Chapter 10. Energy

This pole vaulter can lift herself nearly 6 m (20 ft) off the ground by transforming the kinetic energy of her run into gravitational potential energy.

Chapter Goal: To introduce the ideas of kinetic and potential energy and to learn a new problem-solving strategy based on conservation of energy.

Chapter 10.1-5 Energy

Topics:

- A "Natural Money" Called Energy
- Kinetic Energy and Gravitational Potential Energy
- A Closer Look at Gravitational Potential Energy
- Restoring Forces and Hooke's Law
- Elastic Potential Energy
- Energy Diagrams

Chapter 10. Reading Quizzes

Energy is a physical quantity with properties somewhat similar to

A. money.
B. heat.
C. a liquid.
D. work.
E. momentum.

Energy is a physical quantity with properties somewhat similar to

✔ **A. money.**

B. heat.

C. a liquid.

D. work.

E. momentum.

Hooke's law describes the force of

A. gravity.
B. a spring.
C. collisions.
D. tension.
E. none of the above.

Hooke's law describes the force of

A. gravity.
B. a spring.
C. collisions.
D. tension.
E. none of the above.

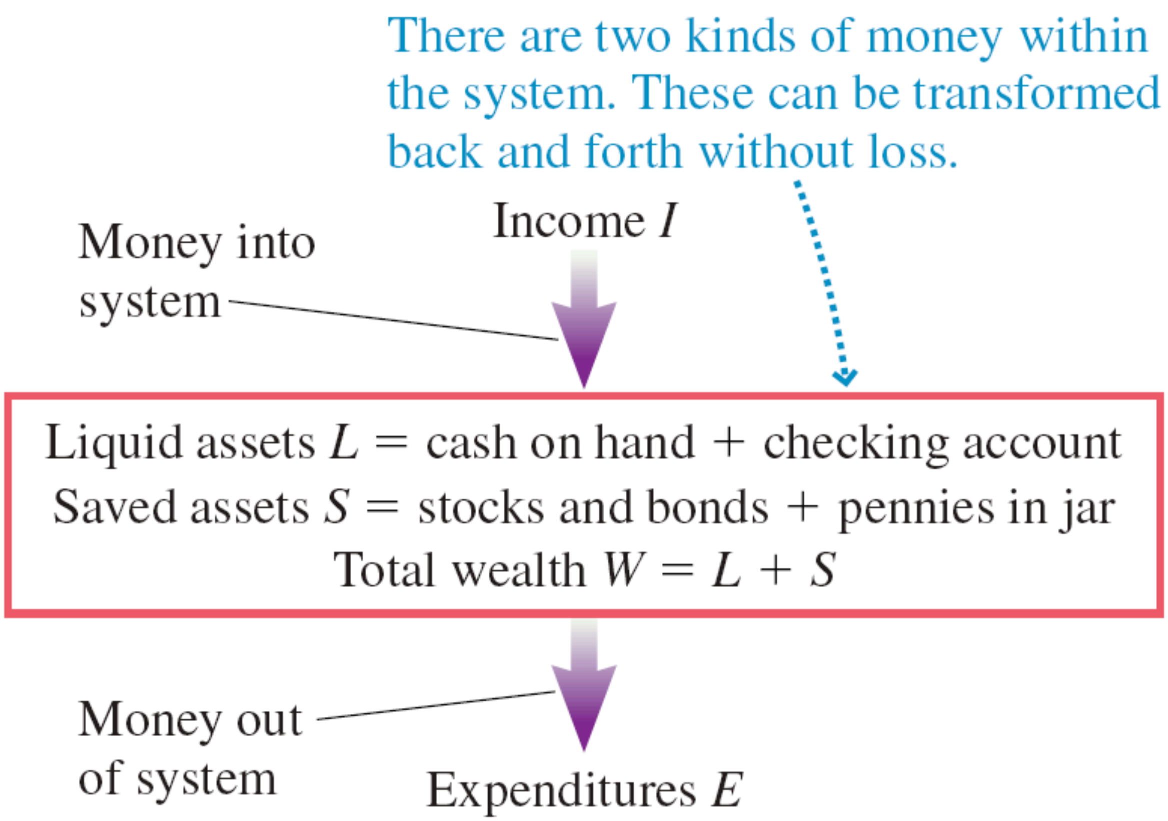
Learning Objectives

- Understand and use the concepts of kinetic and potential energy
- Use energy bar graphs
- Use and interpret energy diagrams
- Solve problems using the law of conservation of mechanical energy

Money-Energy Analogy

From the *Parable of the Lost Penny*

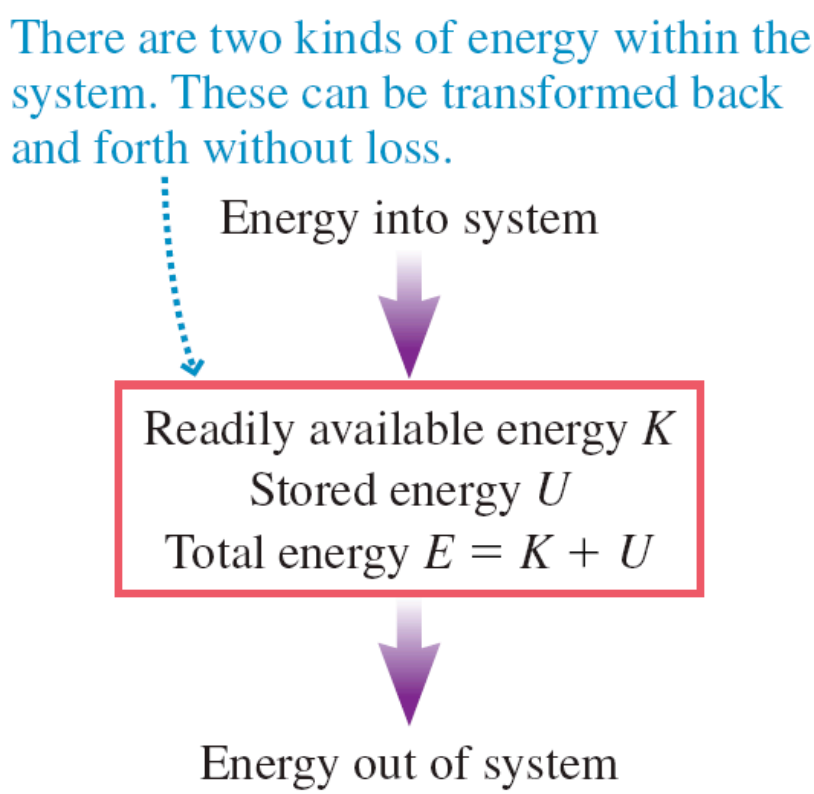
FIGURE 10.1 John's model of the monetary system.



Money—Energy Analogy

From the *law of conservation of energy*

FIGURE 10.2 An initial model of energy. Compare this model to Figure 10.1.



Kinetic and Potential Energy

There are two basic forms of energy. Kinetic energy is an energy of motion

$$K = \frac{1}{2}mv^2 \qquad \text{(kinetic energy)}$$

Gravitational potential energy is an energy of position

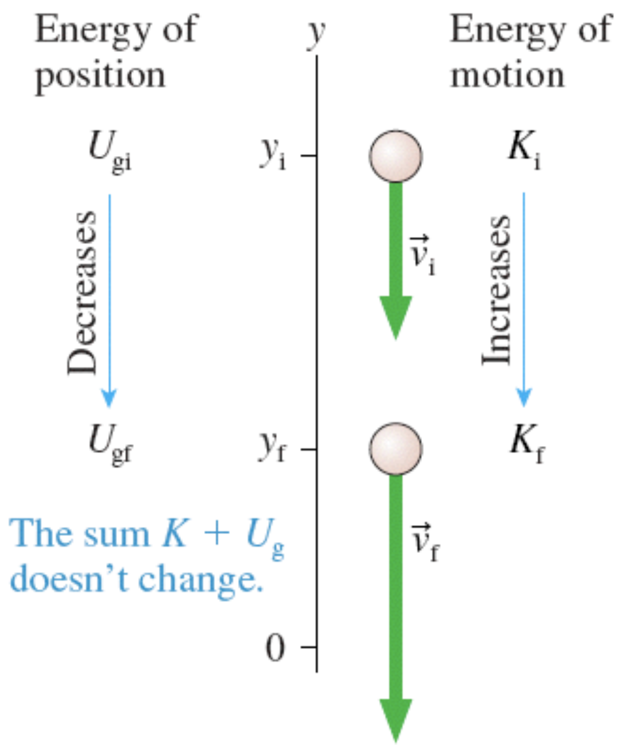
$$U_g = mgy \qquad \text{(gravitational potential energy)}$$

The sum $K + U_g$ is not changed when an object is in freefall. Its initial and final values are equal

$$K_f + U_{gf} = K_i + U_{gi}$$

Kinetic and Potential Energy

FIGURE 10.4 Kinetic energy and gravitational potential energy.



EXAMPLE 10.1 Launching a pebble

QUESTION:

EXAMPLE 10.1 Launching a pebble

Bob uses a slingshot to shoot a 20 g pebble straight up with a speed of 25 m/s. How high does the pebble go?

EXAMPLE 10.1 Launching a pebble

MODEL This is free-fall motion, so the sum of the kinetic and gravitational potential energy does not change as the pebble rises.

EXAMPLE 10.1 Launching a pebble

VISUALIZE **FIGURE 10.5** shows a before-and-after pictorial representation. The pictorial representation for energy problems is essentially the same as the pictorial representation you learned in Chapter 9 for momentum problems. We'll use numerical subscripts 0 and 1 for the initial and final points.

FIGURE 10.5 Pictorial representation of a pebble shot upward from a slingshot.

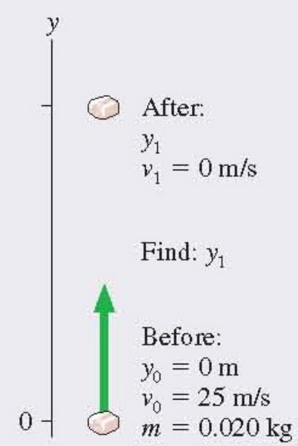
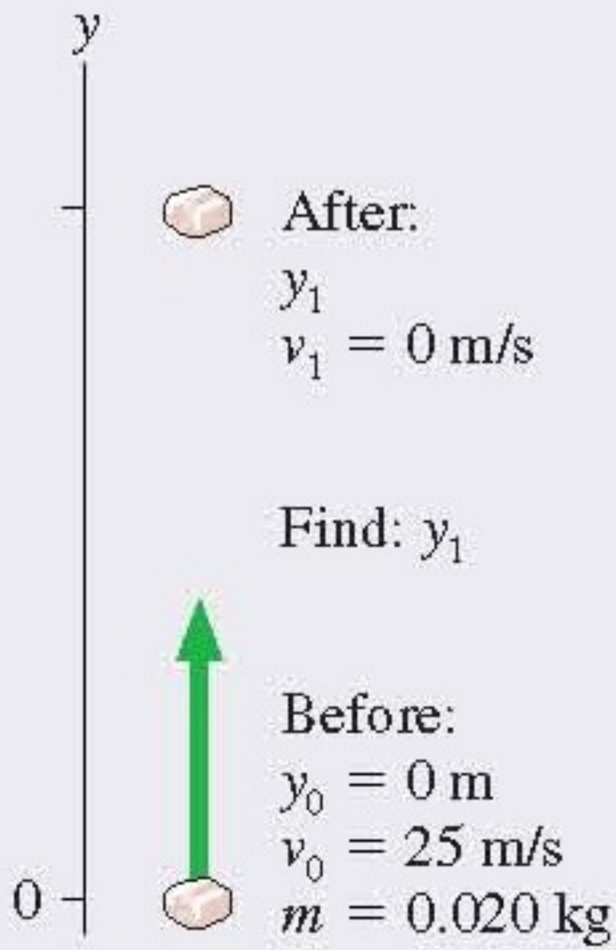


FIGURE 10.5 Pictorial representation of a pebble shot upward from a slingshot.



SOLVE Equation 10.13,

$$K_1 + U_{g1} = K_0 + U_{g0}$$

tells us that the sum $K + U_g$ is not changed by the motion.

