Review of Quiz4 Difficult Questions

Two 10.0 cm diameter charged rings face each other, 20.0 cm apart. Both rings are charged to +20.0 nC. What is the magnitude of the electric field strength at the centre of the left ring?

Hint: In general, the electric field due to a charged ring at a point on the axis is: $E = kzQ/(z^2 + R^2)^{3/2}$

A sphere of radius R has a charge Q. The electric field strength at a distance $r > R$ is E_i.

What is the ratio E_f/E_i of the final to initial electric field strength if R is halved?

A flat disk 1.0 m in diameter is oriented so that the plane of the disk makes an angle of $\pi/6$ radians with a uniform electric field.

If the field strength is 491.0 N/C, find the electric flux through the surface.

What is the electric field strength if the flux through a 2.0 m by 1.0 m rectangular surface is 836.0 Nm^2/C, if the electric field is uniform, and if the plane of the surface is at an angle of $\pi/3$ radians with respect to the direction of the field?

Electric Potential and Electric Field

Physical Example

Solar cells - like batteries "generate" electricity

- What does this mean?
- What does a battery really do?

To answer these questions we need to understand the connection between electric potential and electric field

This is what we will learn over the next two lectures

Chapter 30. Potential and Field

Topics:

- Connecting Potential and Field
- Sources of Electric Potential
- Finding the Electric Field from the Potential
- A Conductor in Electrostatic Equilibrium
- Capacitance and Capacitors
- The Energy Stored in a Capacitor
- Dielectrics

Learning Objectives

- To establish the relationship between $\vec{E}$ and V
- To learn more about the properties of a conductor in electrostatic equilibrium
- To introduce batteries as a practical source of potential difference
  - Batteries are NOT a source of constant current
- To find the connection between charge and potential difference for a capacitor
- To analyze simple capacitor circuits

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Next Class

Reading Quizzes

What quantity is represented by the symbol ε ?

A. Electronic potential
B. Excitation potential
C. EMF
D. Electric stopping power
E. Exosphericity

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A. Electronic potential
B. Excitation potential
✔ **C. EMF**
D. Electric stopping power
E. Exosphericity

What is the SI unit of capacitance?

A. Capaciton
B. Faraday
C. Hertz
D. Henry
E. Exciton

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✔ **B. Faraday**
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The electric field

A. is always perpendicular to an equipotential surface.
B. is always tangent to an equipotential surface.
C. always bisects an equipotential surface.
D. makes an angle to an equipotential surface that depends on the amount of charge.

The electric field

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This chapter investigated

A. parallel capacitors
B. perpendicular capacitors
C. series capacitors.
D. Both a and b.
E. Both a and c.

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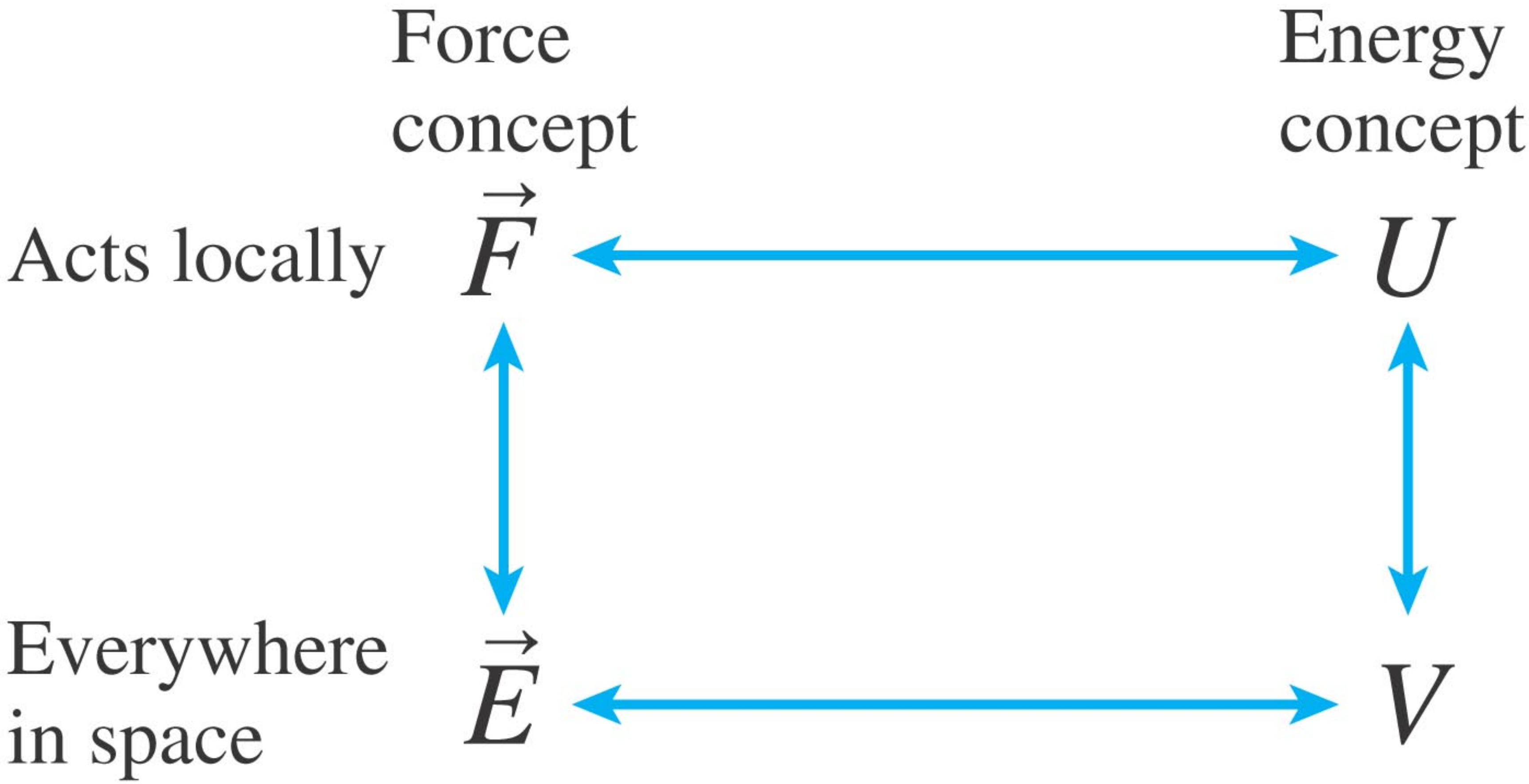

Force concept
Energy concept
Acts locally
$\vec{F}$
U
Everywhere in space
$\vec{E}$
V

Relating Electric Field & Potential

- Last class we defined the electric potential as

$$V \equiv \frac{U_{q+sources}}{q}$$

- Potential energy is defined in terms of work done by force F on charge q as it moves from position i to f $\Delta U = -W(i \rightarrow f) = -\int_{s_i}^{s_f} \vec{F}_s \cdot d\vec{s} = -\int_i^f \vec{F} \cdot d\vec{s}$

- But $\vec{F} = q\vec{E}$ so

$$\boxed{\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}}$$

Finding the Potential from the Electric Field

The potential difference between two points in space is

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s\, ds = -\int_{i}^{f} \vec{E} \cdot d\vec{s}$$

where *s* is the position along a line from point i to point f. That is, we can find the potential difference between two points if we know the electric field.

We can think of an integral as an area under a curve. Thus a graphical interpretation of the equation above is

$$V_f = V_i - (\text{area under the } E_s\text{-versus-}s \text{ curve between } s_i \text{ and } s_f)$$

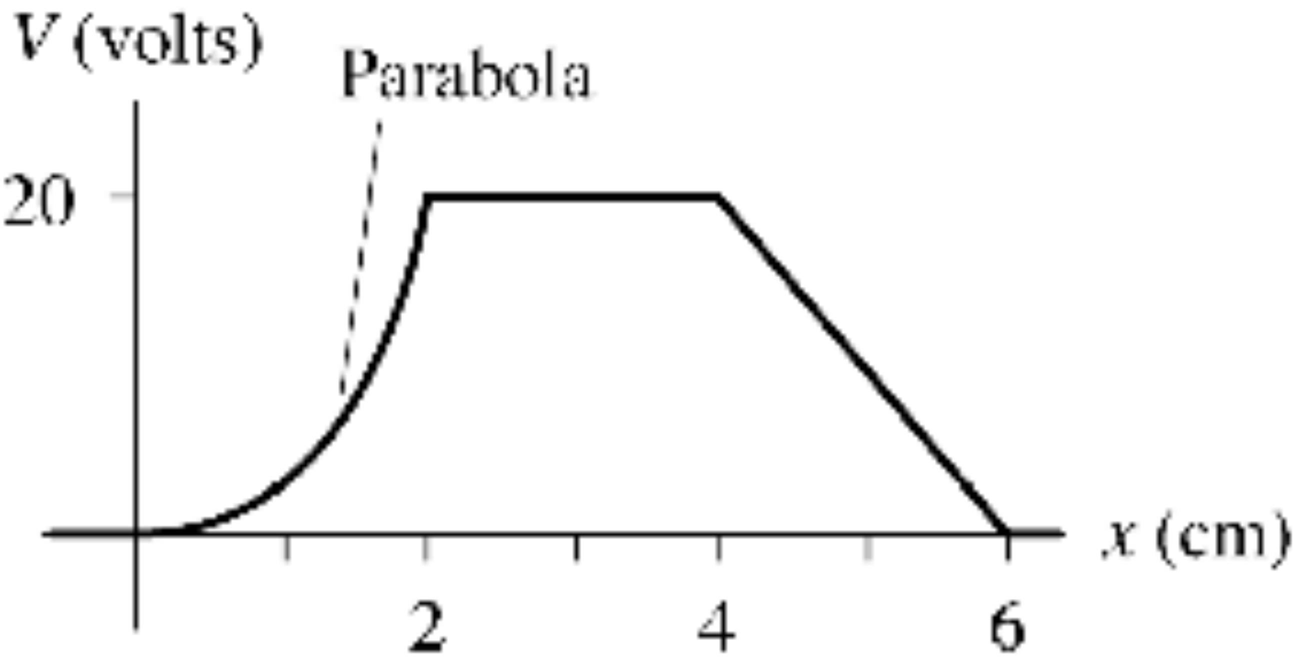
Finding the Electric Field from the Potential

In terms of the potential, the component of the electric field in the *s*-direction is

$$E_s = -\frac{dV}{ds}$$

Now we have reversed Equation 30.3 and have a way to find the electric field from the potential.

