Simon Fraser University
Spring term 2014

MATH 232 - Applied Linear Algebra
Instructor: Kirill Pichon Gostaf

February 3, 2014

Midterm #1.A - 50 minutes

Print in block letters

Student name **Student number**

INSTRUCTIONS

- **Do not open** this booklet until instructed to do so.
- Write your answers below each problem with a **black or blue pen**.
- Your work must be clearly presented.
- You must justify the answers for full credit (unless otherwise stated). Do not expect marks to be given for a correct answer if the intermediate steps are omitted.
- **Neither notes nor books** are allowed.
- **No programmable** and **graphic calculators** or any other computing devices may be used.
- Only non-programmable, scientific calculators are allowed.
- A one-sided cheat sheet is allowed.

Student signature

Score / 25

Question	1	2	3	4
Score				

1. (10 points) Select the right answer; no explanation is required.

◦ There are infinitely many irrational numbers x such that $\sqrt{7} < x < \sqrt{8}$.

☐ TRUE ☐ FALSE

◦ Consider two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$. If $\mathbf{u} + \mathbf{v} = \mathbf{0}$, then $a\,\mathbf{u} + b\,\mathbf{v} = \mathbf{0}$ for all a and b .

☐ TRUE ☐ FALSE

◦ Consider three vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^n$. If $\mathbf{u}$ is orthogonal to $\mathbf{v} + \mathbf{w}$, then $\mathbf{u}$ is orthogonal to $\mathbf{v}$ and $\mathbf{w}$.

☐ TRUE ☐ FALSE

◦ If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors in $\mathbb{R}^2$, then $||\,\mathbf{u} - 3\,\mathbf{v}|| = ||\,\mathbf{u}|| - 3\,||\,\mathbf{v}||$.

☐ TRUE ☐ FALSE

◦ If $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, then $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$.

☐ TRUE ☐ FALSE

◦ If two linear systems are equivalent, then they have the same augmented matrix.

☐ TRUE ☐ FALSE

◦ A linear system with seven equations and two unknowns must be inconsistent.

☐ TRUE ☐ FALSE

◦ If the last column of an augmented matrix consists entirely of zeros, then the corresponding linear system must be consistent.

☐ TRUE ☐ FALSE

◦ A linear system of two equations in three unknowns can have a unique solution.

☐ TRUE ☐ FALSE

◦ A non-homogeneous linear system of equations with more equations than unknowns must be inconsistent.

☐ TRUE ☐ FALSE

2. (5 points) Select the right answer; briefly **explain your choice**.

◦ For which values of k, if any, are the vectors $\mathbf{u} = (k, 1, 3, -2)$ and $\mathbf{v} = (1, 8, k, 6)$ orthogonal ?

☐ $k = 1$ ☐ $k = 1, -2$ ☐ $k = 2, -3$ ☐ $k = -2$ ☐ non of the above

◦ If M is an augmented 4×3 matrix, then the number of pivot columns in any of its echelon forms is at most

☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 7 ☐ 12 ☐ non of the above

◦ For which values of m, if any, the following system has exactly one solution ?

$$\begin{cases} 5x + 4y = m \\ 3x - 2y = 1 \end{cases}$$

☐ 1 ☐ 2 ☐ $\emptyset$ ☐ $(-\infty, \infty)$ ☐ non of the above

◦ Consider the following matrices

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 2 & 2 \\ 6 & -1 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 2 & 1 \end{bmatrix}$$

Whenever the sum and product are defined, compute: $A + B$, AB, BA and $A^T C$.

3. (4 points) Solve the following system.

$$\begin{cases} x + 2y + z = 2 \\ 2x + 3y + 3z = 3 \\ -3x - 4y - 5z = -5 \end{cases}$$

State your answers clearly.

4. (6 points) Consider the following points: $A(2,0)$, $B(5,0)$, $C(4,3)$ and $D(3,2)$.

- Find the components of the vector $\mathbf{u} = \overrightarrow{AC}$. Compute $||\mathbf{u}||$. Find the midpoint of the line segment $\overrightarrow{AC}$.

- Find vector and parametric equations of the line that passes through the points A and C. Determine whether the point $S(1,-2)$ belongs to this line.

- Determine whether the vectors $\mathbf{u} = \overrightarrow{AC}$ and $\mathbf{v} = \overrightarrow{BD}$ are parallel, orthogonal or neither.

- Find the angle $\measuredangle BAD$ in degrees (round to one decimal place).

- Find the unit vector $\mathbf{w}$ that has the same direction as $\overrightarrow{BC}$.

- Find a nonzero vector $\mathbf{n}$ that is perpendicular to $\overrightarrow{BC}$.