Using the definitions of K and U_g,

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$$

Here $y_0 = 0$ m and $v_1 = 0$ m/s, so the equation simplifies to

$$mgy_1 = \frac{1}{2}mv_0^2$$

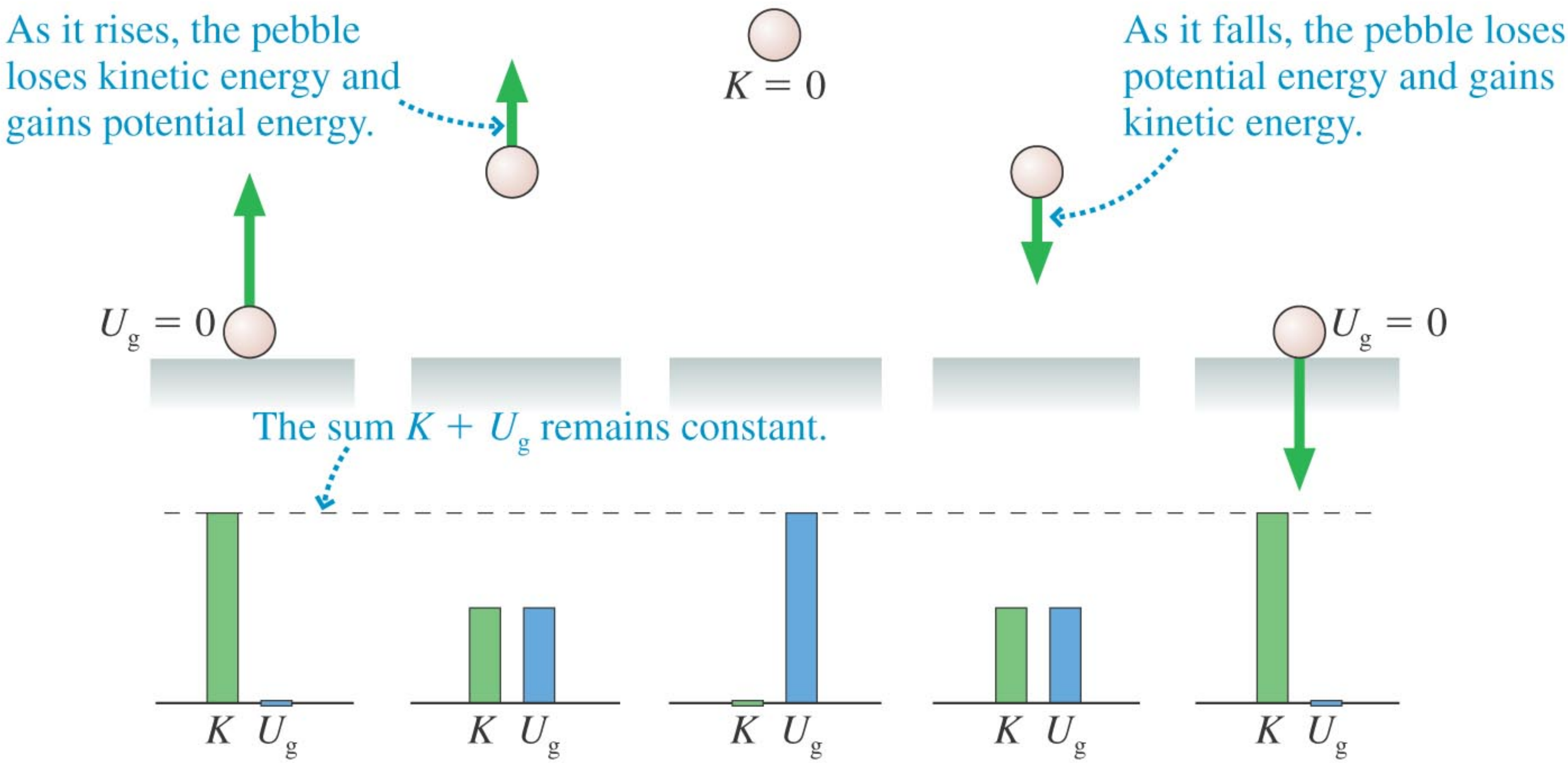
This is easily solved for the height y_1:

$$y_1 = \frac{v_0^2}{2g} = \frac{(25\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 32\text{ m}$$

EXAMPLE 10.1 Launching a pebble

ASSESS Notice that the mass canceled and wasn't needed, a fact about free fall that you should remember from Chapter 2.

Energy Bar Charts



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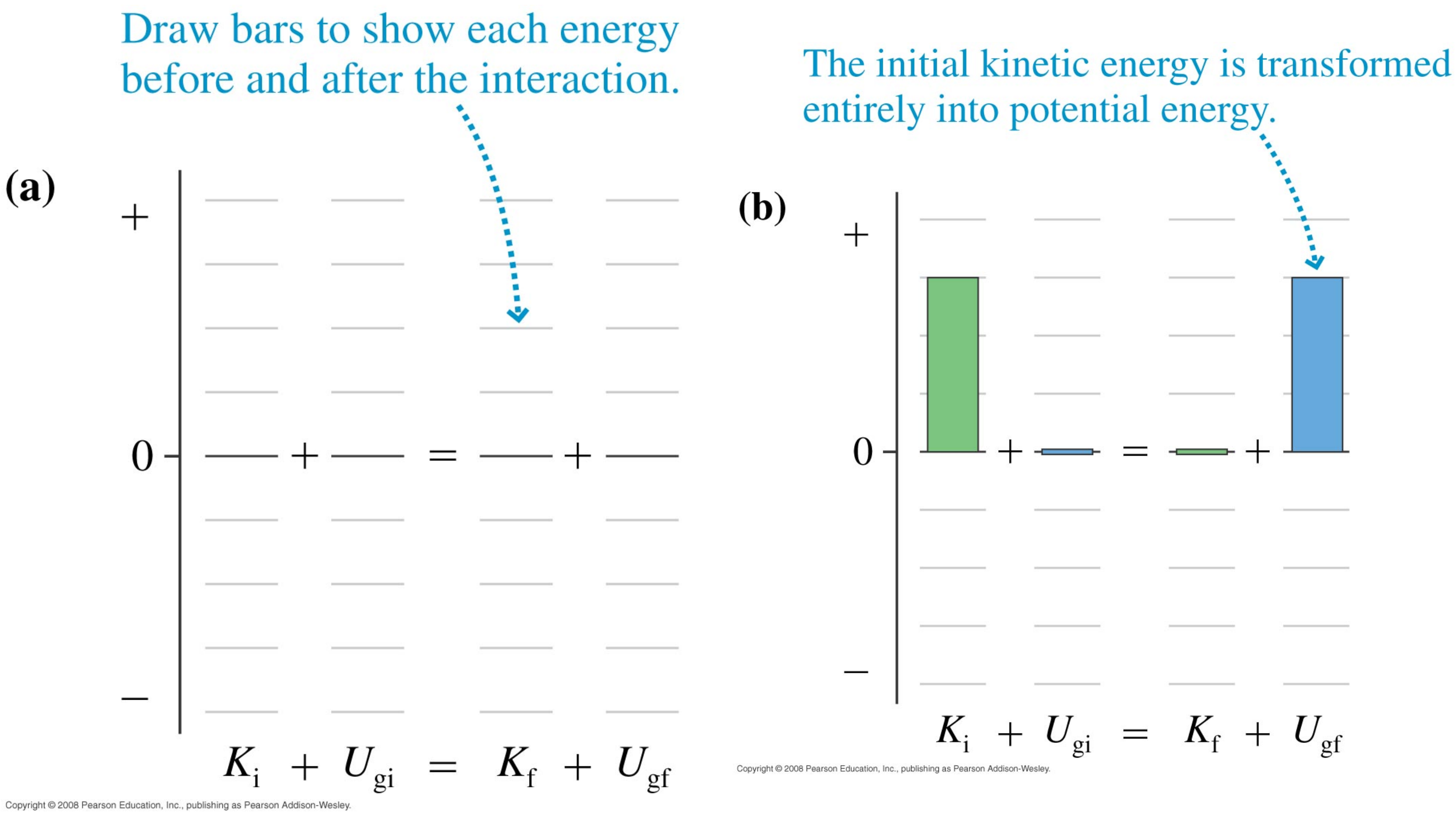
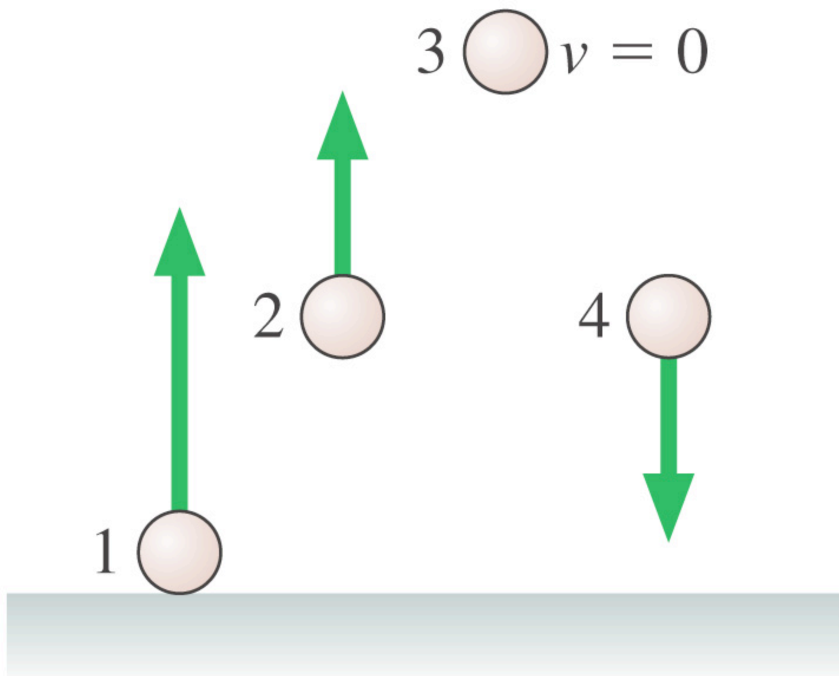
Energy Bar Charts – for problem solving



Rank in order, from largest to smallest, the gravitational potential energies of balls 1 to 4.



A. $(U_g)_1 > (U_g)_2 = (U_g)_4 > (U_g)_3$

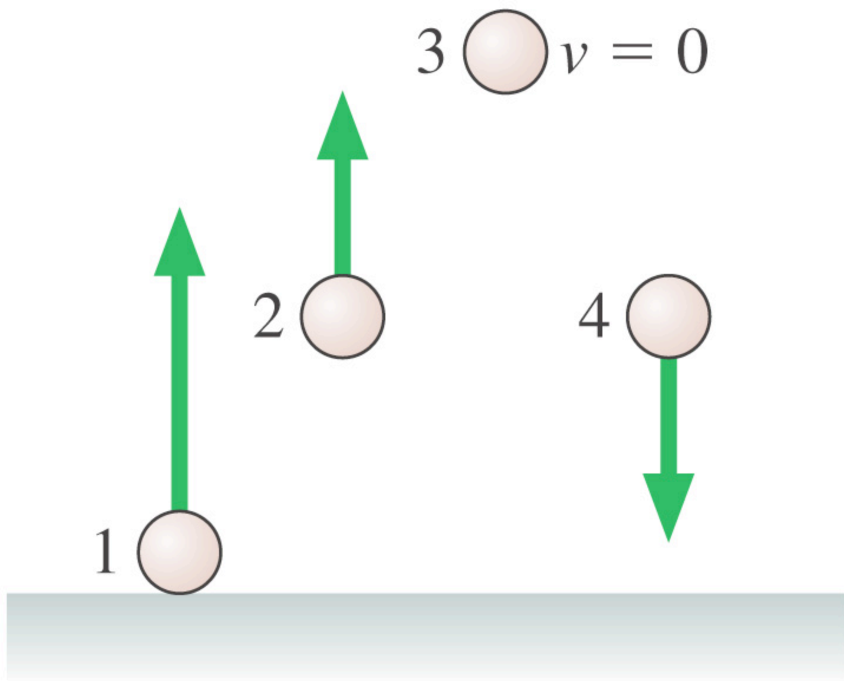
B. $(U_g)_4 > (U_g)_3 > (U_g)_2 > (U_g)_1$

C. $(U_g)_1 > (U_g)_2 > (U_g)_3 > (U_g)_4$

D. $(U_g)_4 = (U_g)_2 > (U_g)_3 > (U_g)_1$

E. $(U_g)_3 > (U_g)_2 = (U_g)_4 > (U_g)_1$

Rank in order, from largest to smallest, the gravitational potential energies of balls 1 to 4.