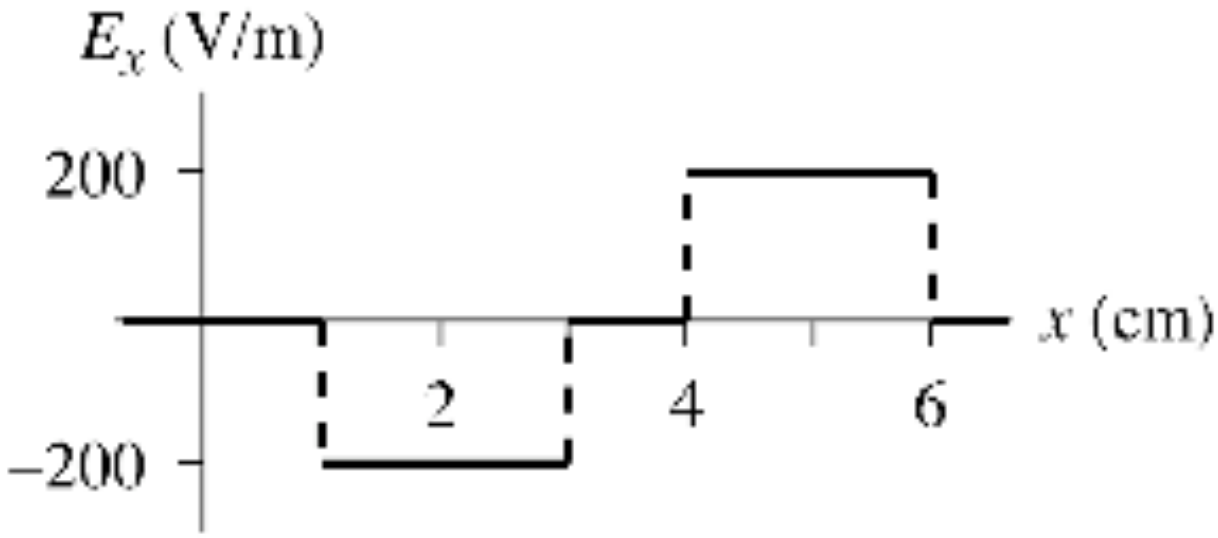
What is the Electric Field?



Graph E_x

$$E_s = -\frac{dV}{ds}$$

What is the Electric Potential?



Graph V (use $V = 0$ at $x = 0$)

$$\boxed{\Delta V = -\int_i^f \vec{E} \cdot d\vec{s}}$$

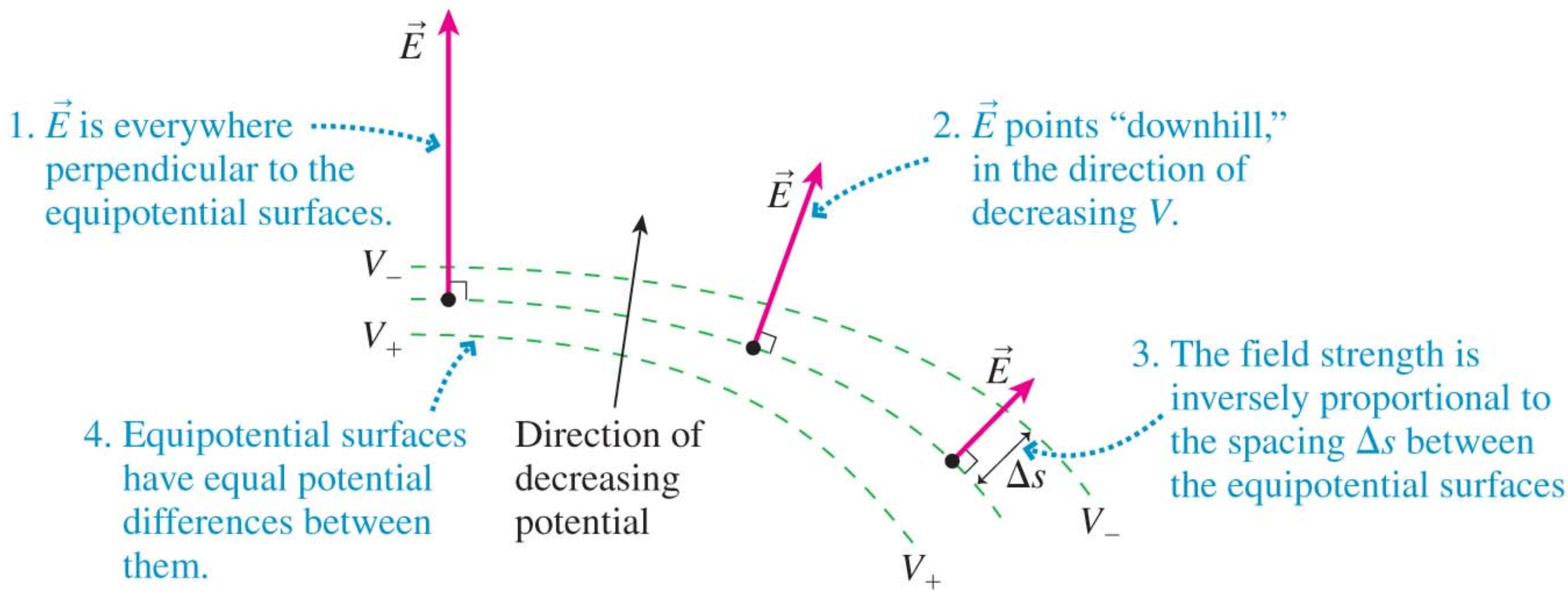
Use V=0 at x=0

How would you make such an electric field?

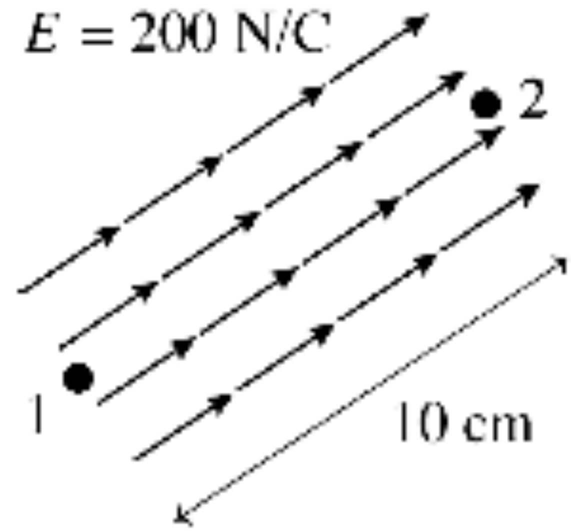
Consider a point charge with q = 5/9 C

- Determine the values of r at which the potential is 500, 400, 300, 200, and 100 volts
- Graph V vs x along an x-axis passing through the charge
- Determine E_x
- Draw contour map of the potential
- Draw electric field vectors on contour map
- Estimate electric field strength

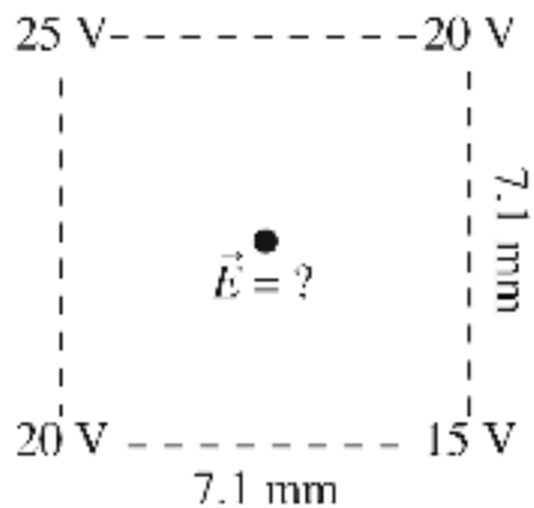
FIGURE 30.14 The geometry of the potential and the field.

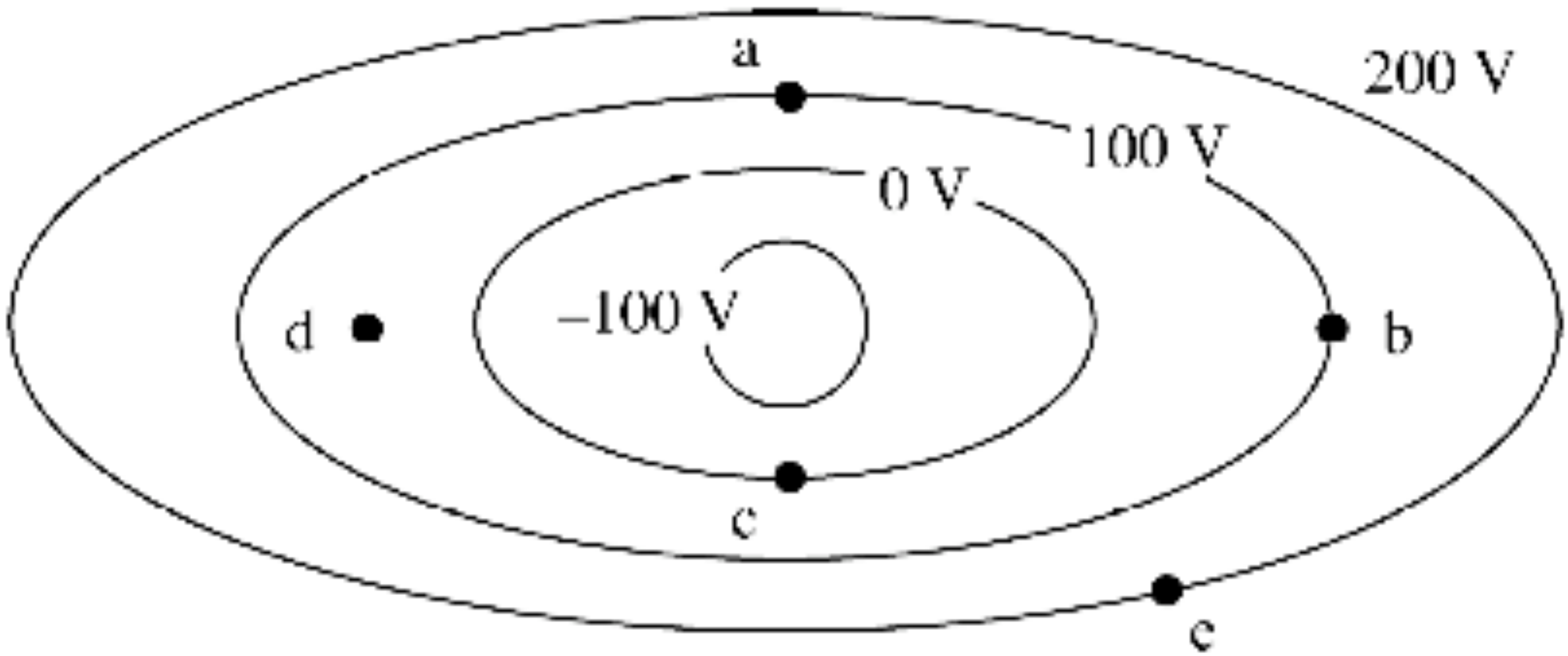


- Is point 1 or point 2 at a higher potential?
- What is ΔV_{12}?



