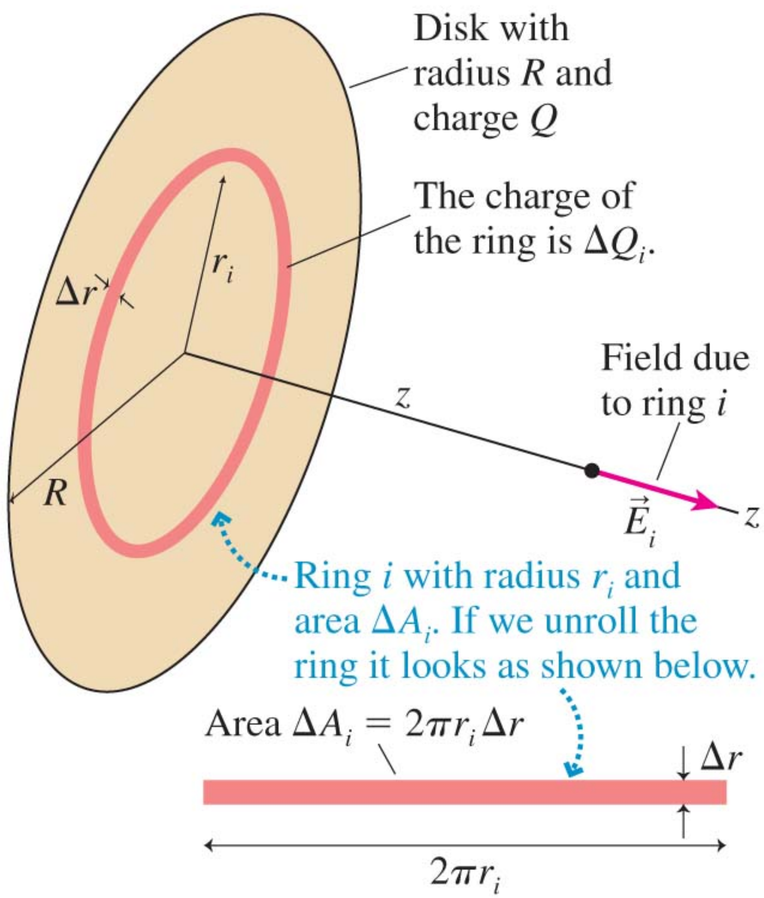
Which of the following actions will increase the electric field strength at the position of the dot?



A. Make the rod longer without changing the charge.
B. Make the rod fatter without changing the charge.
✔ **C. Make the rod shorter without changing the charge.**
D. Remove charge from the rod.
E. Make the rod narrower without changing the charge.

A Disk of Charge

FIGURE 27.17 Calculating the on-axis field of a charged disk.



A Disk of Charge

The on-axis electric field of a charged disk of radius R, centered on the origin with axis parallel to z, and surface charge density $\eta = Q/\pi R^2$ is

$$(E_{\text{disk}})_z = \frac{\eta}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

NOTE: This expression is only valid for $z > 0$. The field for $z < 0$ has the same magnitude but points in the opposite direction.

A Sphere of Charge

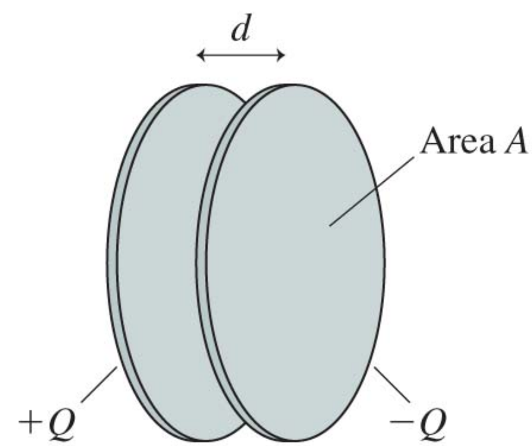
A sphere of charge Q and radius R, be it a uniformly charged sphere or just a spherical shell, has an electric field *outside* the sphere that is exactly the same as that of a point charge Q located at the center of the sphere:

$$\vec{E}_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad \text{for } r \geq R$$