A. $(U_g)_1 > (U_g)_2 = (U_g)_4 > (U_g)_3$

B. $(U_g)_4 > (U_g)_3 > (U_g)_2 > (U_g)_1$

C. $(U_g)_1 > (U_g)_2 > (U_g)_3 > (U_g)_4$

D. $(U_g)_4 = (U_g)_2 > (U_g)_3 > (U_g)_1$

✔ **E. $(U_g)_3 > (U_g)_2 = (U_g)_4 > (U_g)_1$**

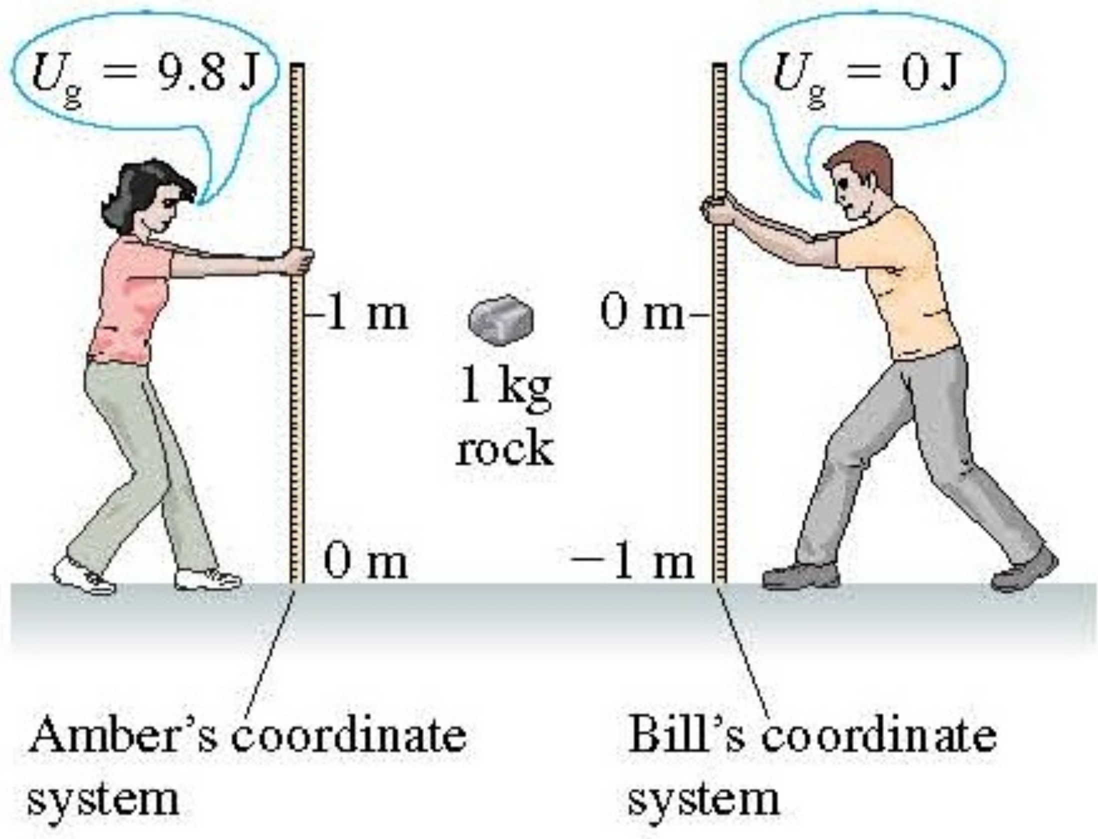
The Zero of Potential Energy

- You can place the origin of your coordinate system, and thus the "zero of potential energy," wherever you choose and be assured of getting the correct answer to a problem.

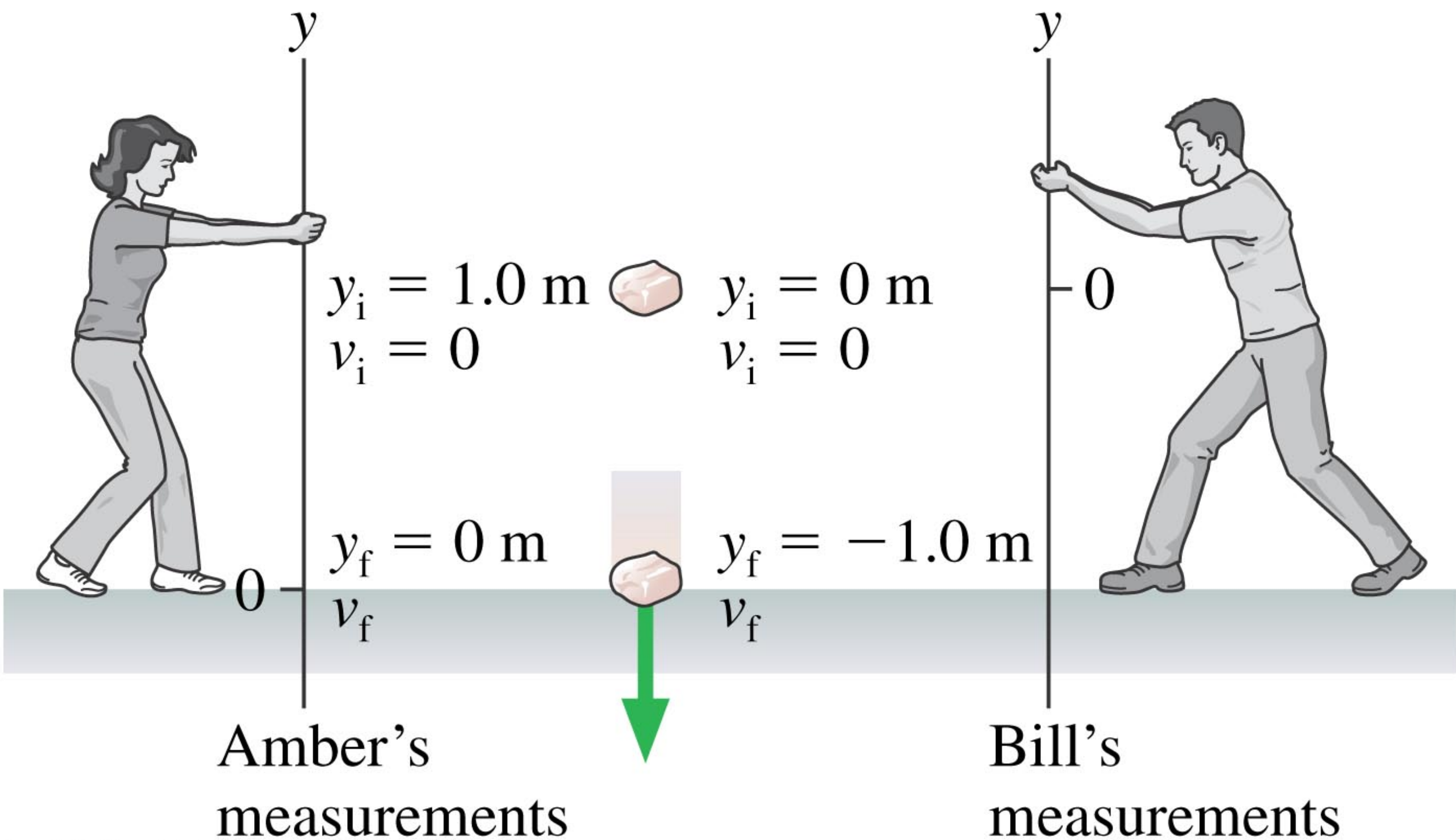
- The reason is that only ΔU has physical significance, not U_g itself.

The Zero of Potential Energy

FIGURE 10.8 Amber and Bill use coordinate systems with different origins to determine the potential energy of a rock.



The Zero of Potential Energy



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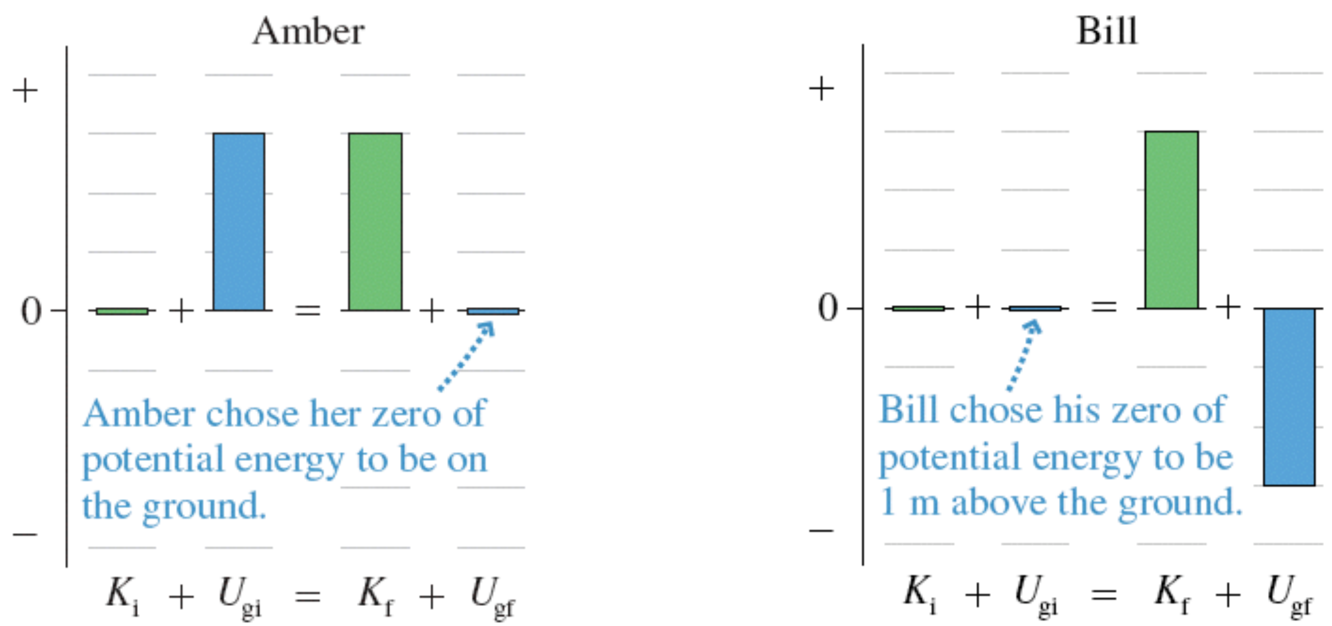
The Zero of Potential Energy

FIGURE 10.10 Amber's and Bill's energy bar charts for the falling rock.



EXAMPLE 10.3 The speed of a sled

QUESTION:

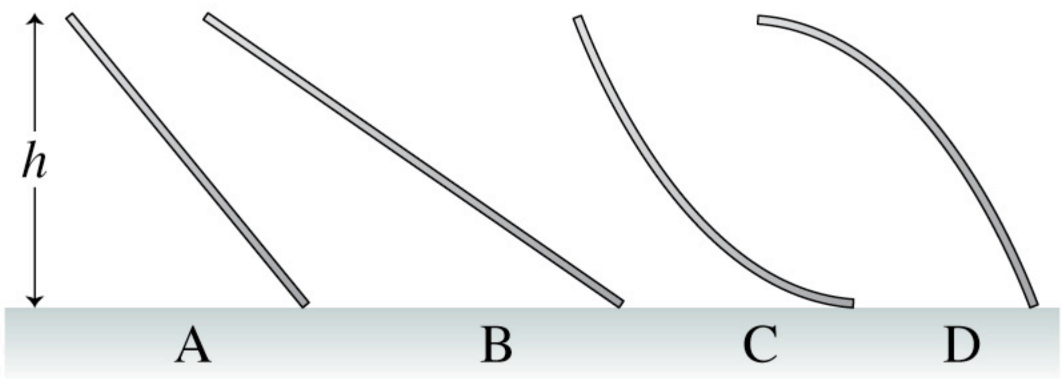
EXAMPLE 10.3 The speed of a sled

Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0-m-high, very slippery slope. What is her speed at the bottom?

EXAMPLE 10.3 The speed of a sled

MODEL Model Christine and the sled as a particle. Assume the slope is frictionless. In that case, the sum of her kinetic and gravitational potential energy does not change as she slides down.

A small child slides down the four frictionless slides A–D. Each has the same height. Rank in order, from largest to smallest, her speeds v_A to v_D at the bottom.



A. $v_C > v_A = v_B > v_D$

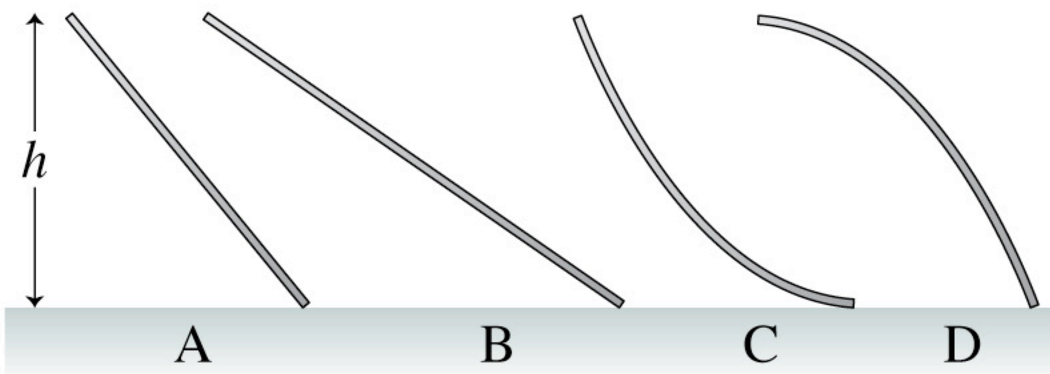
B. $v_C > v_B > v_A > v_D$

C. $v_D > v_A > v_B > v_C$

D. $v_A = v_B = v_C = v_D$

E. $v_D > v_A = v_B > v_C$

A small child slides down the four frictionless slides A–D. Each has the same height. Rank in order, from largest to smallest, her speeds v_A to v_D at the bottom.