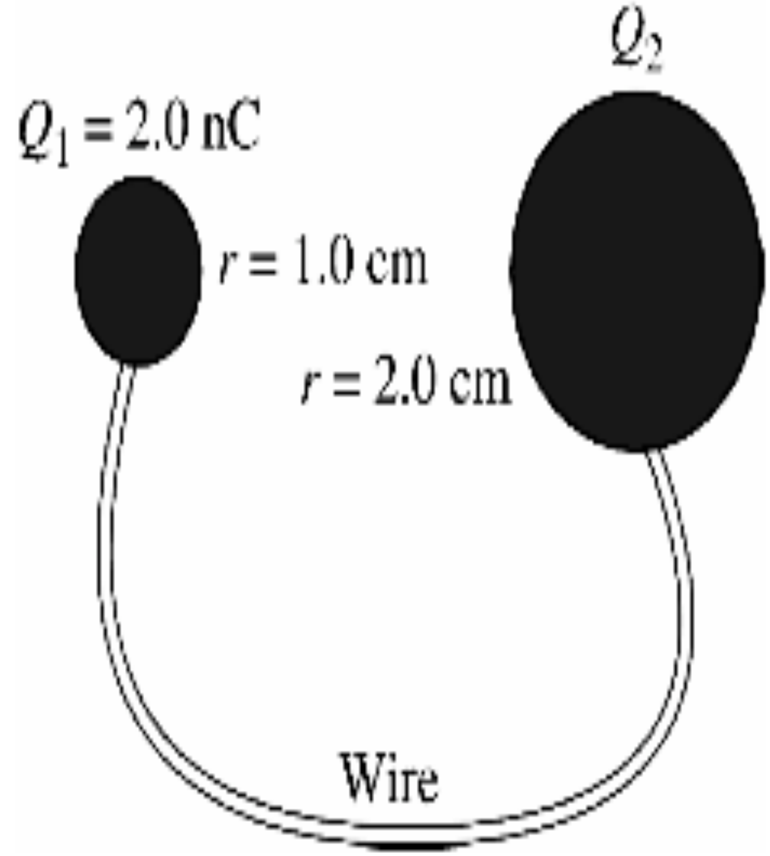
- What is the electric field in the center?





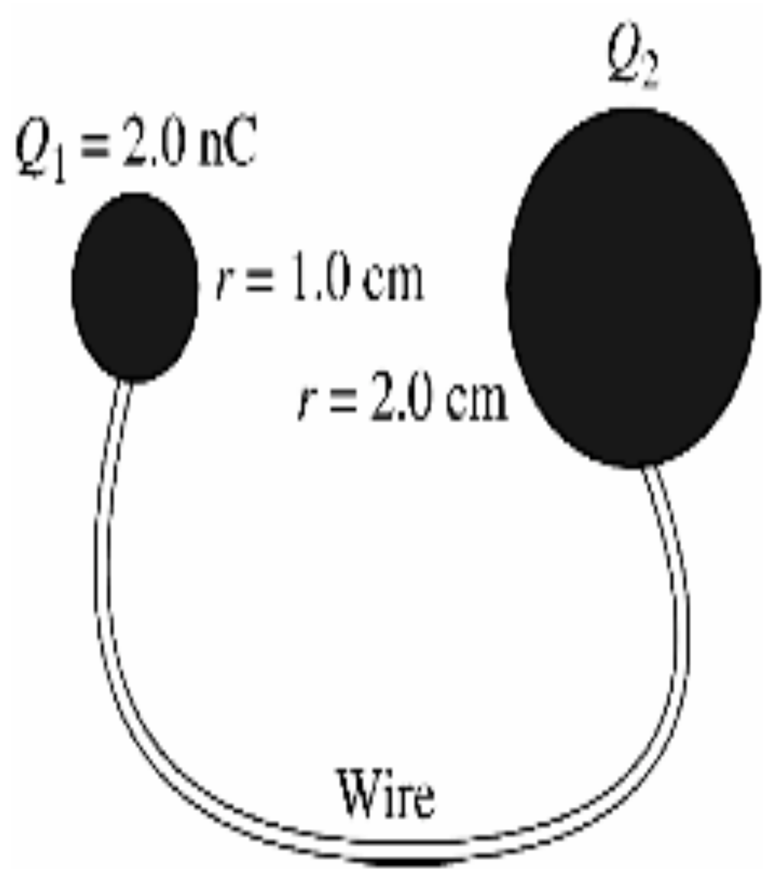
- Compare the field strengths at a and b
- Compare the field strengths at c and d
- Draw the electric field vector at points a through e

What is Q_2?



What is Q_2?

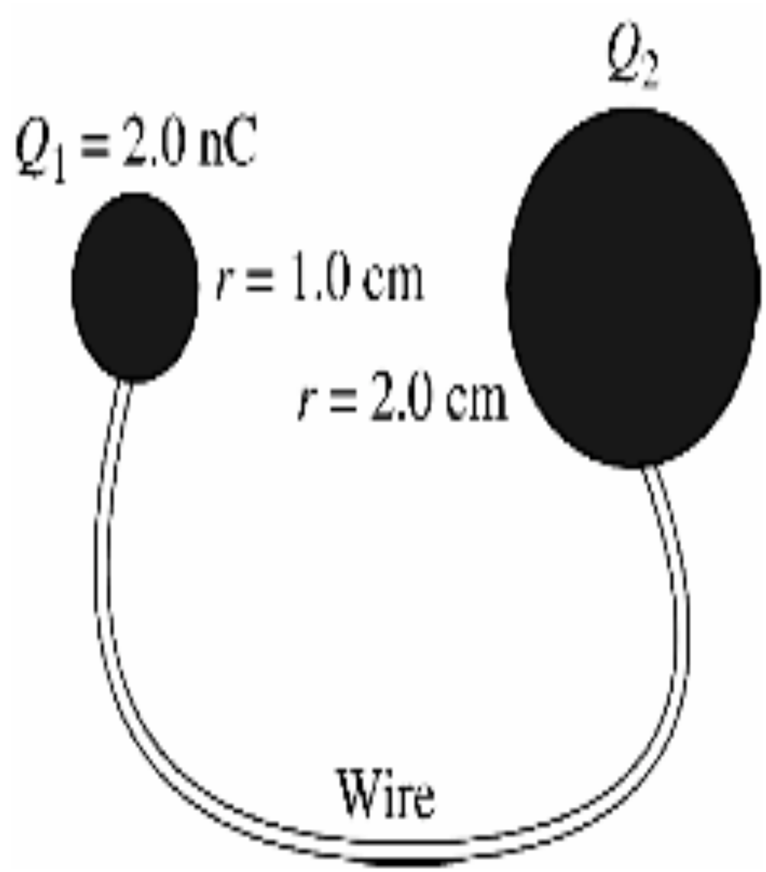
Is this a single conductor or
two independent conductors?



What is Q_2?

Is this a single conductor or two independent conductors?

Is this an equilibrium situation?

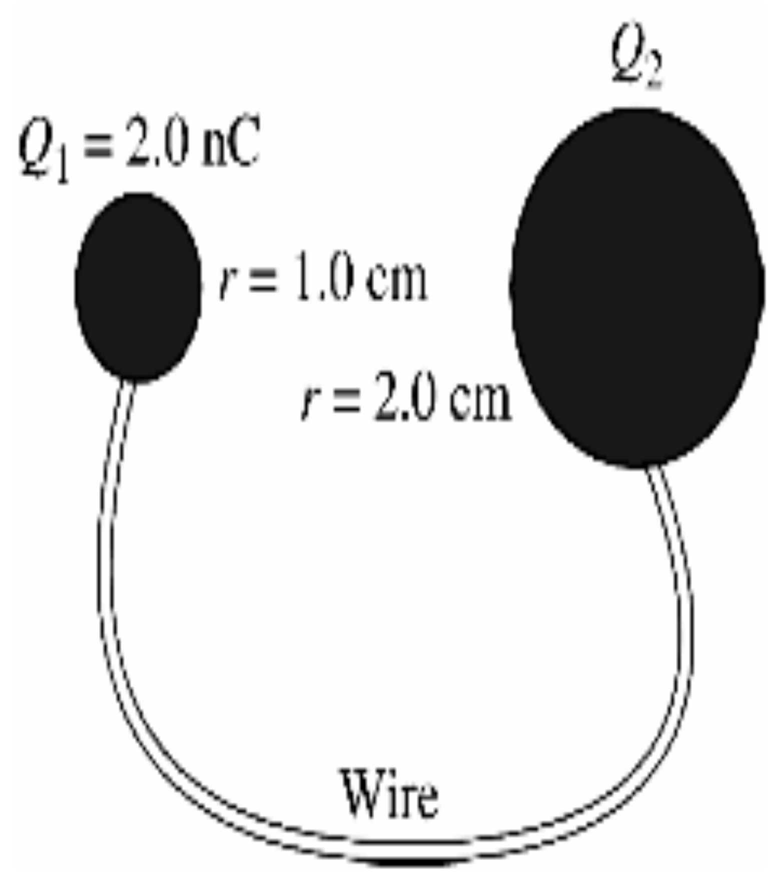


What is Q_2?