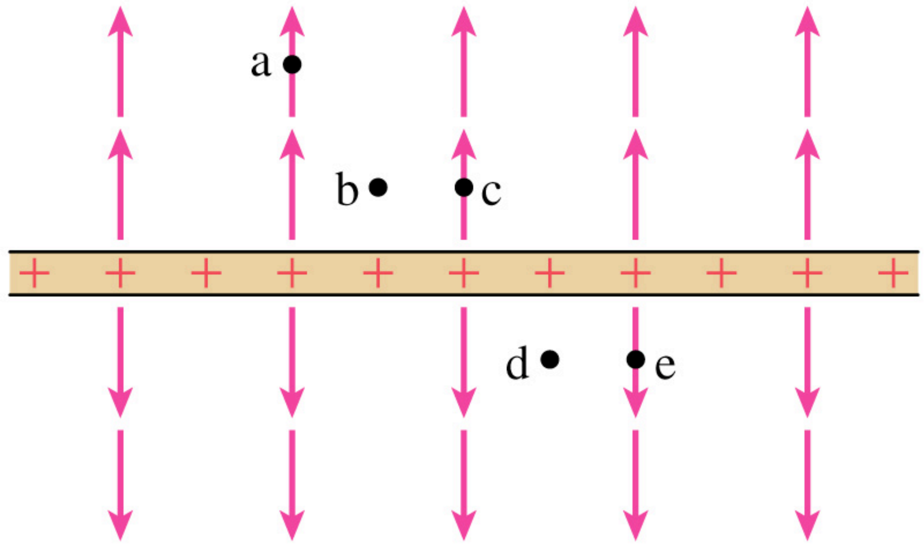
The Parallel-Plate Capacitor

- The figure shows two electrodes, one with charge $+Q$ and the other with $-Q$ placed face-to-face a distance d apart.
- This arrangement of two electrodes, charged equally but oppositely, is called a **parallel-plate capacitor.**
- Capacitors play important roles in many electric circuits.

FIGURE 27.20 A parallel-plate capacitor.

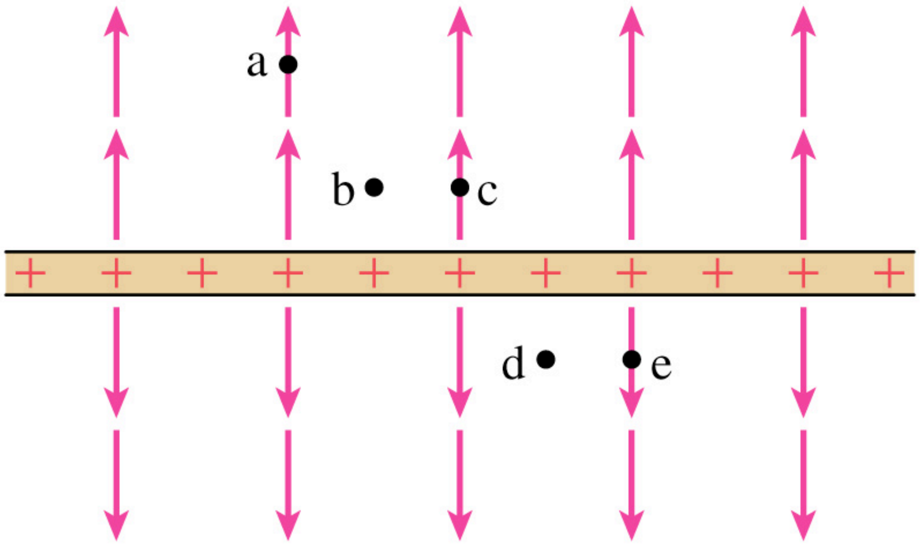


Rank in order, from largest to smallest, the electric field strengths E_a to E_e at these five points near a plane of charge.



A. $E_a > E_c > E_b > E_e > E_d$
B. $E_a = E_b = E_c = E_d = E_e$
C. $E_a > E_b = E_c > E_d = E_e$
D. $E_b = E_c = E_d = E_e > E_a$
E. $E_e > E_d > E_c > E_b > E_a$

Rank in order, from largest to smallest, the electric field strengths E_a to E_e at these five points near a plane of charge.