A. $v_C > v_A = v_B > v_D$

B. $v_C > v_B > v_A > v_D$

C. $v_D > v_A > v_B > v_C$

D. $v_A = v_B = v_C = v_D$

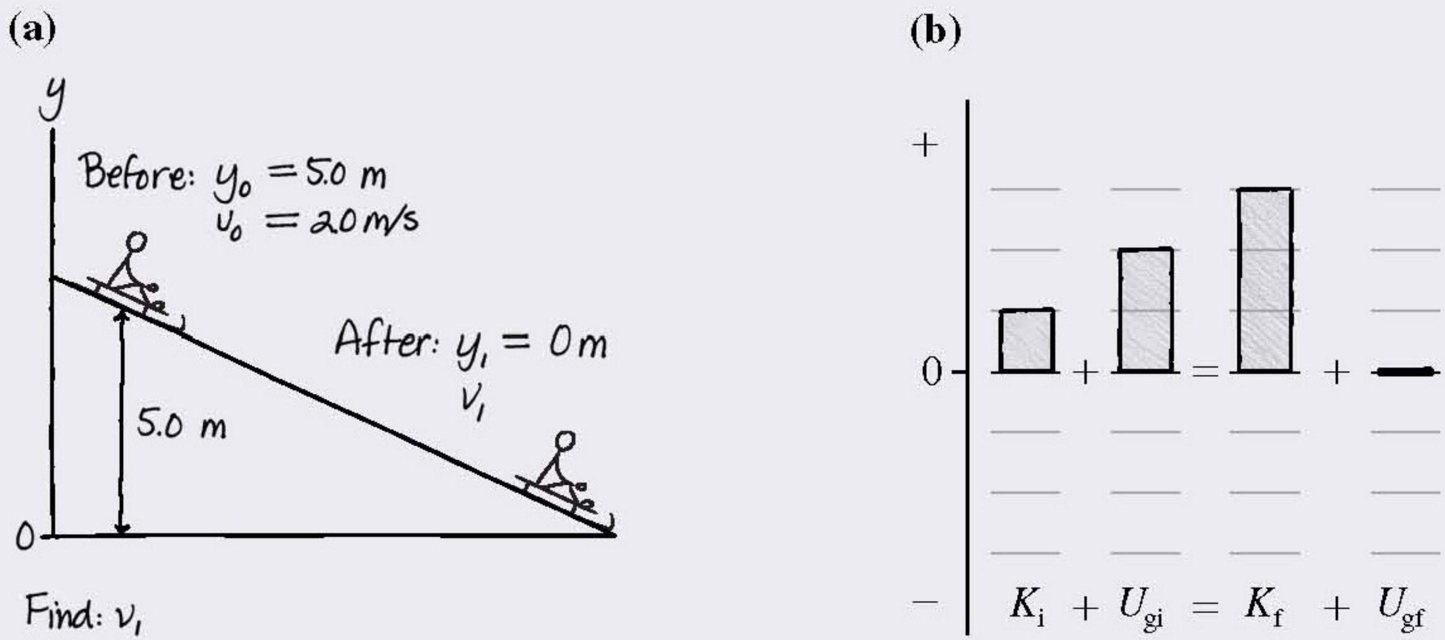
E. $v_D > v_A = v_B > v_C$

EXAMPLE 10.3 The speed of a sled

VISUALIZE **FIGURE 10.12a** shows a before-and-after pictorial representation. We are not told the angle of the slope, or even if it is a straight slope, but the *change* in potential energy depends only on the height Christine descends and *not* on the shape of the hill. **FIGURE 10.12b** shows an energy bar chart in which we see an initial kinetic *and* potential energy being transformed into entirely kinetic energy as she goes down the slope. The purpose of the bar chart is to visualize how the energy changes, and we can show that without knowing any numerical values.

EXAMPLE 10.3 The speed of a sled

FIGURE 10.12 Pictorial representation and energy bar chart of Christine sliding down the hill.



EXAMPLE 10.3 The speed of a sled

SOLVE The quantity $K + U_g$ is the same at the bottom of the hill as it was at the top. Thus

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$$

This is easily solved for Christine's speed at the bottom:

$$v_1 = \sqrt{v_0^2 + 2g(y_0 - y_1)} = \sqrt{v_0^2 + 2gh} = 10 \text{ m/s}$$

ASSESS We did not need the mass of either Christine or the sled.

Conservation of Mechanical Energy

The sum of the kinetic energy and the potential energy of a system is called the **mechanical energy**.

$$E_{\text{mech}} = K + U$$

Here, *K* is the total kinetic energy of all the particles in the system and *U* is the potential energy stored in the system. The kinetic energy and the potential energy can change, as they are transformed back and forth into each other, but their sum remains constant.

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

The Basic Energy Model

1) Kinetic energy is associated with the motion of a particle. Potential energy is associated with its position
2) Kinetic energy can be transformed into potential energy & potential energy can be transformed into kinetic energy
3) Under some circumstances mechanical energy –

   $E_{mech} = K + U$ is conserved

Three critical questions:

Under what conditions is E_{mech} conserved?

What happens to the energy when E_{mech} is not conserved?

How do you calculate the potential energy U for forces other than gravity?

PROBLEM-SOLVING STRATEGY 10.1

Conservation of mechanical energy

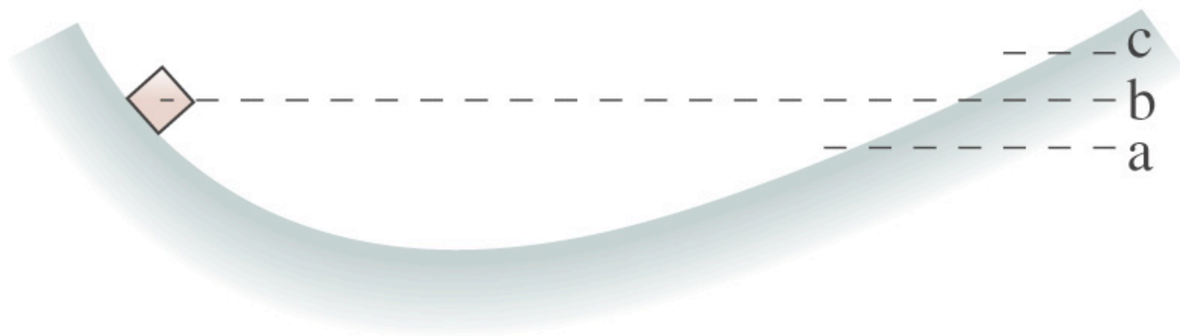
MODEL Choose a system without friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + U_f = K_i + U_i$$

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

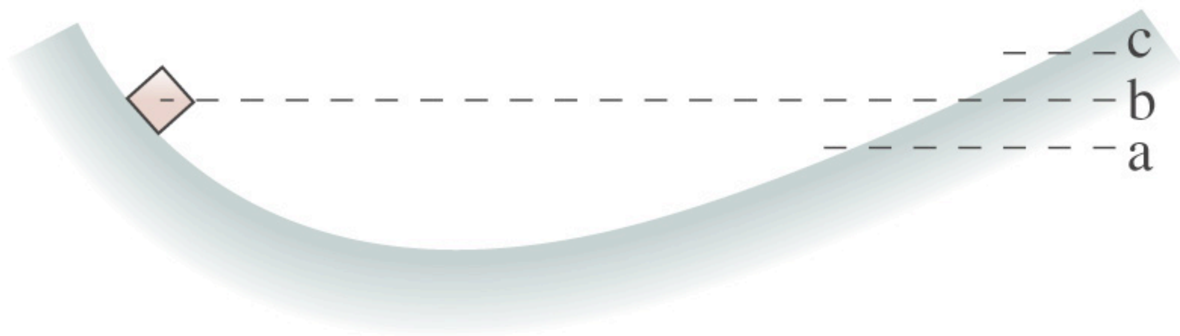


A box slides along the frictionless surface shown in the figure. It is released from rest at the position shown. Is the highest point the box reaches on the other side at level a, at level b, or level c?

A. At level a

B. At level b

C. At level c



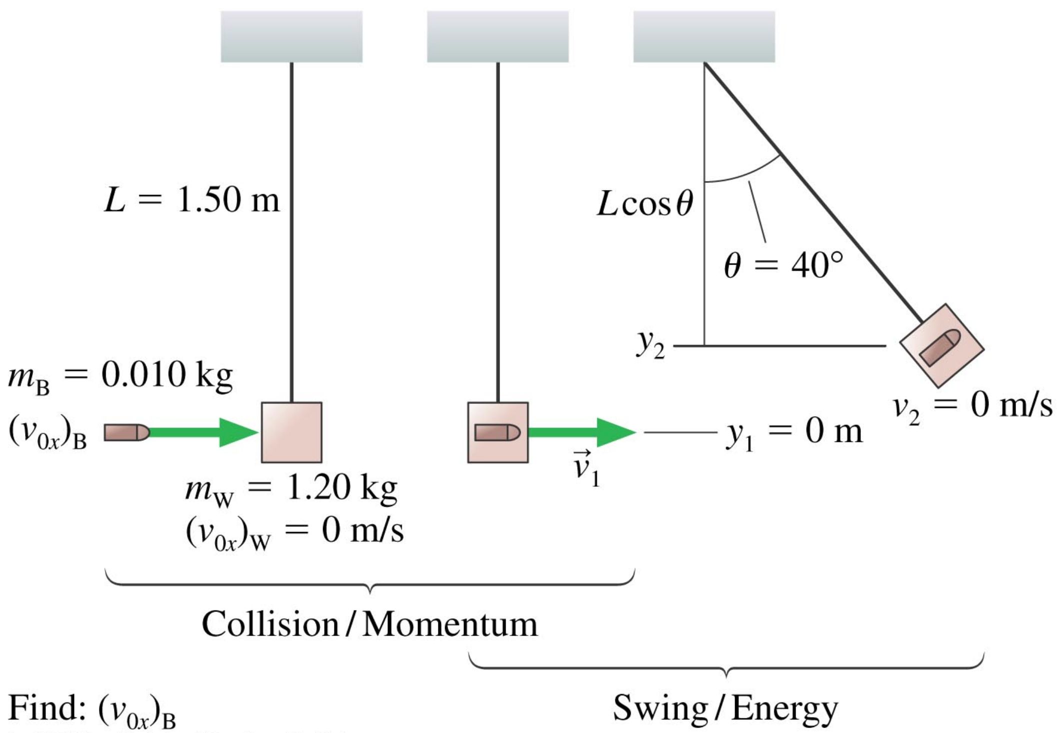
A box slides along the frictionless surface shown in the figure. It is released from rest at the position shown. Is the highest point the box reaches on the other side at level a, at level b, or level c?