Is this a single conductor or two independent conductors?

Is this an equilibrium situation?

What do you know about conductors in electrostatic equilibrium?



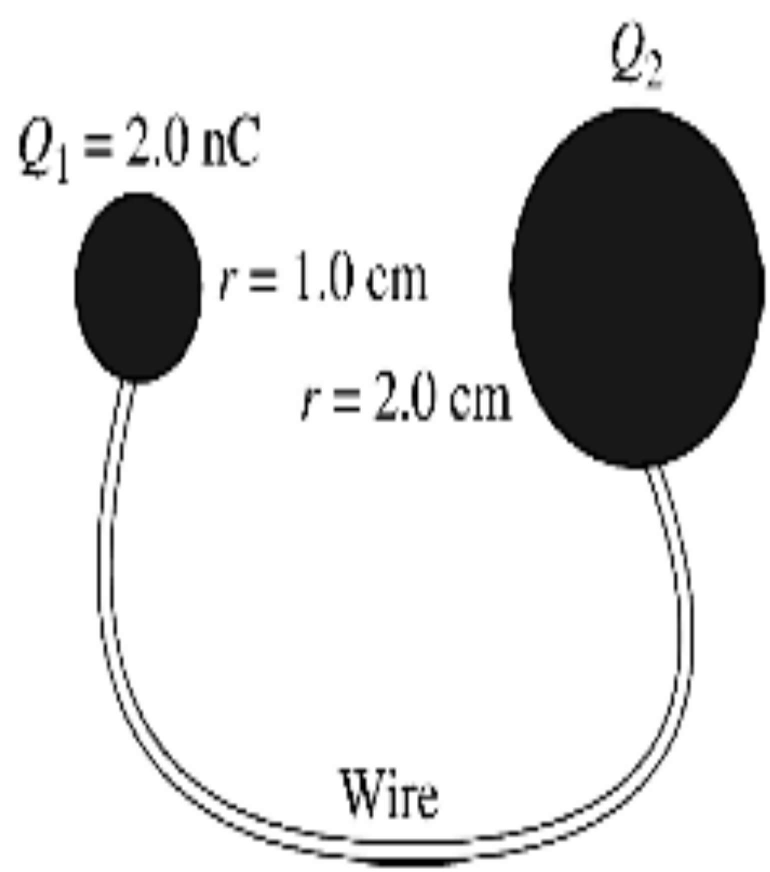
What is Q_2?

Is this a single conductor or two independent conductors?

Is this an equilibrium situation?

What do you know about conductors in electrostatic equilibrium?

What do you know about the potential at the surface of a sphere?



EXAMPLE 30.2 The potential of a parallel-plate capacitor

EXAMPLE 30.2 The potential of a parallel-plate capacitor

In Chapter 27, the electric field inside a capacitor was found to be

$$\vec{E} = \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative} \right)$$

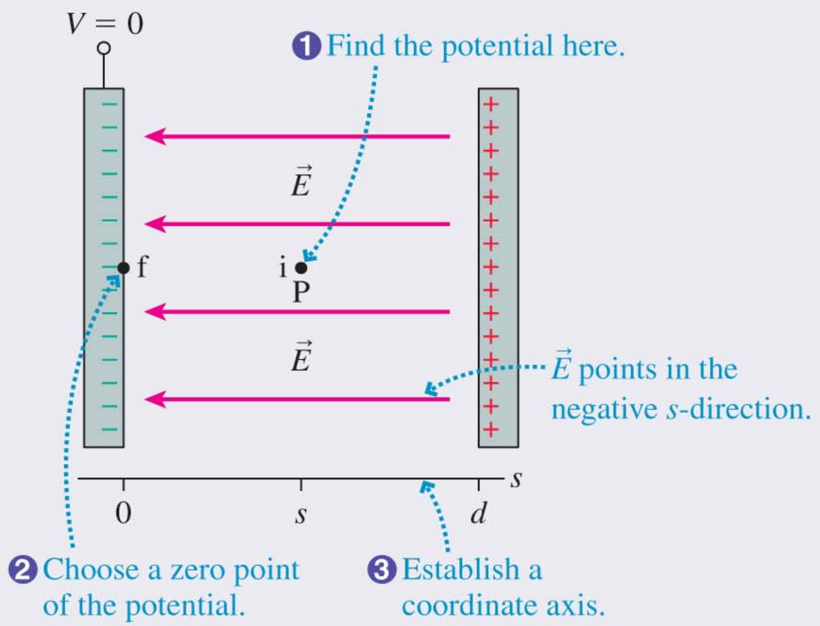
Find the electric potential inside the capacitor. Let $V = 0$ at the negative plate.

MODEL The electric field inside a capacitor is a uniform field.

EXAMPLE 30.2 The potential of a parallel-plate capacitor

VISUALIZE **FIGURE 30.5** shows the capacitor and establishes a point P where we want to find the potential. We've chosen an *s*-axis measured from the negative plate, which is the zero point of the potential.

FIGURE 30.5 Finding the potential inside a capacitor.



Continued

EXAMPLE 30.2 The potential of a parallel-plate capacitor

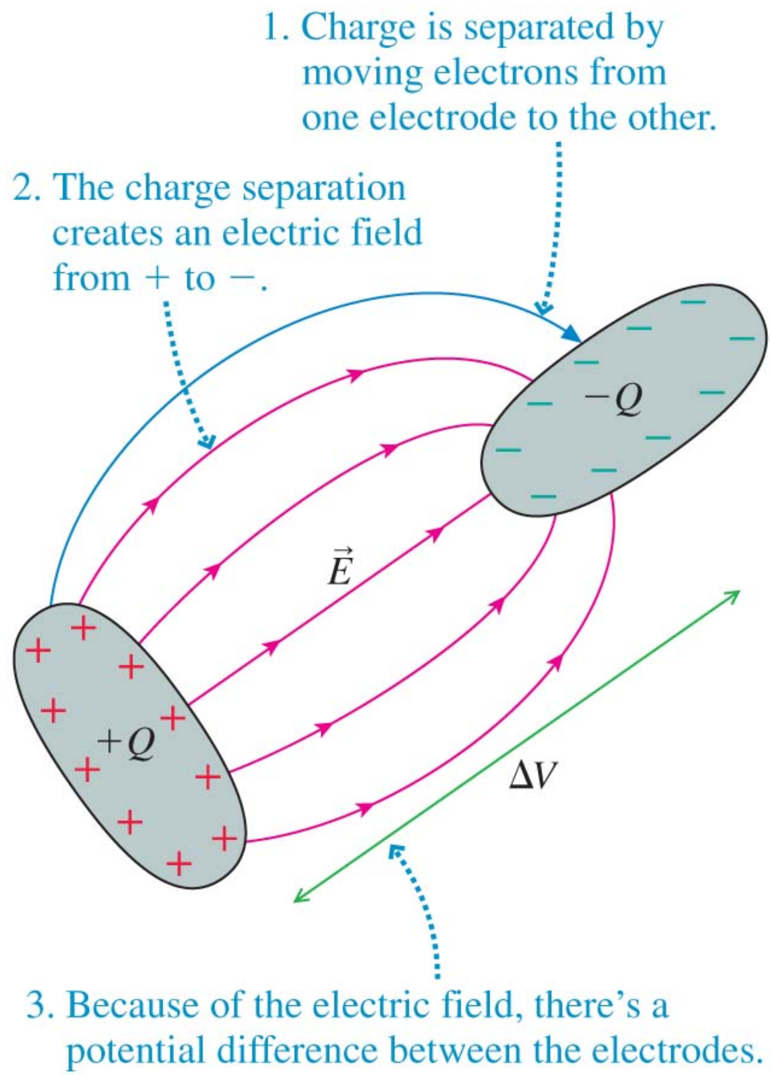
SOLVE We'll integrate along the s-axis from $s_i = s$ to $s_f = 0$ (where $V_f = 0$ V). Notice that $\vec{E}$ points in the negative s-direction, so $E_s = -Q/\epsilon_0 A$. $Q/\epsilon_0 A$ is a constant, so

$$V_i = V(s) = 0 + \int_s^0 E_s\,ds = \left(-\frac{Q}{\epsilon_0 A}\right)\int_s^0 ds = \frac{Q}{\epsilon_0 A}s = Es$$

EXAMPLE 30.2 The potential of a parallel-plate capacitor

ASSESS $V = Es$ is the capacitor potential we deduced in Chapter 29 by working directly with the potential energy. The potential increases linearly from $V = 0$ at the negative plate to $V = Ed$ at the positive plate. Here we found the potential by explicitly recognizing the connection between the potential and the field.

FIGURE 30.6 A charge separation creates a potential difference.