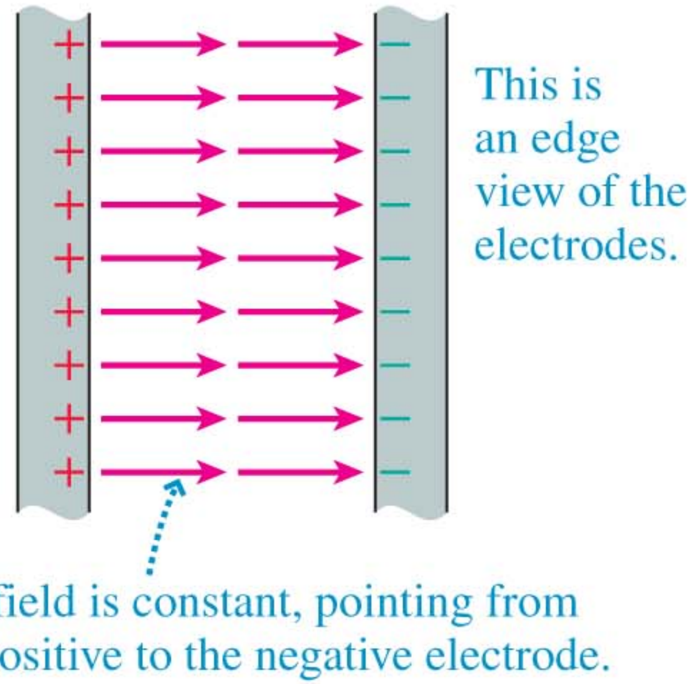


A. $E_a > E_c > E_b > E_e > E_d$
✔ **B. $E_a = E_b = E_c = E_d = E_e$**
C. $E_a > E_b = E_c > E_d = E_e$
D. $E_b = E_c = E_d = E_e > E_a$
E. $E_e > E_d > E_c > E_b > E_a$

Chapter 27 – Part 2

FIGURE 27.22 The electric field of a capacitor.

(a) Ideal capacitor



The Parallel-Plate Capacitor

The electric field inside a capacitor is

$$\vec{E}_{\text{capacitor}} = \vec{E}_+ + \vec{E}_- = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$

$$= \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right)$$

where A is the surface area of each electrode. Outside the capacitor plates, where E_+ and E_- have equal magnitudes but *opposite* directions, the electric field is zero.

EXAMPLE 27.7 The electric field inside a capacitor

QUESTIONS:

EXAMPLE 27.7 The electric field inside a capacitor

Two 1.0 cm $\times$ 2.0 cm rectangular electrodes are 1.0 mm apart. What charge must be placed on each electrode to create a uniform electric field of strength 2.0×10^6 N/C? How many electrons must be moved from one electrode to the other to accomplish this?

EXAMPLE 27.7 The electric field inside a capacitor

MODEL The electrodes can be modeled as a parallel-plate capacitor because the spacing between them is much smaller than their lateral dimensions.

EXAMPLE 27.7 The electric field inside a capacitor

SOLVE The electric field strength inside the capacitor is $E = Q/\epsilon_0 A$. Thus the charge to produce a field of strength E is

$$Q = (8.85 \times 10^{-12}\ \text{C}^2/\text{N m}^2)(2.0 \times 10^{-4}\ \text{m}^2)(2.0 \times 10^{6}\ \text{N/C})$$
$$= 3.5 \times 10^{-9}\ \text{C} = 3.5\ \text{nC}$$

The positive plate must be charged to $+3.5$ nC and the negative plate to -3.5 nC. In practice, the plates are charged by using a *battery* to move electrons from one plate to the other. The number of electrons in 3.5 nC is

$$N = \frac{Q}{e} = \frac{3.5 \times 10^{-9}\ \text{C}}{1.60 \times 10^{-19}\ \text{C/electron}} = 2.2 \times 10^{10}\ \text{electrons}$$

Thus 2.2×10^{10} electrons are moved from one electrode to the other. Note that the capacitor *as a whole* has no net charge.