A. At level a

✔ **B. At level b**

C. At level c

Ballistic Pendulum



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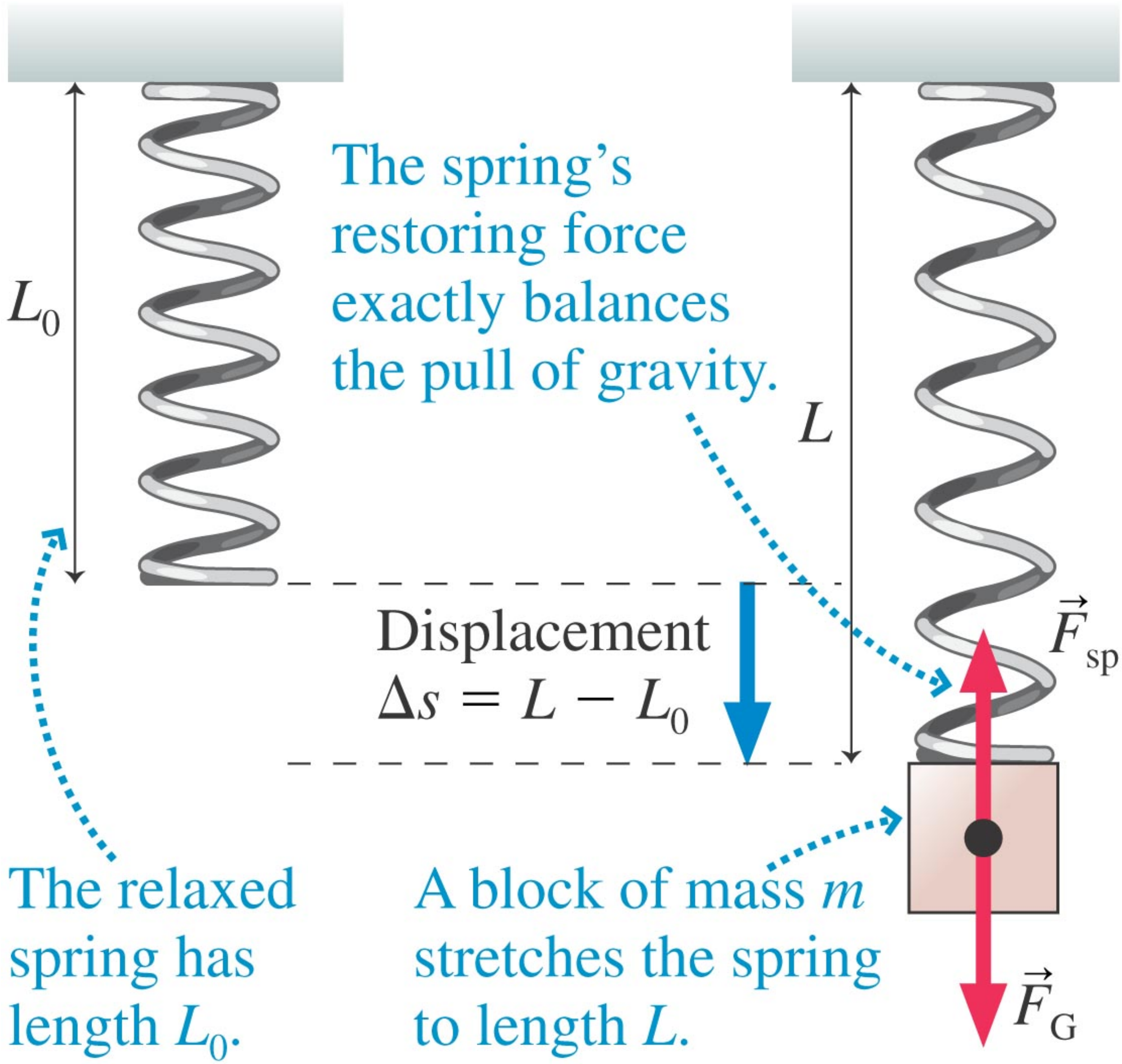
- A 10 g bullet is fired into a 1200 g wood block hanging from a 150 cm long string. The bullet embeds itself in the wood and block swings out to an angle of 40°. What is the speed of the bullet?



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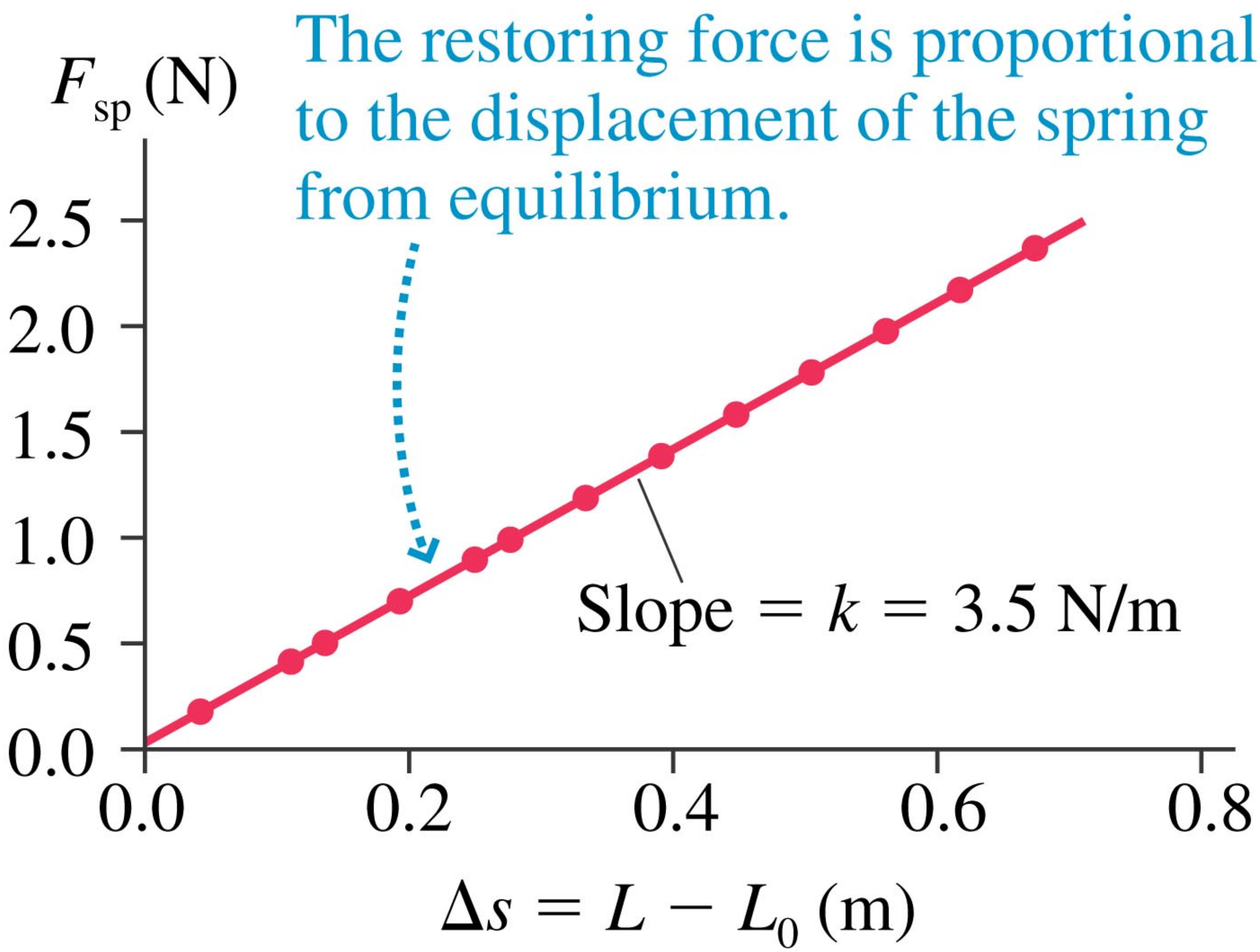

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Hooke's Law

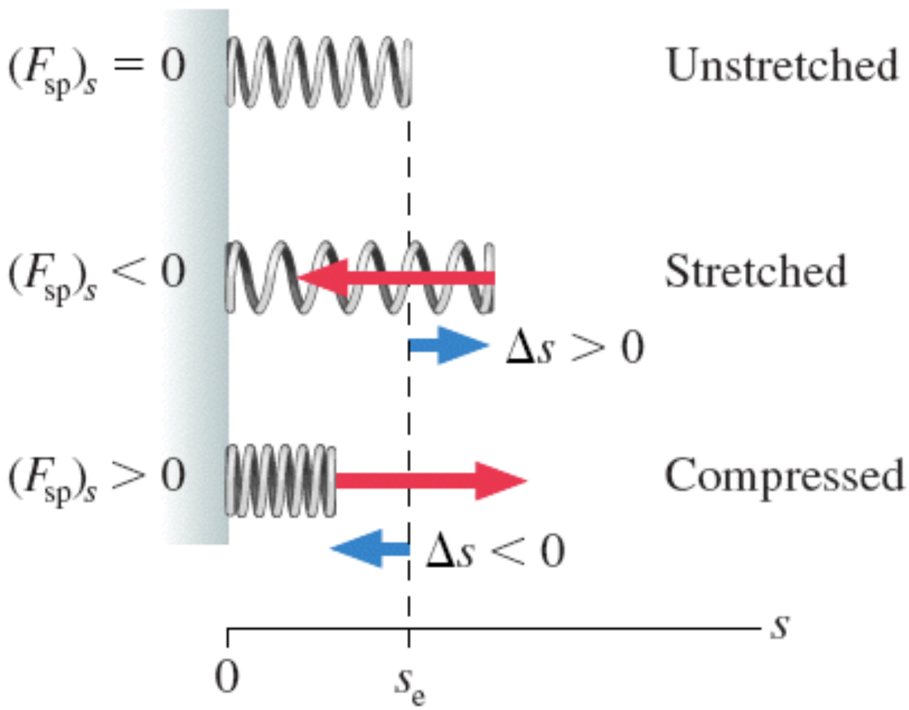
If you stretch a rubber band, a force appears that tries to pull the rubber band back to its equilibrium, or unstretched, length. A force that restores a system to an equilibrium position is called a **restoring force**. If s is the position of the end of a spring, and s_e is the equilibrium position, we define $\Delta s = s - s_e$. If $(F_{sp})_s$ is the s-component of the restoring force, and k is the spring constant of the spring, then Hooke's Law states that

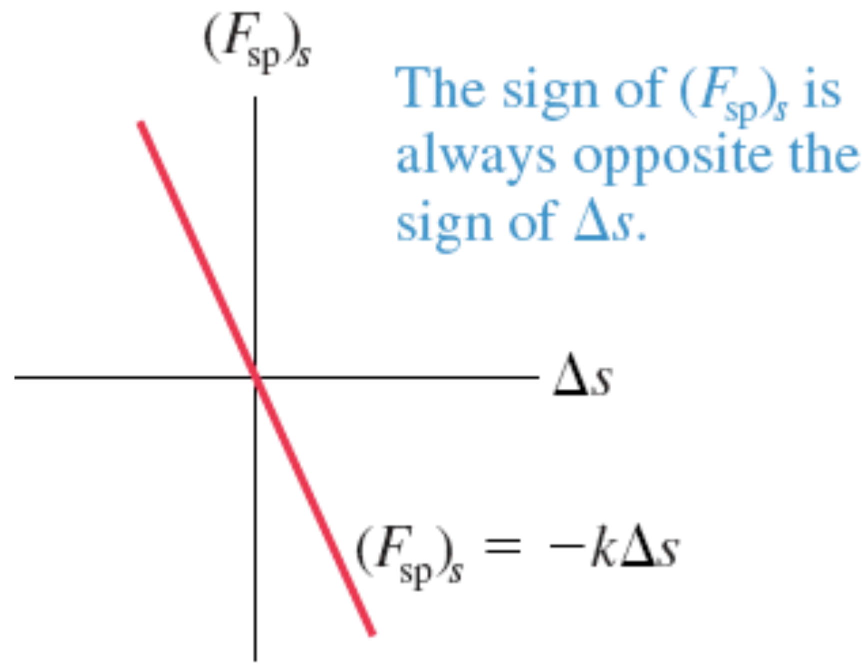
$$(F_{sp})_s = -k\Delta s \qquad \text{(Hooke's law)}$$

The minus sign is the mathematical indication of a *restoring force*.

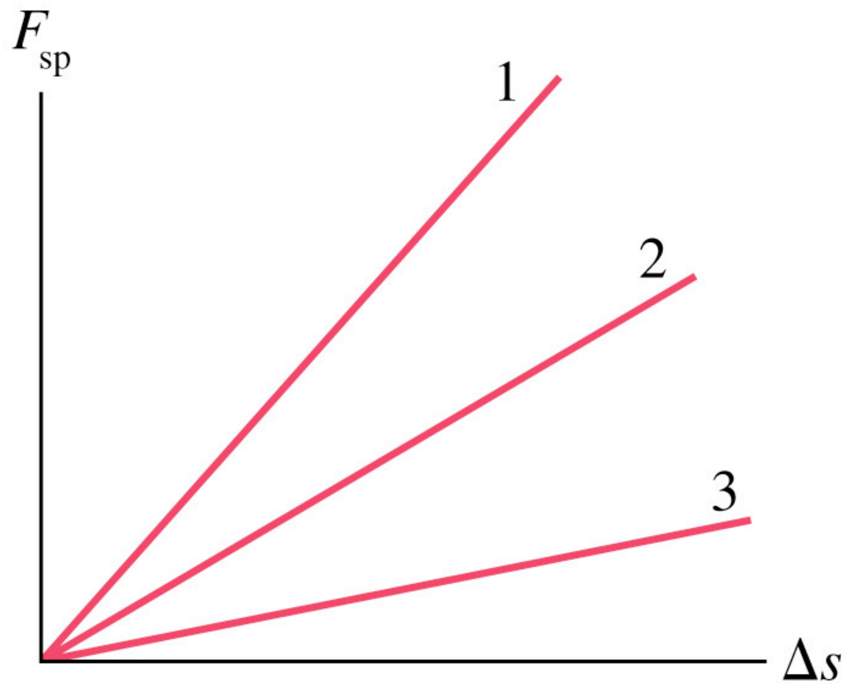
Hooke's Law

FIGURE 10.16 The direction of $\vec{F}_{sp}$ is always opposite the displacement $\Delta\vec{s}$.





The graph shows force versus displacement for three springs. Rank in order, from largest to smallest, the spring constants k_1, k_2, and k_3.