(a)

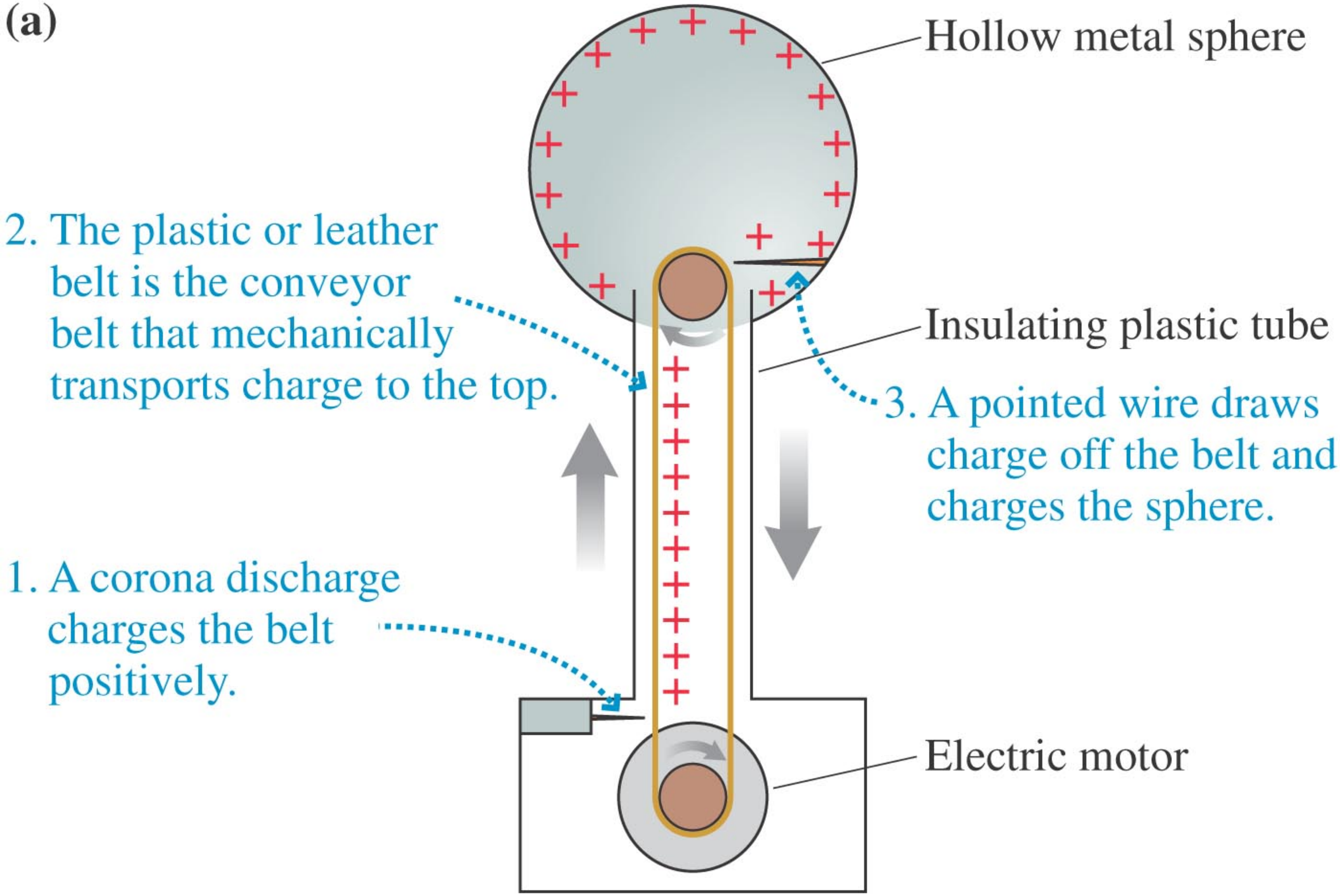
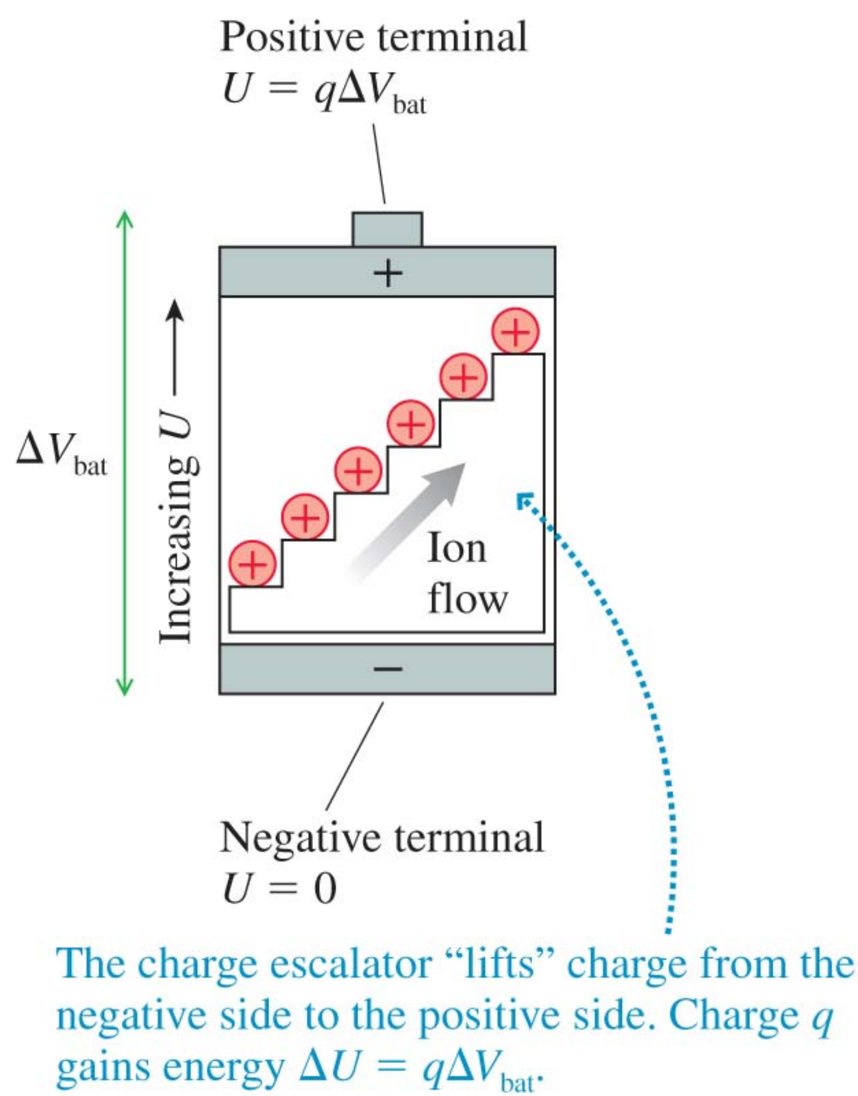

Hollow metal sphere
2. The plastic or leather belt is the conveyor belt that mechanically transports charge to the top.
Insulating plastic tube
3. A pointed wire draws charge off the belt and charges the sphere.
1. A corona discharge charges the belt positively.
Electric motor

FIGURE 30.8 The charge escalator model of a battery.



Batteries and emf

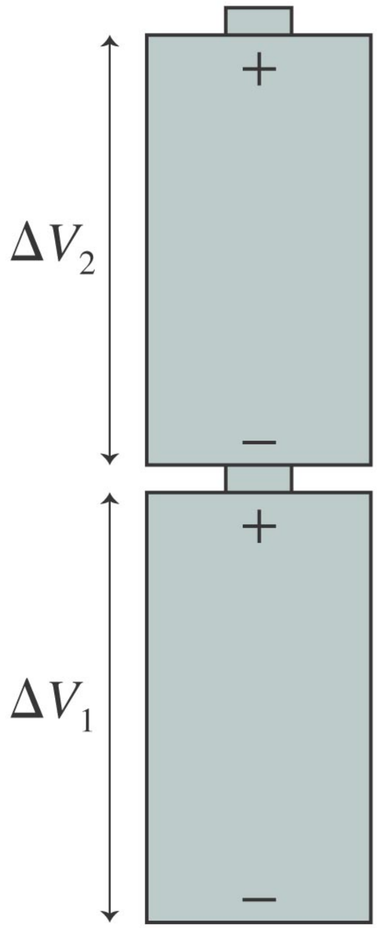
The potential difference between the terminals of an ideal battery is

$$\Delta V_{\text{bat}} = \frac{W_{\text{chem}}}{q} = \mathcal{E} \qquad \text{(ideal battery)}$$

In other words, a battery constructed to have an emf of 1.5V creates a 1.5 V potential difference between its positive and negative terminals.
The total potential difference of batteries in series is simply the sum of their individual terminal voltages:

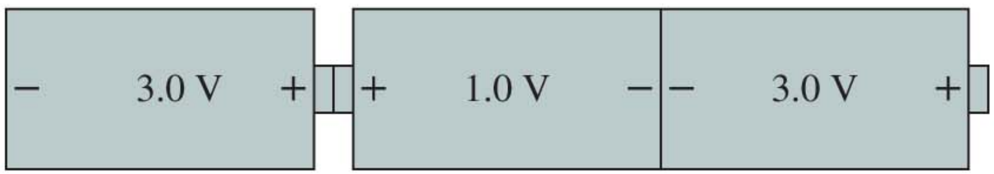
$$\Delta V_{\text{series}} = \Delta V_1 + \Delta V_2 + \cdots \qquad \text{(batteries in series)}$$

Batteries in Series



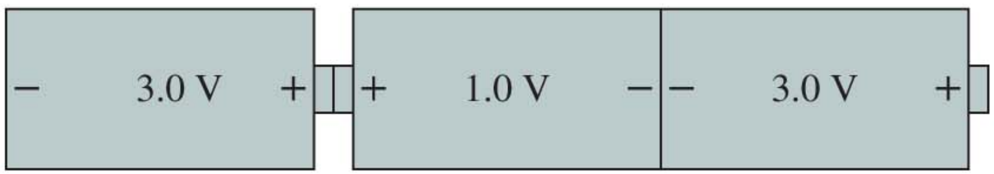
$$\Delta V_{series} = \Delta V_1 + \Delta V_2$$

What total potential difference is created by these three batteries?



A. 1.0 V
B. 2.0 V
C. 5.0 V
D. 6.0 V
E. 7.0 V

What total potential difference is created by these three batteries?