EXAMPLE 27.7 The electric field inside a capacitor

ASSESS The plate spacing does not enter the result. As long as the spacing is much smaller than the plate dimensions, as is true in this example, the field is independent of the spacing.

Motion of a Charged Particle in an Electric Field

The electric field exerts a force

$$\vec{F}_{\text{on } q} = q\vec{E}$$

on a charged particle. If this is the only force acting on q, it causes the charged particle to accelerate with

$$\vec{a} = \frac{\vec{F}_{\text{on } q}}{m} = \frac{q}{m}\vec{E}$$

In a uniform field, the acceleration is constant:

$$a = \frac{qE}{m} = \text{constant}$$

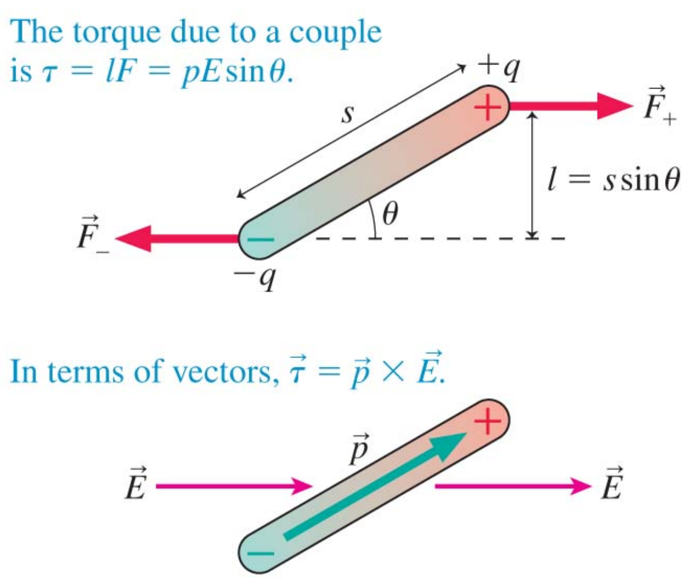
Dipoles in an Electric Field

The torque on a dipole in an electric field is

$$\tau = lF = (s\sin\theta)(qE) = pE\sin\theta$$

where θ is the angle the dipole makes with the electric field.

FIGURE 27.30 The torque on a dipole.



Chapter 27. Summary Slides

General Principles

Sources of $\vec{E}$

Electric fields are created by charges.

Two major tools for calculating $\vec{E}$ are

- The field of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{r}$$

- The principle of superposition

Multiple point charges

Use superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$

Continuous distribution of charge

- Divide the charge into segments ΔQ for which you already know the field.
- Find the field of each ΔQ.
- Find $\vec{E}$ by summing the fields of all ΔQ.

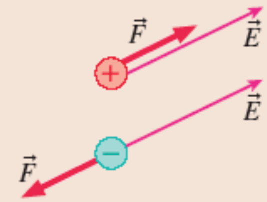
The summation usually becomes an integral. A critical step is replacing ΔQ with an expression involving a **charge density** (λ or η) and an integration coordinate.

General Principles

Consequences of $\vec{E}$

The electric field exerts a force on a charged particle:

$$\vec{F} = q\vec{E}$$



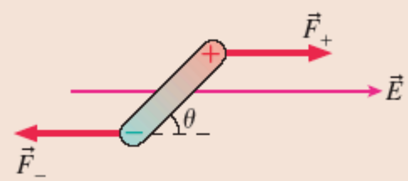
The force causes acceleration:

$$\vec{a} = (q/m)\vec{E}$$

Trajectories of charged particles are calculated with kinematics.

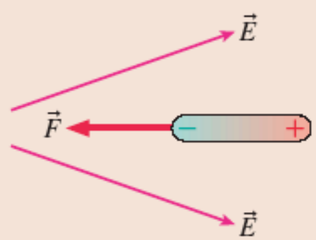
The electric field exerts a torque on a dipole:

$$\tau = pE\sin\theta$$



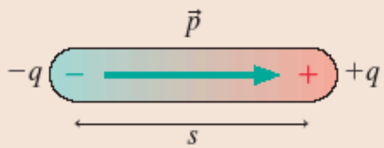
The torque tends to align the dipoles with the field.