A. $k_1 > k_3 > k_2$

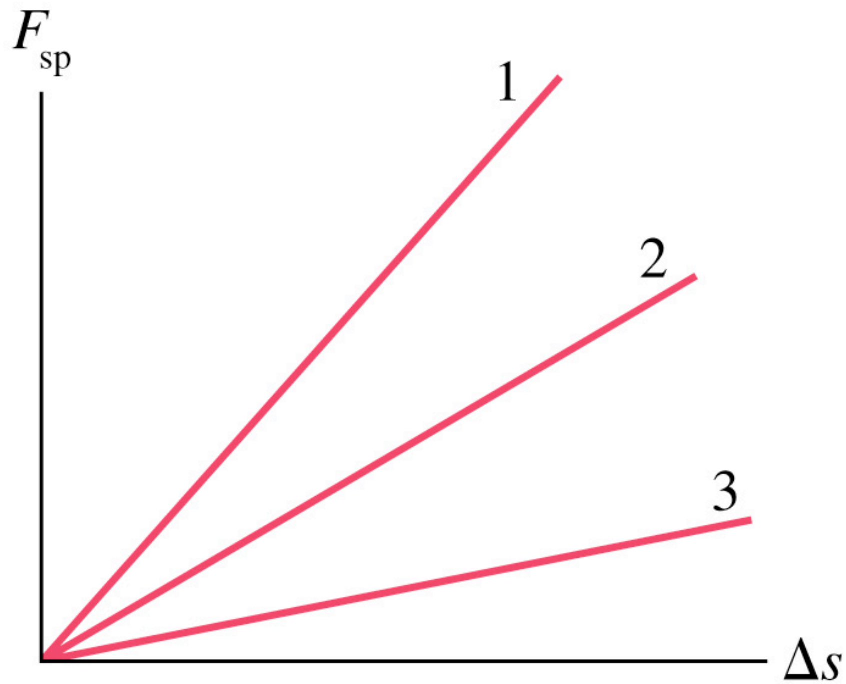
B. $k_3 > k_2 > k_1$

C. $k_1 = k_3 > k_2$

D. $k_2 > k_1 = k_3$

E. $k_1 > k_2 > k_3$

The graph shows force versus displacement for three springs. Rank in order, from largest to smallest, the spring constants k_1, k_2, and k_3.



A. $k_1 > k_3 > k_2$

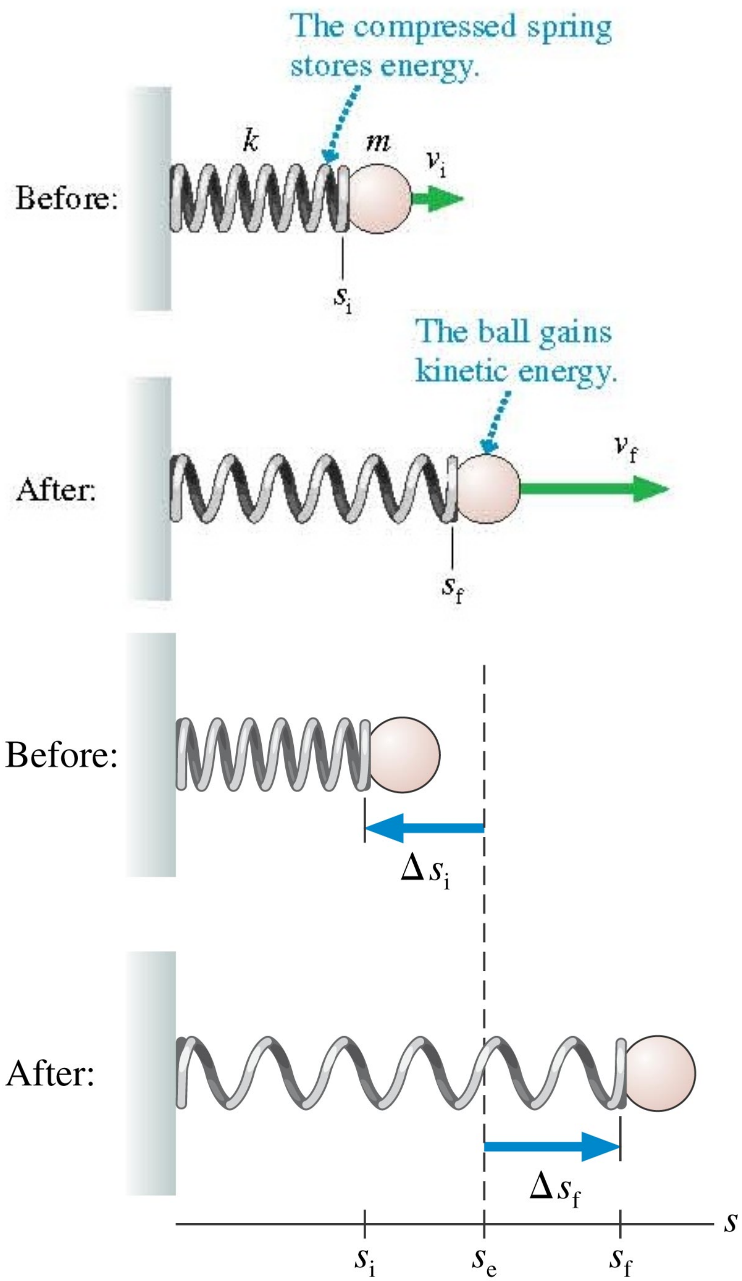
B. $k_3 > k_2 > k_1$

C. $k_1 = k_3 > k_2$

D. $k_2 > k_1 = k_3$

E. $k_1 > k_2 > k_3$

FIGURE 10.19 Before and after a spring launches a ball.



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Elastic Potential Energy

Consider a before-and-after situation in which a spring launches a ball. The compressed spring has "stored energy," which is then transferred to the kinetic energy of the ball. We define the **elastic potential energy** U_s of a spring to be

$$U_s = \frac{1}{2}k(\Delta s)^2 \qquad \text{(elastic potential energy)}$$

EXAMPLE 10.6 A spring-launched plastic ball

QUESTION:

EXAMPLE 10.6 A spring-launched plastic ball

A spring-loaded toy gun launches a 10 g plastic ball. The spring, with spring constant 10 N/m, is compressed by 10 cm as the ball is pushed into the barrel. When the trigger is pulled, the spring is released and shoots the ball back out. What is the ball's speed as it leaves the barrel? Assume friction is negligible.

EXAMPLE 10.6 A spring-launched plastic ball

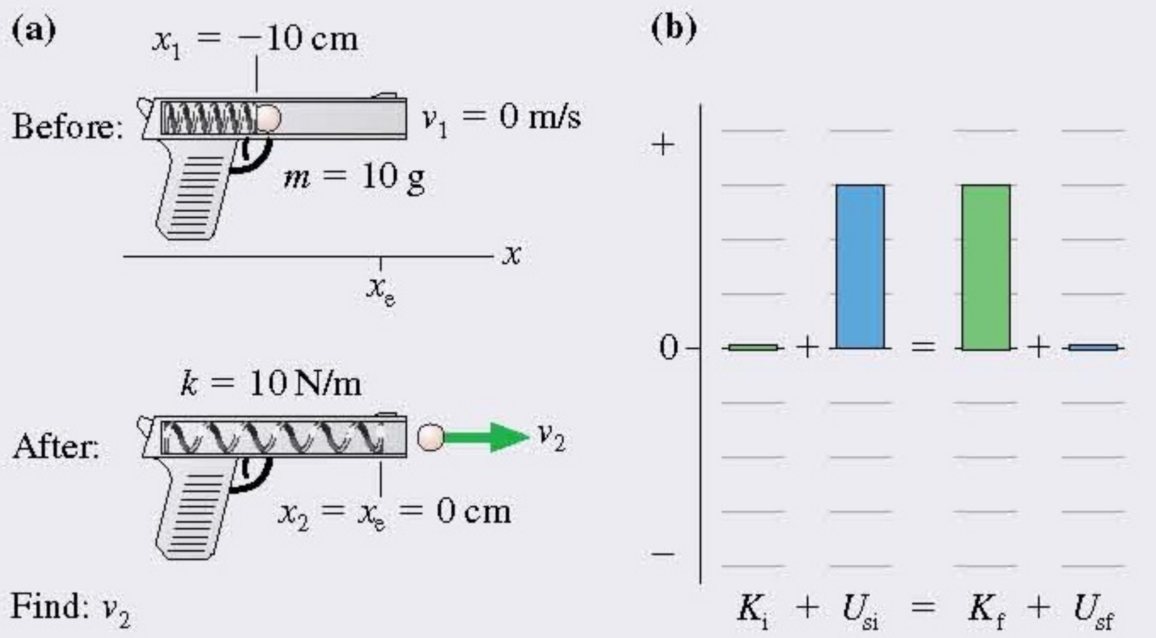
MODEL Assume an ideal spring that obeys Hooke's law. Also assume that the gun is held firmly enough to prevent recoil. There's no friction; hence the mechanical energy $K + U_s$ is conserved.

EXAMPLE 10.6 A spring-launched plastic ball

VISUALIZE **FIGURE 10.21a** shows a before-and-after pictorial representation. The compressed spring will push on the ball until the spring has returned to its equilibrium length. We have chosen to put the origin of the coordinate system at the equilibrium position of the free end of the spring, making $x_1 = -10$ cm and $x_2 = x_e = 0$ cm. It's also useful to look at an energy bar chart. The bar chart of **FIGURE 10.21b** shows the potential energy stored in the compressed spring being entirely transformed into the kinetic energy of the ball.

EXAMPLE 10.6 A spring-launched plastic ball

FIGURE 10.21 Pictorial representation and energy bar chart of a ball being shot from a spring-loaded toy gun.



EXAMPLE 10.6 A spring-launched plastic ball

SOLVE The energy conservation equation is $K_2 + U_{s2} = K_1 + U_{s1}$. We can use the elastic potential energy of the spring, Equation 10.36, to write this as

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Notice that we used x, rather than the generic s, and that we explicitly wrote out the meaning of Δx_1 and Δx_2. Using $x_2 = x_e = 0$ m and $v_1 = 0$ m/s simplifies this to

$$\frac{1}{2}mv_2^2 = \frac{1}{2}kx_1^2$$

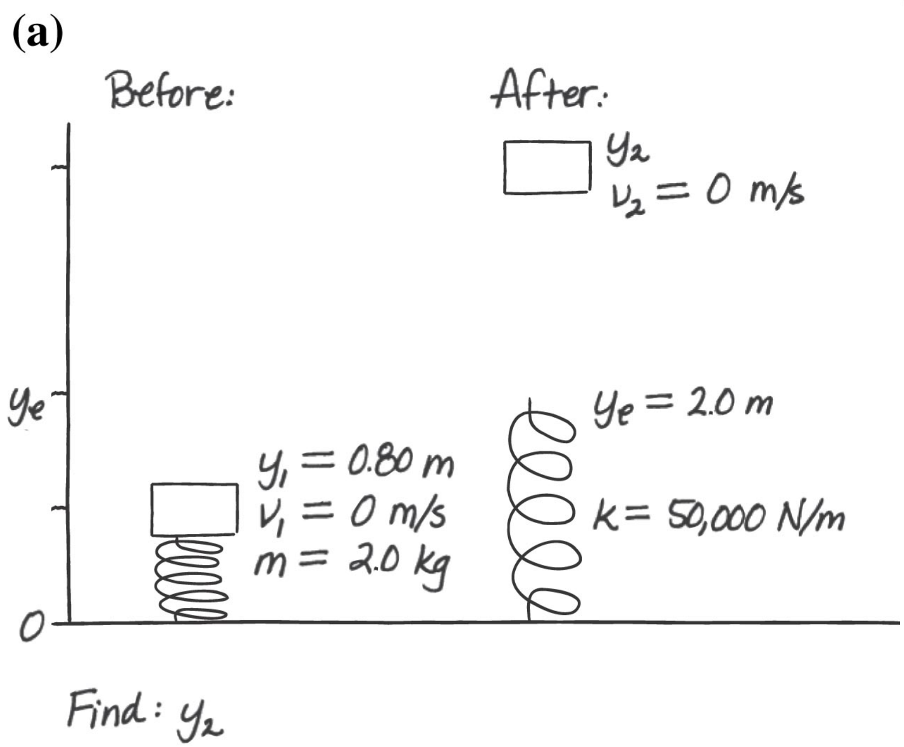
It is now straightforward to solve for the ball's speed:

$$v_2 = \sqrt{\frac{kx_1^2}{m}} = \sqrt{\frac{(10\text{ N/m})(-0.10\text{ m})^2}{0.010\text{ kg}}} = 3.2\text{ m/s}$$

EXAMPLE 10.6 A spring-launched plastic ball

ASSESS This is a problem that we could *not* have solved with Newton's laws. The acceleration is not constant, and we have not learned how to handle the kinematics of nonconstant acceleration. But with conservation of energy—it's easy!

Spring launched satellite: A 2.0 kg payload is placed on top of a very stiff 2.0 m long spring with spring constant 50,000 N/m. The spring is winched down to a length of 80 cm. How high does the payload go?