A. 1.0 V
B. 2.0 V
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D. 6.0 V
E. 7.0 V

Finding the Electric Field from the Potential

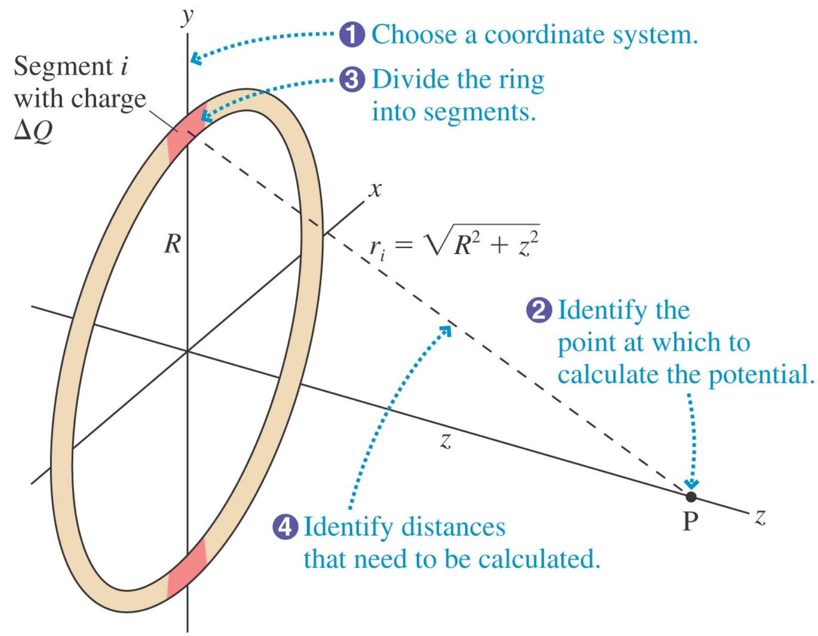
In terms of the potential, the component of the electric field in the *s*-direction is

$$E_s = -\frac{dV}{ds}$$

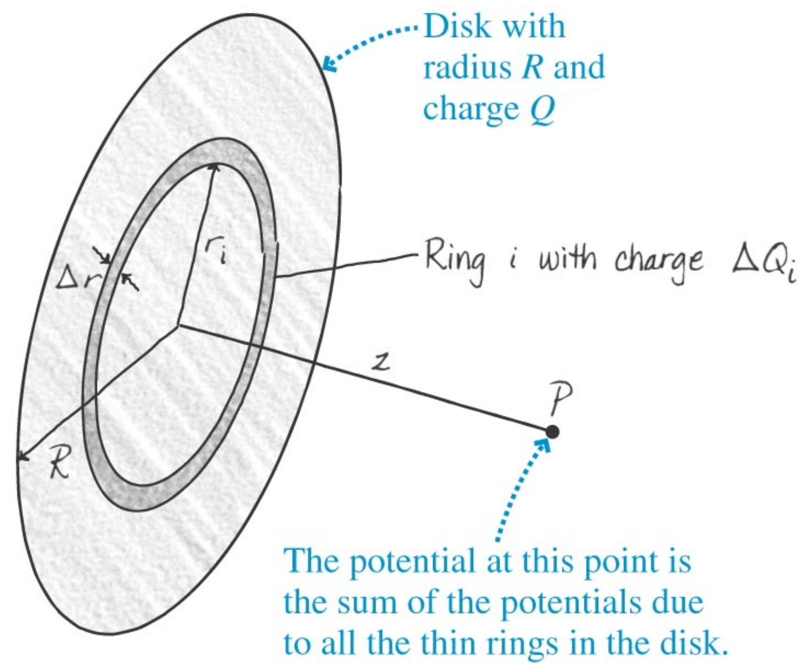
Now we have reversed Equation 30.3 and have a way to find the electric field from the potential.

Recall from Chapter 29

- Compute the potential of a ring of charge



- Compute the potential of a disk of charge



Electric Field of a Ring of Charge

- In Chapter 29 – we found $$V_{ring} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{z^2+R^2}}$$
- Find on axis electric field of a ring of charge.

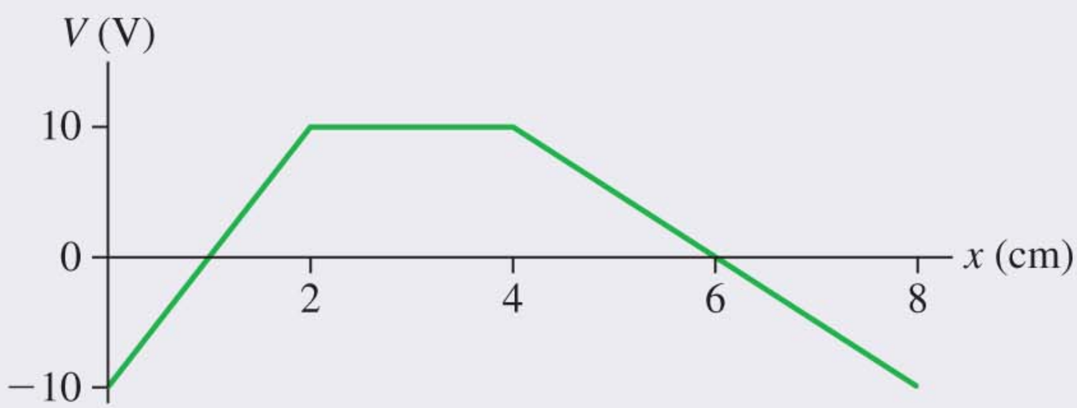
$$E_Z = -\frac{dV}{dz} = -\frac{d}{dz}\left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{z^2+R^2}}\right) = \frac{1}{4\pi\varepsilon_0}\frac{zQ}{\left(z^2+R^2\right)^{3/2}}$$

- This is the same results as chap 27 but the calculation was MUCH easier – no angles

EXAMPLE 30.4 Finding E from the slope of V

FIGURE 30.11 is a graph of the electric potential in a region of space where $\vec{E}$ is parallel to the x-axis. (a) A proton is released from rest at $x = 6$ cm. Will it move? If so, which way? (b) Draw a graph of E_x versus x.

FIGURE 30.11 Graph of V versus position x.



MODEL The proton will accelerate if there's an electric field at $x = 6$ cm. The electric field is the *negative* of the slope of the potential graph.

EXAMPLE 30.4 Finding *E* from the slope of *V*

SOLVE a. The proton will move if a force acts on it, there will be a force if there's an electric field, and there will be an electric field if the potential changes with position—which it does. The potential graph has a negative slope at $x = 6$ cm (the fact that $V = 0$ is not relevant). The electric field component E_x is the *negative* of the slope, so $E_x > 0$. Thus $\vec{E}$ and $\vec{F} = e\vec{E}$ point in the positive x-direction, causing the proton to move to the right. Alternatively, you learned in Chapter 29 that a positive charged particle moves in the direction of decreasing potential—to the right at $x = 6$ cm—as it converts electric potential energy to kinetic energy.

EXAMPLE 30.4 Finding *E* from the slope of *V*

b. There are three regions of different slope:

$$0 < x < 2\text{ cm} \quad \begin{cases} \Delta V/\Delta x = (20\text{ V})/(0.020\text{ m}) = 1000\text{ V/m} \\ E_x = -1000\text{ V/m} \end{cases}$$

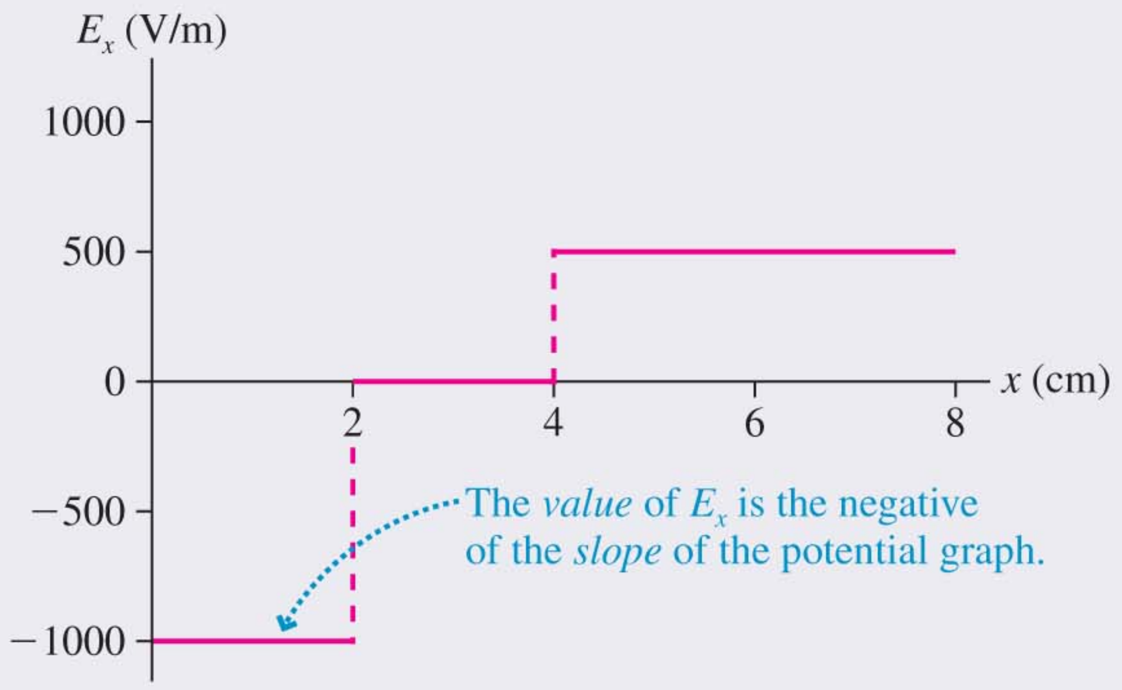
$$2 < x < 4\text{ cm} \quad \begin{cases} \Delta V/\Delta x = 0\text{ V/m} \\ E_x = 0\text{ V/m} \end{cases}$$

$$4 < x < 8\text{ cm} \quad \begin{cases} \Delta V/\Delta x = (-20\text{ V})/(0.040\text{ m}) = -500\text{ V/m} \\ E_x = 500\text{ V/m} \end{cases}$$

The results are shown in **FIGURE 30.12**.

EXAMPLE 30.4 Finding *E* from the slope of *V*

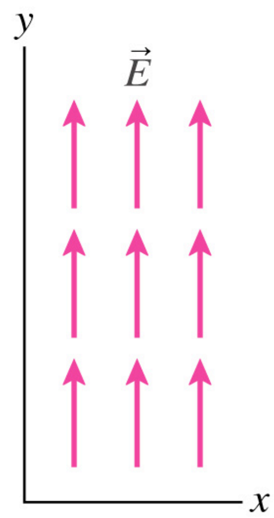
FIGURE 30.12 Graph of E_x versus position x.