In a nonuniform electric field, a dipole has a net force in the direction of increasing field strength.



Applications

Electric dipole



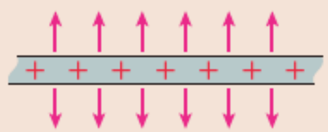
The electric dipole moment is

$\vec{p} = (qs, \text{from negative to positive})$

Field on axis: $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$

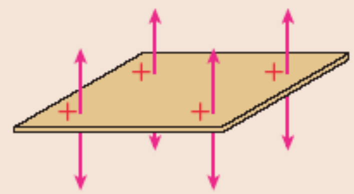
Field in bisecting plane: $\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$

Infinite line of charge with linear charge density λ



$$\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}, \text{perpendicular to line}\right)$$

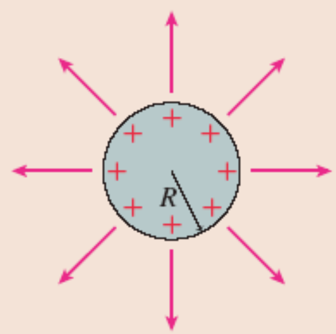
Infinite plane of charge with surface charge density η



$$\vec{E} = \left(\frac{\eta}{2\epsilon_0}, \text{perpendicular to plane}\right)$$

Sphere of charge

Same as a point charge Q for $r > R$

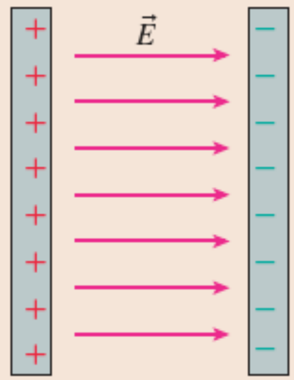


Applications

Parallel-plate capacitor

The electric field inside an ideal capacitor is a **uniform electric field:**

$$\vec{E} = \left(\frac{\eta}{\epsilon_0}, \text{ from positive to negative}\right)$$



A real capacitor has a weak **fringe field** around it.

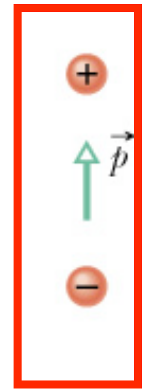
Electric field due to an electric dipole(cont)

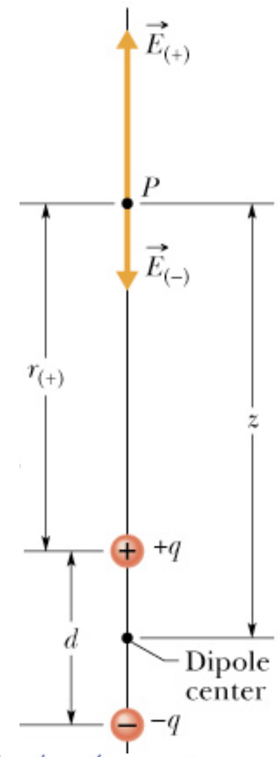
Electric dipole moment p of the dipole.

Vector quantity

Magnitude is product qd   [units C-m]

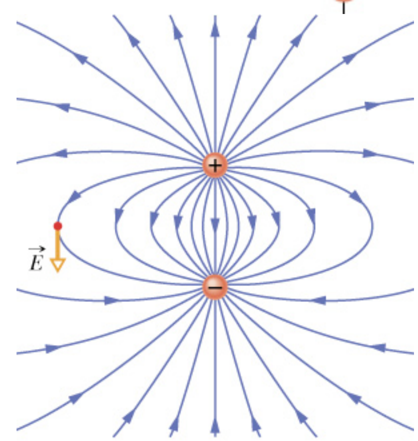
Directed from negative to positive end of the dipole.





$$E = \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$$

Eqn holds only for distant points on axis of dipole.



E for a dipole varies as $1/r^3$ for all distant points, even off the axis (not proven here)

Direction of E on axis is same as p when outside charges.