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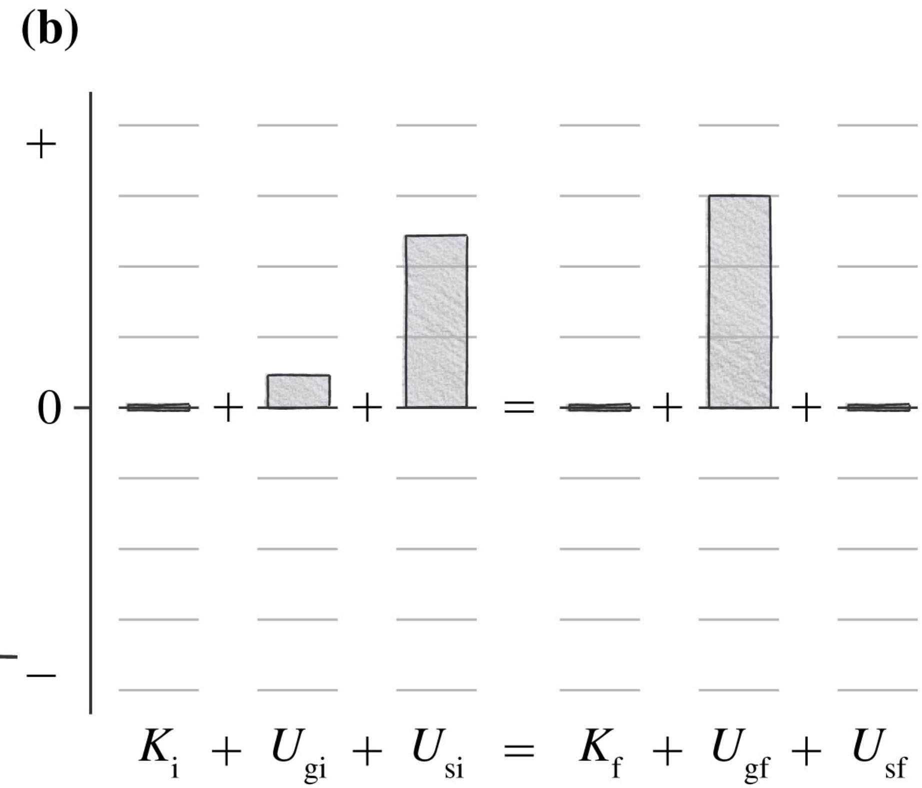

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A spring-loaded gun shoots a plastic ball with a speed of 4 m/s. If the spring is compressed twice as far, the ball's speed will be

A. 1 m/s.
B. 2 m/s.
C. 4 m/s.
D. 8 m/s.
E. 16 m/s.

A spring-loaded gun shoots a plastic ball with a speed of 4 m/s. If the spring is compressed twice as far, the ball's speed will be

A. 1 m/s.

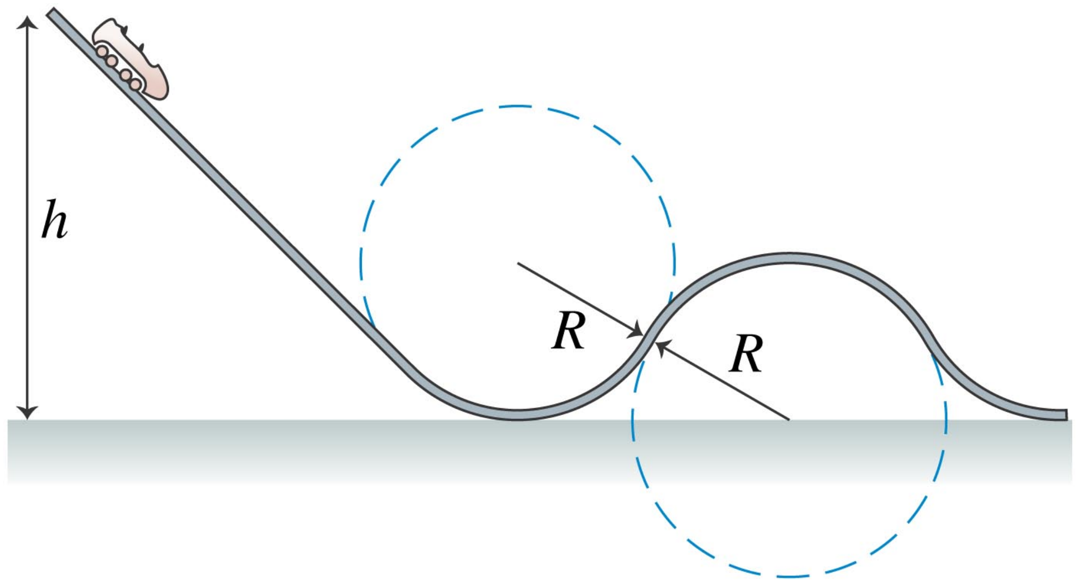
B. 2 m/s.

C. 4 m/s.

✔ **D. 8 m/s.**

E. 16 m/s.

A roller coaster car on a frictionless track shown in figure starts from rest at height h. The track's valley and hill consist of circular shaped segments of radius R.



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What is the maximum height h_{max} from which the car can start so as to not fly off the track when going over the hill?

Evaluate h_{max} for a roller coaster that has $R = 10$ m.

General Principles

Law of Conservation of Mechanical Energy

If there are no friction or other energy-loss processes (to be explored more thoroughly in Chapter 11), then the mechanical energy $E_{mech} = K + U$ of a system is conserved. Thus

$$K_f + U_f = K_i + U_i$$

- K is the sum of the kinetic energies of all particles.
- U is the sum of all potential energies.

General Principles

Solving Energy Conservation Problems

MODEL Choose a system without friction or other losses of mechanical energy.

VISUALIZE Draw a before-and-after pictorial representation.

SOLVE Use the law of conservation of energy:

$$K_f + U_f = K_i + U_i$$

ASSESS Is the result reasonable?

Important Concepts

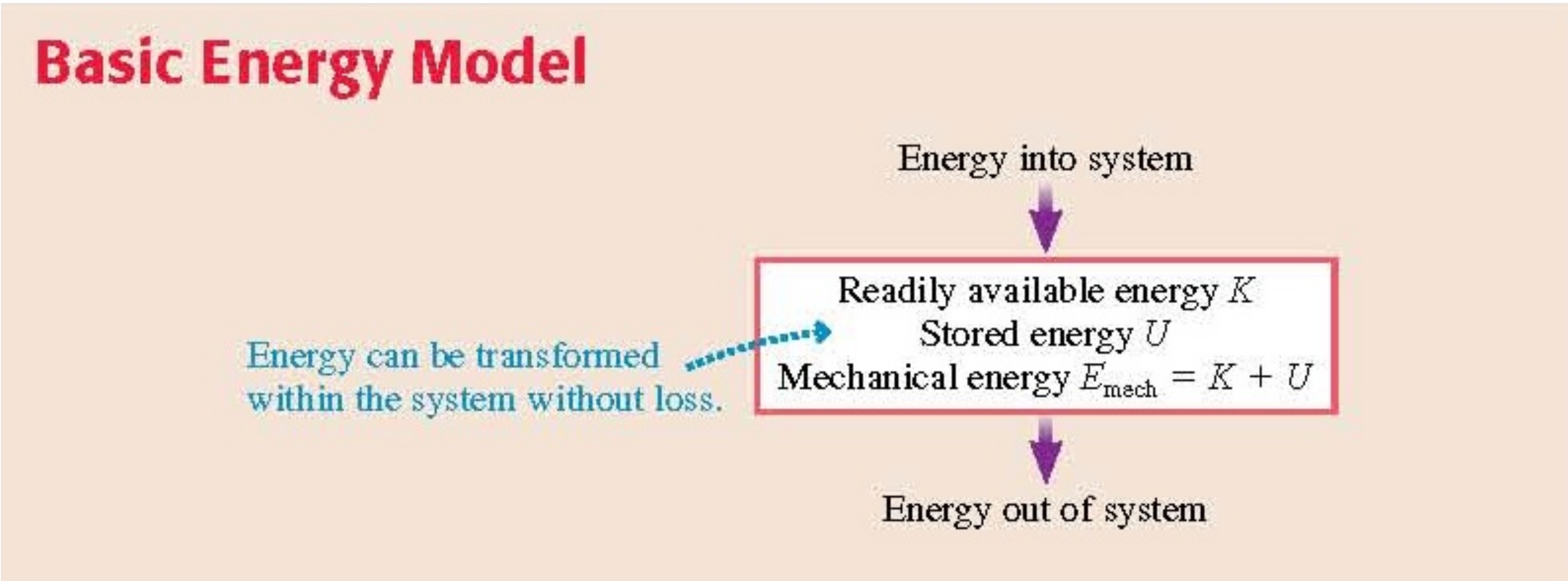
Kinetic energy is an energy of motion:

$$K = \frac{1}{2}mv^2$$

Potential energy is an energy of position

- **Gravitational:** $U_g = mgy$
- **Elastic:** $U_s = \frac{1}{2}k(\Delta s)^2$

Important Concepts



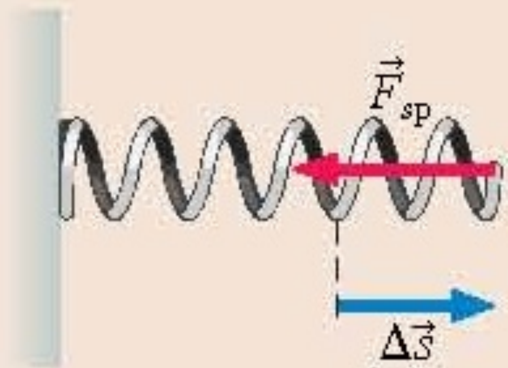
Applications

Hooke's law

The restoring force of an ideal spring is

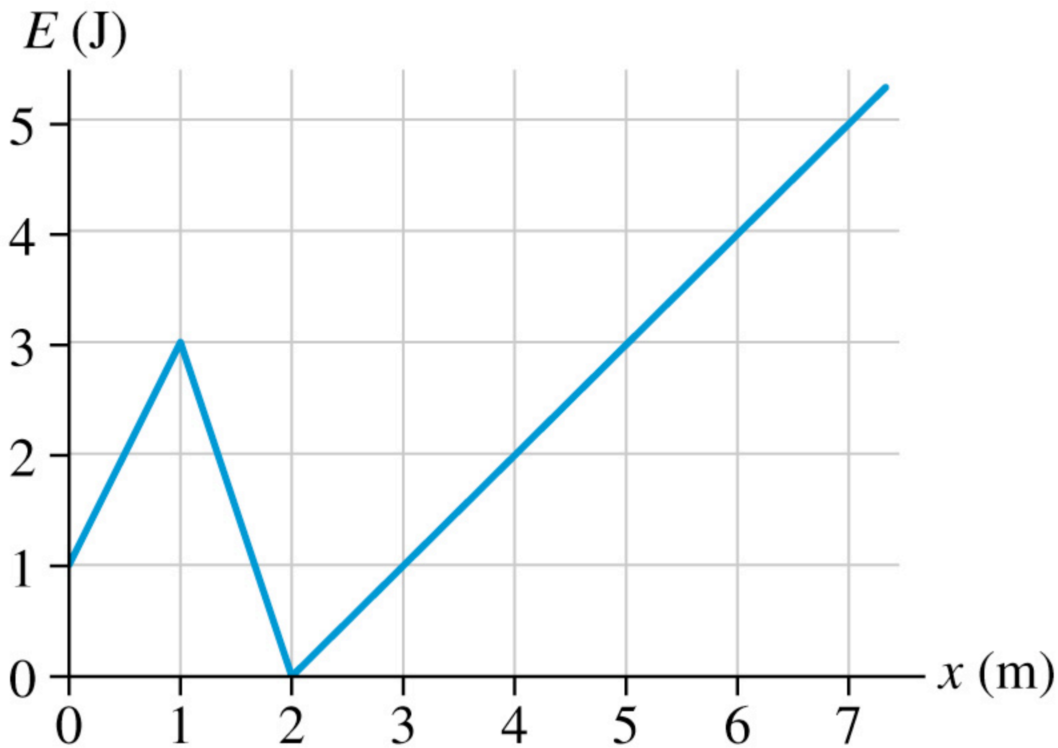
$$(F_{sp})_s = -k\,\Delta s$$

where k is the spring constant and $\Delta s = s - s_e$ is the displacement from equilibrium.



Chapter 10. Clicker Questions

A particle with the potential energy shown in the graph is moving to the right. It has 1 J of kinetic energy at x = 1 m. Where is the particle's turning point?