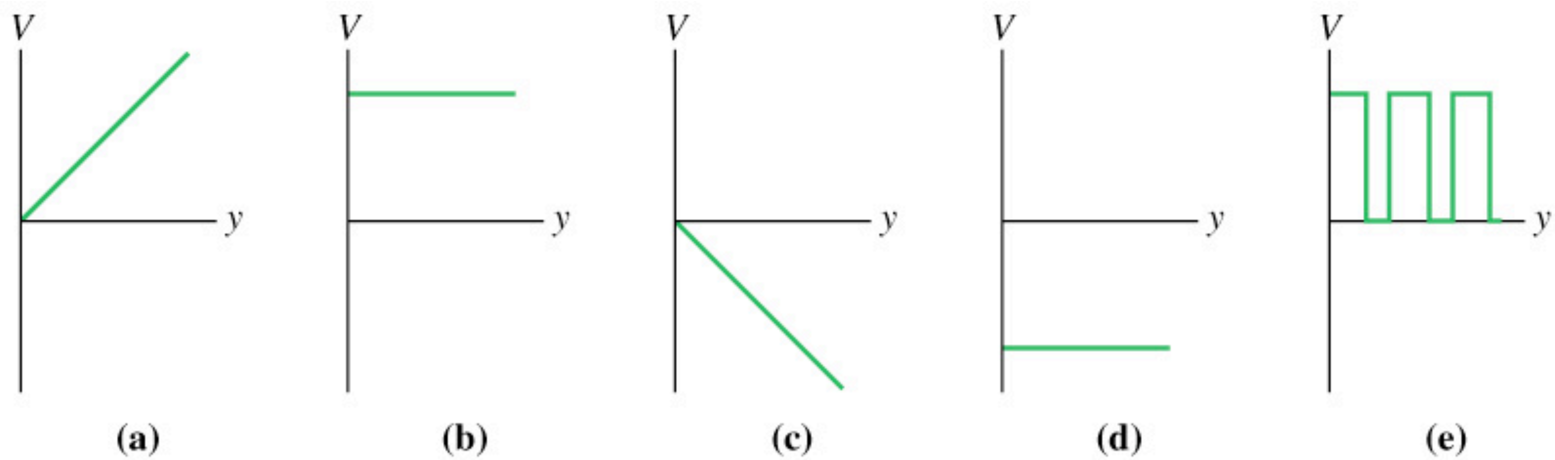


EXAMPLE 30.4 Finding *E* from the slope of *V*

ASSESS The electric field $\vec{E}$ points to the left (E_x is negative) for $0 < x < 2$ cm and to the right (E_x is positive) for $4 < x < 8$ cm. Notice that **the electric field is zero in a region of space where the potential is not changing.**

Which potential-energy graph describes this electric field?





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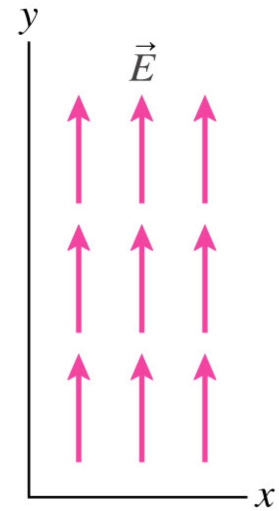
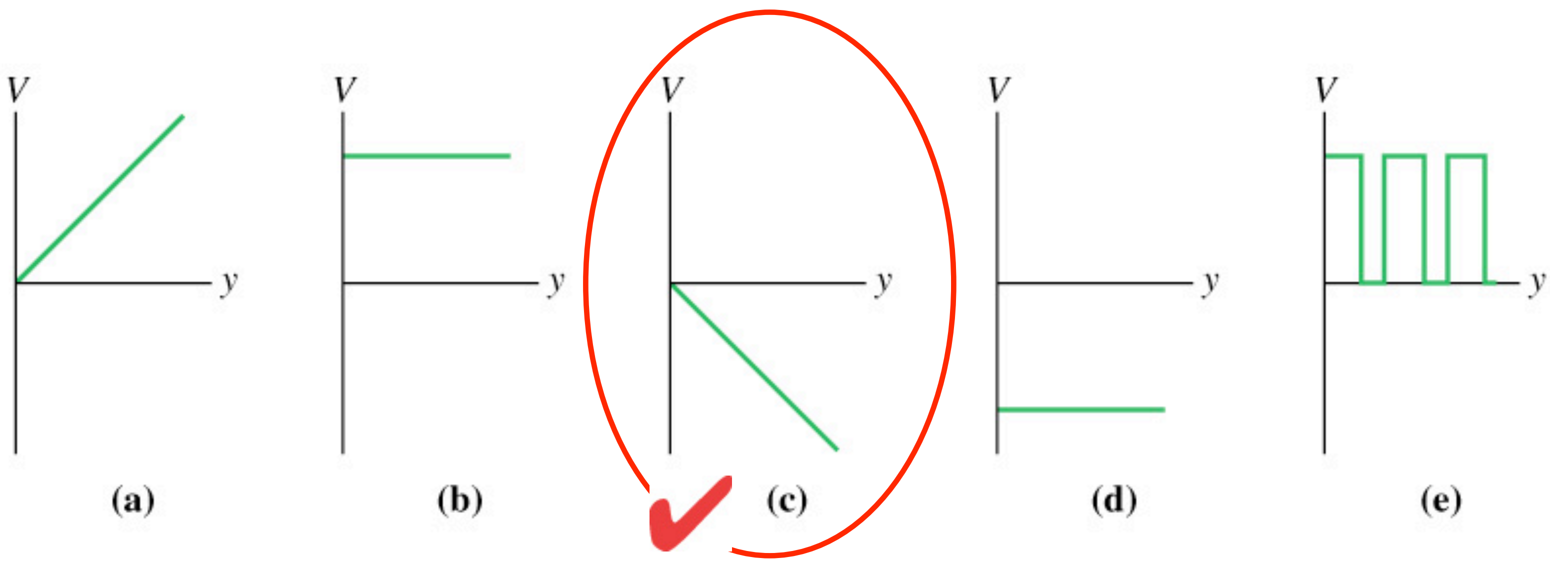
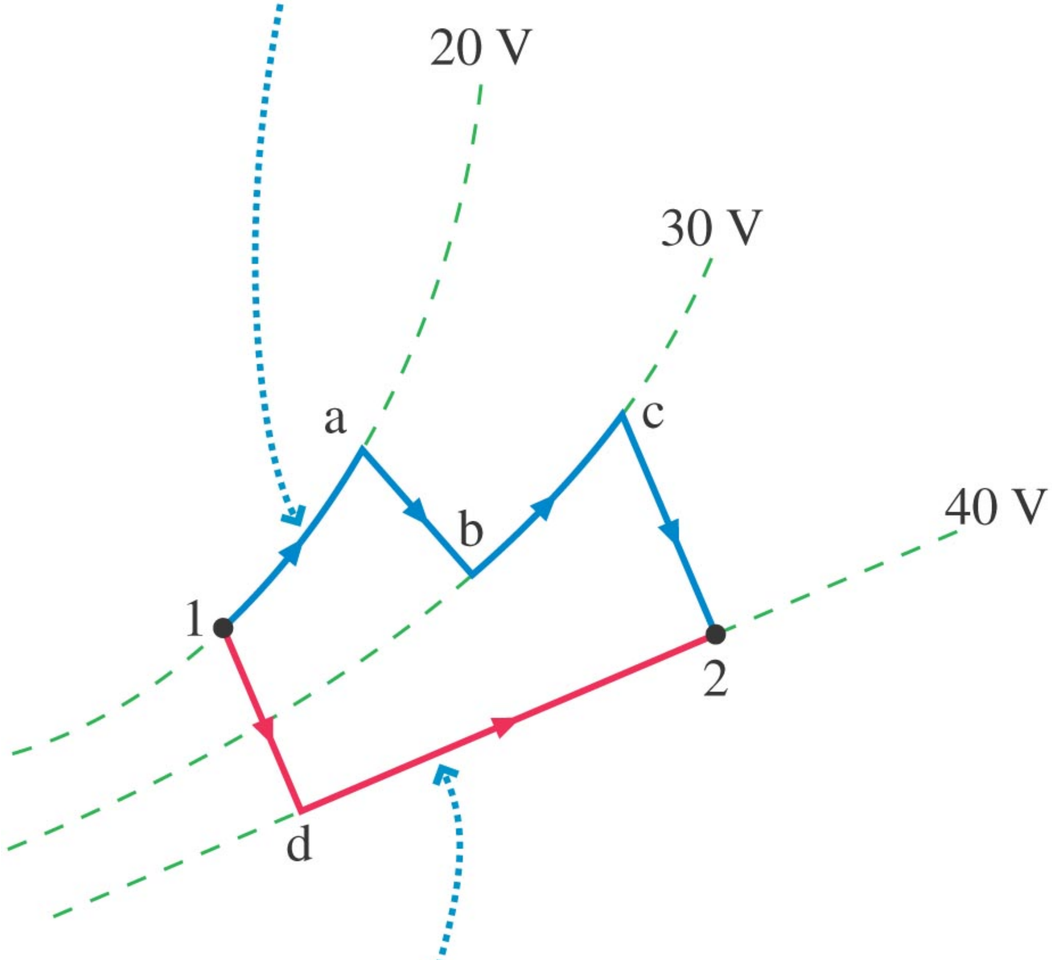




The potential difference along path 1-a-b-c-2 is $\Delta V = 0\text{ V} + 10\text{ V} + 0\text{ V} + 10\text{ V} = 20\text{ V}$.



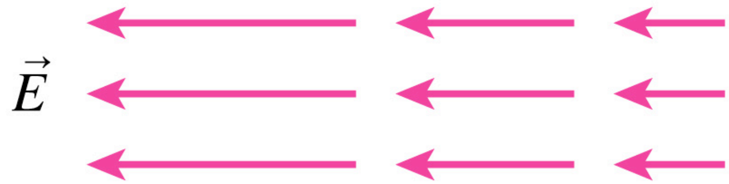
The potential difference along path 1-d-2 is $\Delta V = 20\text{ V} + 0\text{ V} = 20\text{ V}$.