A. $x = 1$ m

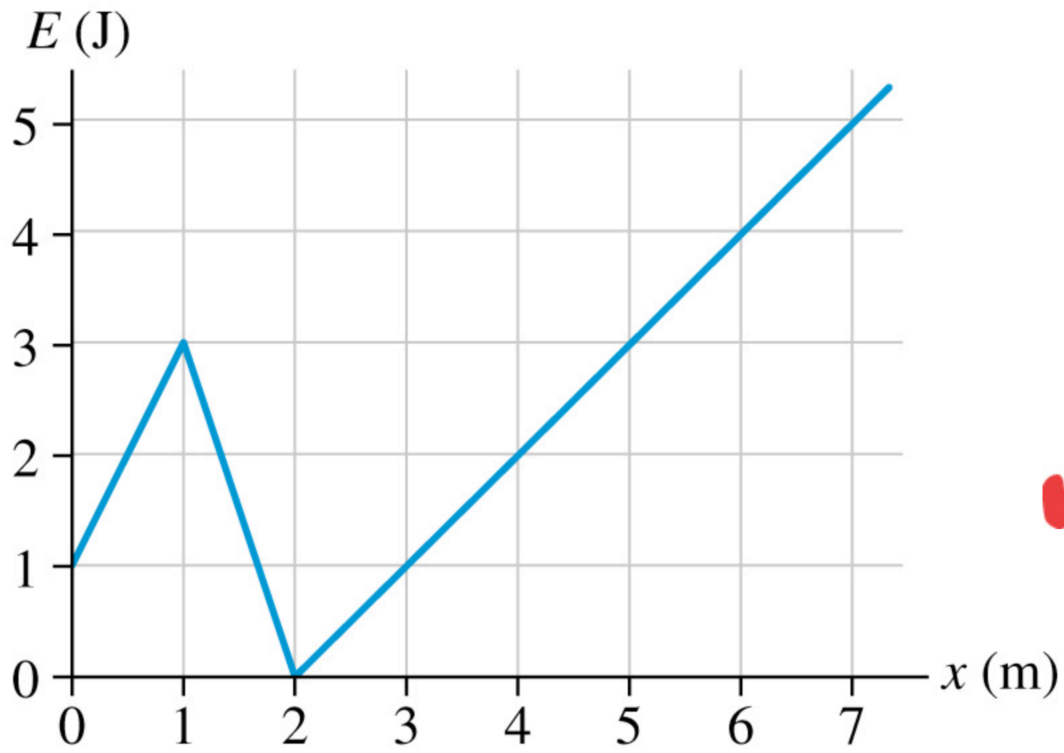
B. $x = 2$ m

C. $x = 5$ m

D. $x = 6$ m

E. $x = 7.5$ m

A particle with the potential energy shown in the graph is moving to the right. It has 1 J of kinetic energy at x = 1 m. Where is the particle's turning point?



A. x = 1 m

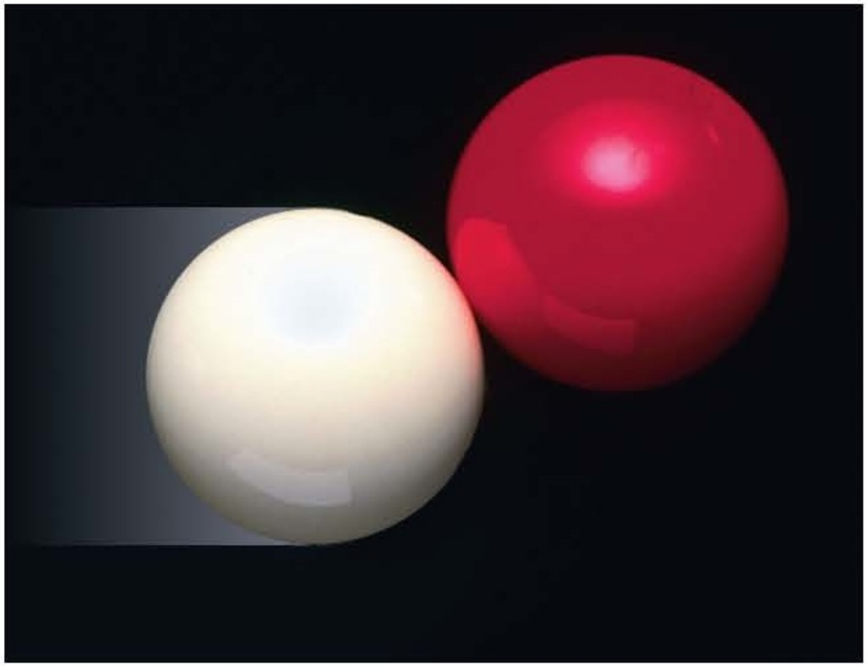
B. x = 2 m

C. x = 5 m

✔ **D. x = 6 m**

E. x = 7.5 m

Elastic Collisions

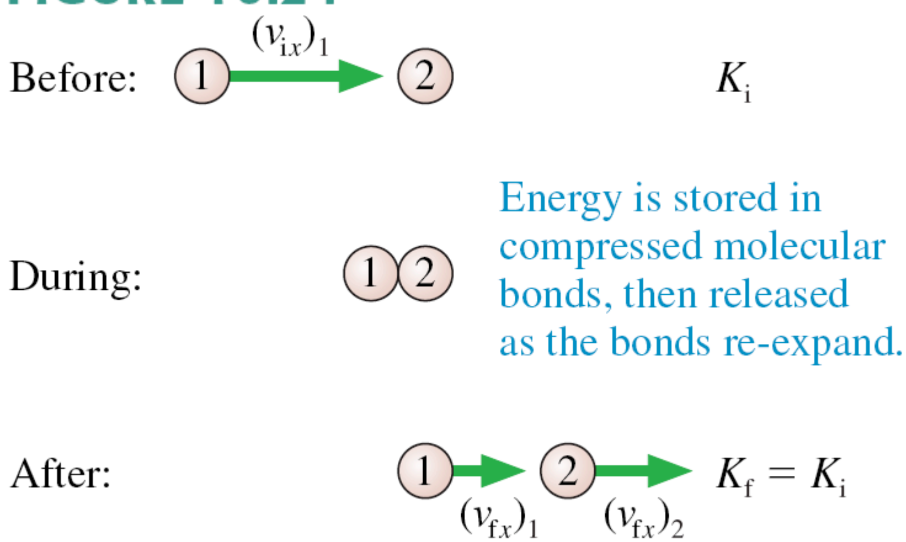


A perfectly elastic collision conserves both momentum and mechanical energy.

Elastic Collisions

Consider a head-on, perfectly elastic collision of a ball of mass m_1 having initial velocity $(v_{ix})_1$, with a ball of mass m_2 that is initially at rest. The balls' velocities after the collision are $(v_{fx})_1$ and $(v_{fx})_2$.These are velocities, not speeds, and have signs. Ball 1, in particular, might bounce backward and have a negative value for $(v_{fx})_1$.

FIGURE 10.24



Elastic Collisions

Consider a head-on, perfectly elastic collision of a ball of mass m_1 having initial velocity $(v_{ix})_1$, with a ball of mass m_2 that is initially at rest.

momentum conservation: $m_1(v_{fx})_1 + m_2(v_{fx})_2 = m_1(v_{ix})_1$

energy conservation: $\frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2 = \frac{1}{2}m_1(v_{ix})_1^2$

The solution, worked out in the text, is

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

Tactics: Analyzing elastic collisions

TACTICS BOX 10.1 **Analyzing elastic collisions**

1. Use the Galilean transformation to transform the initial velocities of balls 1 and 2 from the "lab frame" S to a reference frame S′ in which ball 2 is at rest.
2. Use Equations 10.43 to determine the outcome of the collision in frame S′.
3. Transform the final velocities back to the "lab frame" S.

Applications

Perfectly elastic collisions

Both mechanical energy and momentum are conserved.

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

If ball 2 is moving, transform to a reference frame in which ball 2 is at rest.

A *perfectly elastic collision* is a collision

A. between two springs.
B. that conserves thermal energy.
C. that conserves kinetic energy.
D. that conserves potential energy.
E. that conserves mechanical energy.

A *perfectly elastic collision* is a collision

A. between two springs.

B. that conserves thermal energy.

C. that conserves kinetic energy.

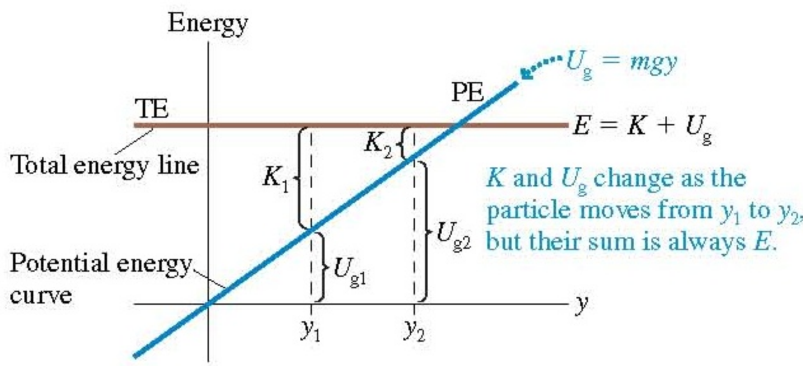
D. that conserves potential energy.

✔ **E. that conserves mechanical energy.**

Energy Diagrams

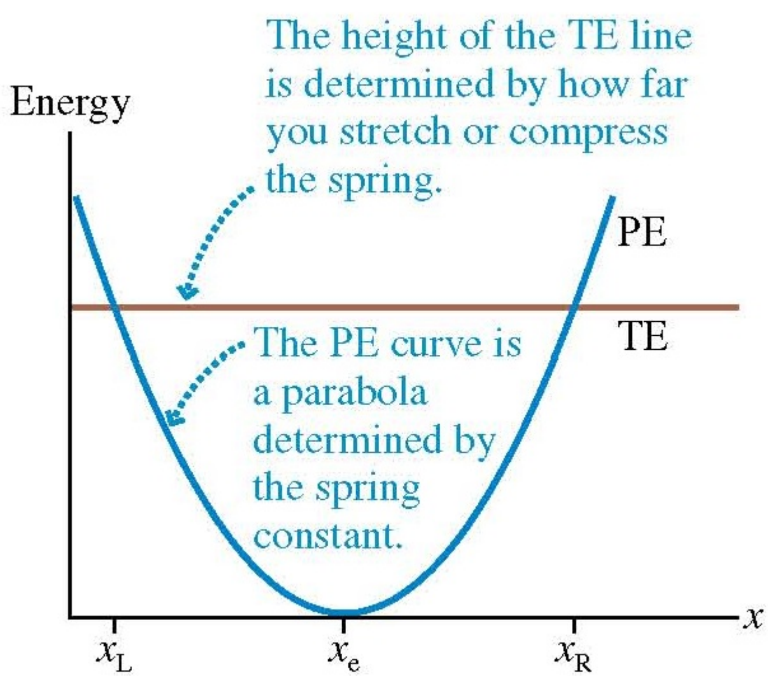
A graph showing a system's potential energy and total energy as a function of position is called an **energy diagram**.

FIGURE 10.30 The energy diagram of a particle in free fall.



Energy Diagrams

FIGURE 10.32 The energy diagram of a mass on a horizontal spring.



Tactics: Interpreting an energy diagram

TACTICS BOX 10.2 **Interpreting an energy diagram**

MP

1. The distance from the axis to the PE curve is the particle's potential energy. The distance from the PE curve to the TE line is its kinetic energy. These are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ doesn't change.
2. A point where the TE line crosses the PE curve is a turning point. The particle reverses direction.
3. The particle cannot be at a point where the PE curve is above the TE line.

Tactics: Interpreting an energy diagram

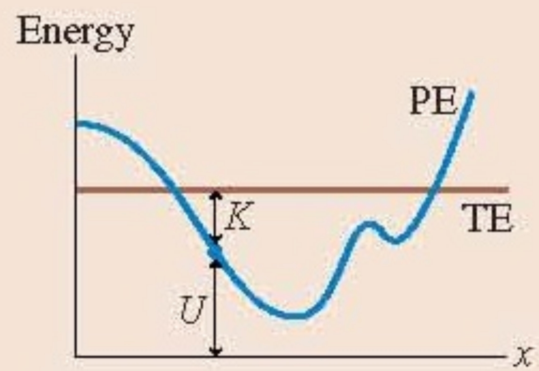
4. The PE curve is determined by the properties of the system—mass, spring constant, and the like. You cannot change the PE curve. However, you can raise or lower the TE line simply by changing the initial conditions to give the particle more or less total energy.
5. A minimum in the PE curve is a point of stable equilibrium. A maximum in the PE curve is a point of unstable equilibrium.

Exercises 18–20

Important Concepts

Energy diagrams

These diagrams show the potential-energy curve PE and the total mechanical energy line TE.



- The distance from the axis to the curve is PE.
- The distance from the curve to the TE line is KE.
- A point where the TE line crosses the PE curve is a **turning point.**
- Minima in the PE curve are points of **stable equilibrium.** Maxima are points of **unstable equilibrium.**

Chapter 10. Summary Slides