Kirchhoff's Loop Law

For any path that starts and ends at the same point

$$\Delta V_{\text{loop}} = \sum_i (\Delta V)_i = 0$$

Stated in words, **the sum of all the potential differences encountered while moving around a loop or closed path is zero.**
This statement is known as **Kirchhoff's loop law.**



Which set of equipotential surfaces matches this electric field?

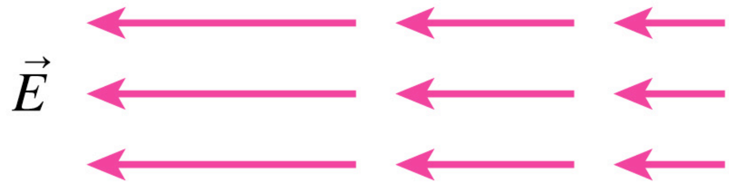
(a)

(b)

(c)

(d)

(e)



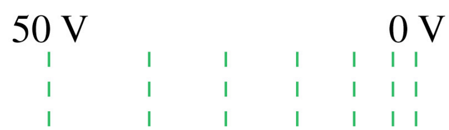
Which set of equipotential surfaces matches this electric field?

(a)

(b)

(c)

(d)



(e)

General Principles

Connecting V and $\vec{E}$

The electric potential and the electric field are two different perspectives of how source charges alter the space around them. V and $\vec{E}$ are related by

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s\, ds$$

where s is measured from point i to point f and E_s is the component of $\vec{E}$ parallel to the line of integration.

Graphically

$\Delta V =$ the negative of the area under the E_s graph

and

$$E_s = -\frac{dV}{ds}$$

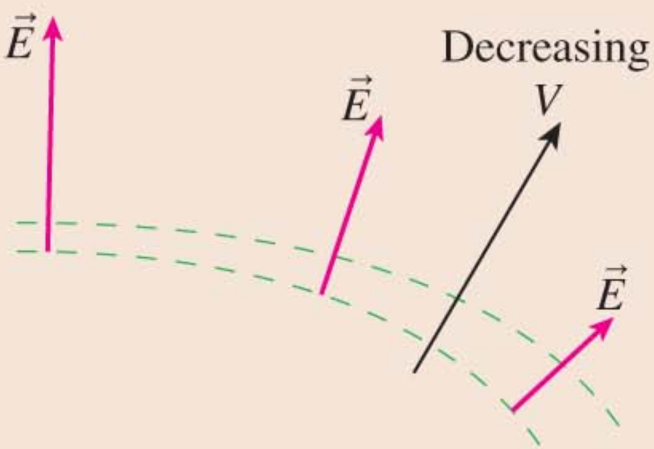
$=$ the negative of the slope of the potential graph

General Principles

The Geometry of Potential and Field

The electric field

- Is perpendicular to the equipotential surfaces.
- Points "downhill" in the direction of decreasing V.
- Is inversely proportional to the spacing Δs between the equipotential surfaces.



General Principles

Conservation of Energy

The sum of all potential differences around a closed path is zero.
$\sum(\Delta V)_i = 0.$

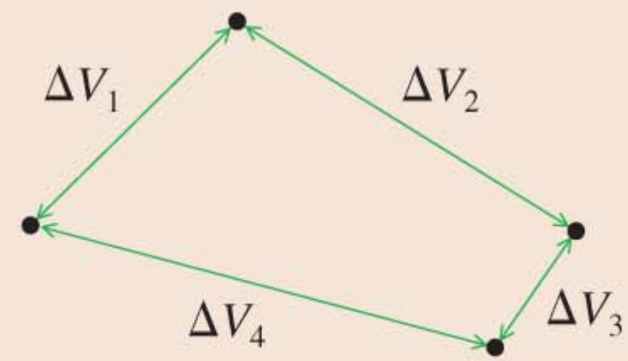
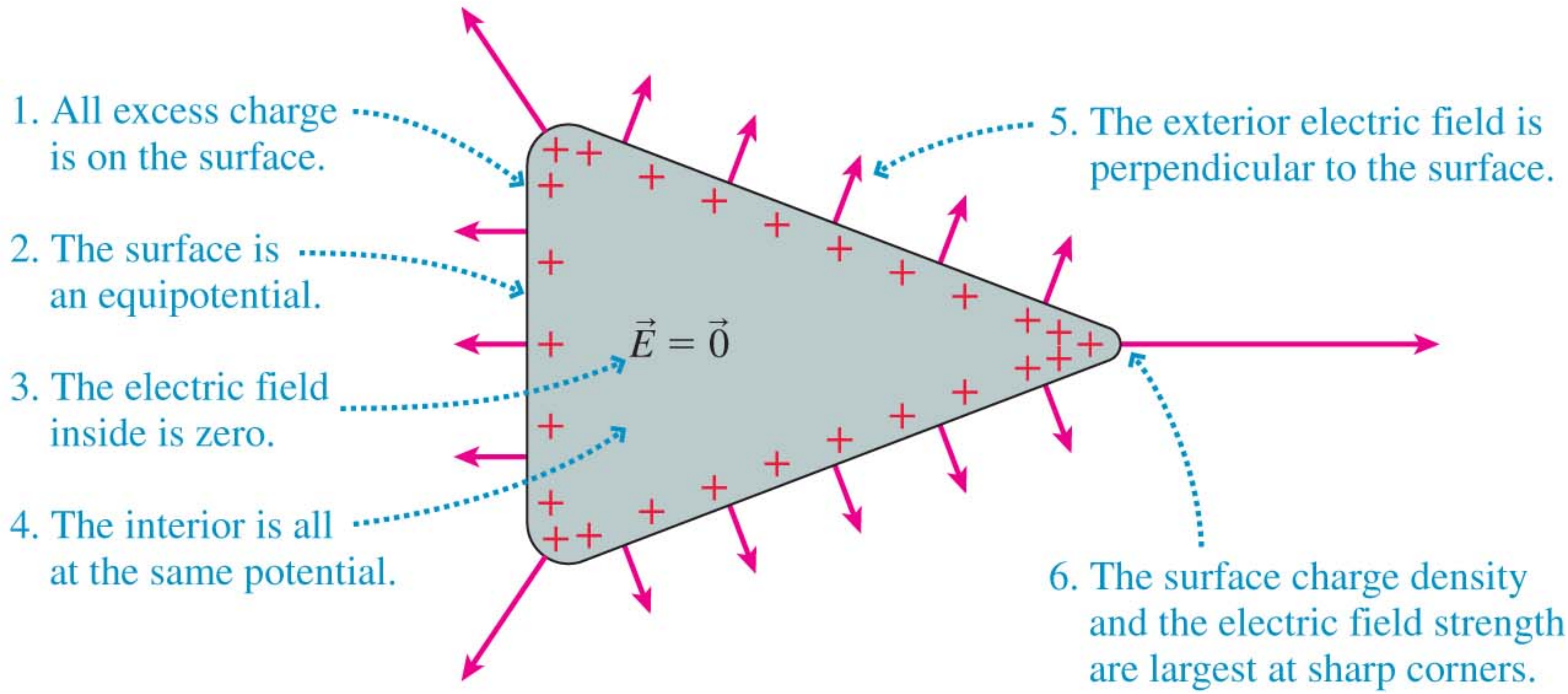
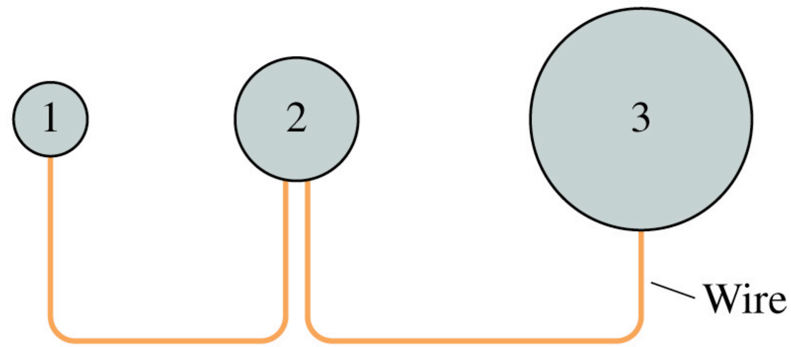


FIGURE 30.19 Electric properties of a conductor in electrostatic equilibrium.

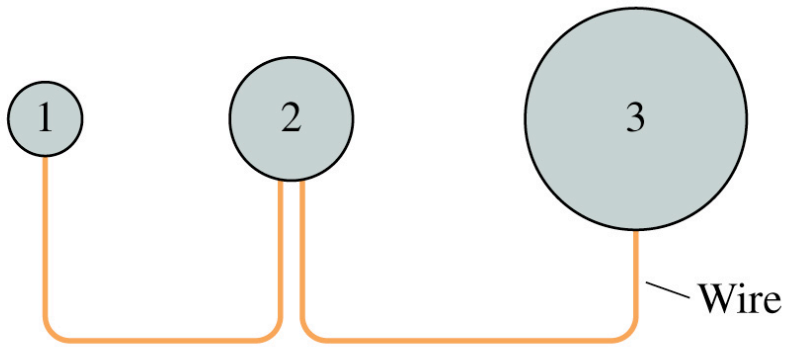


Three charged, metal spheres of different radii are connected by a thin metal wire. The potential and electric field at the surface of each sphere are *V* and *E*. Which of the following is true?



A. $V_1 = V_2 = V_3$ and $E_1 > E_2 > E_3$
B. $V_1 > V_2 > V_3$ and $E_1 = E_2 = E_3$
C. $V_1 = V_2 = V_3$ and $E_1 = E_2 = E_3$
D. $V_1 > V_2 > V_3$ and $E_1 > E_2 > E_3$
E. $V_3 > V_2 > V_1$ and $E_1 = E_2 = E_3$

Three charged, metal spheres of different radii are connected by a thin metal wire. The potential and electric field at the surface of each sphere are *V* and *E*. Which of the following is true?



✔ **A. $V_1 = V_2 = V_3$ and $E_1 > E_2 > E_3$**
B. $V_1 > V_2 > V_3$ and $E_1 = E_2 = E_3$
C. $V_1 = V_2 = V_3$ and $E_1 = E_2 = E_3$
D. $V_1 > V_2 > V_3$ and $E_1 > E_2 > E_3$
E. $V_3 > V_2 > V_1$ and $E_1 = E_2 = E_3$