UNIVERSITY OF OTTAWA

Department of Economics **ECO 2145A** **Fall 2014**

Dr. M. Rafiquzzaman **Test # 2** **Solution**

Multiple choice:

#1 A
#2 B
#3 A
#4 B
#5 A

Problem part:

1. Given market demand is: $p_1 = 10 - 0.5y$; $y = y_1 + y_2$; $C(y) = 2y; MC_1(y_1) = 2$; $MC(y_2) = 2.$

(a) From the demand equation, the residual demand function for Firm 1 is:

$p = 10 - 0.5(y_1 + y_2) = (10 - 0.5y) - 0.5y_1 \Rightarrow MR_1 = (10 - 0.5y_2) - y_1$

Profit maximizing condition: $MR_1 = MC(y_1) \Rightarrow (10 - 0.5y_2) - y_1 = 2 \Rightarrow y_1 = 8 - 0.5y_2 ...(1.1)$, which is the best response function of Firm 1.

Similarly, the residual demand function for Firm 2 is:

$p = (10 - 0.5y_1) - 0.5y_2 \Rightarrow MR_2 = (10 - 0.5y_1) - y_2$.

Profit maximizing condition: $MR_2 = MC(y_2) \Rightarrow (10 - 0.5y_1) - y_2 = 2 \Rightarrow y_2 = 8 - 0.5y_1 ...(1.2)$, which is the best response function of Firm 2.

For Nash-Cournot equilibrium, solve (1.1) and (1.2). Substituting (1.2) into (1.1), we have

$$y_1 = 8 - 0.5(8 - 0.5y_1) = 8 - 4 + 0.25y_1 = 4 + 0.25y_1 \Rightarrow (y_1 - 0.25y_1) = 4 \Rightarrow 0.75y_1 = 4 \Rightarrow y_1 = \frac{4}{0.75} \approx 5.33$$

From (1.2), $y_2 = 8 - 0.5(5.33) = 8 - 2.665 \approx 5.33$;

$p = 10 - 0.5(5.33 + 5.33) = 10 - 0.5(10.66) = 10 - 5.33 = \$4.67.$

$\pi_1 = py_1 - C(y_1) = (4.67)(5.33) - 2(5.33) = 24.89 - 10.66 = \$14.23.$

By symmetry, $\pi_2 = \$14.23.$

Thus, $y_1 = 5.33, y_2 = 5.33; p = \$4.67; \pi_1 = \pi_2 = \$14.23.$

(b) If Firm 1 is a Stackelberg leader and Firm 2 is a Stackelberg follower, then Firm 1 substitutes Firm2's beat response function in its own profit equation which it maximizes as if it is a monopolist. Firm1's profit function is:

$$\pi_1 = py_1 - C(y_1) = [10 - 0.5(y_1 + y_2)]y_1 - 2y_1 = 10y_1 - 0.05y_1^2 - 0.05y_1y_2 - 2y_1$$
$$= 8y_1 - 0.5y_1^2 - 0.5y_1y_2, ... (1.3)$$

Substitute (1.2) into (1.3). Then

$$\pi_1 = 8y_1 - 0.5y_1^2 - 0.5y_1(8 - 0.5y_1) = 8y_1 - 0.5y_1^2 - 4y_1 + 0.25y_1^2 = 4y_1 - 0.25y_1^2, ...(1.4)$$

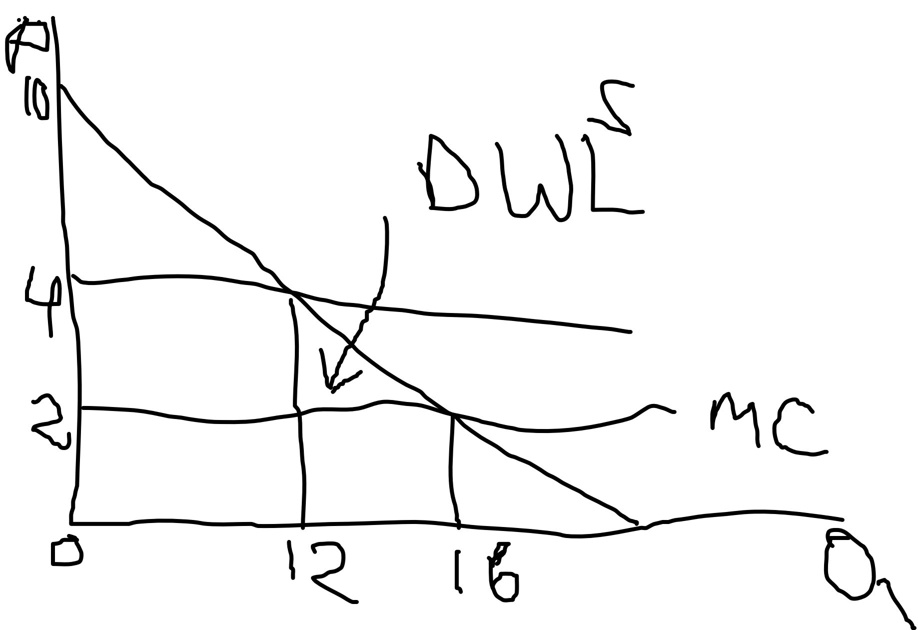
The leader's profit maximization is: $\frac{d\pi_1}{dy_1} = 0$.

From (1.4), $\frac{d\pi_1}{dy_1} = 4 - 0.5y_1 = 0 \Rightarrow y_1^S = \frac{4}{0.5} = 8$.

From (1.2), $y_2^S = 8 - 0.5(8) = 4$.

From the demand equation, $P^S = 10 - 0.5(8 + 4) = 10 - 6 = \$4$

Calculation of CS^S, PS^S, W^S, DWL^S



CS, PS, and W under competition

$$P = MC \Rightarrow 10 - 0.5y = 2 \Rightarrow y^C = \frac{8}{0.5} = 16; P^C = 2$$

$$CS^C = \frac{1}{2}(10 - 2)(16) = 64; \; PS^C = 0; \; W^C = CS^C + PS^C = 64 + 0 = \$64$$

Calculation of CS, PS, W, and DWL in case of Stackelberg solution.

$$y_1^S + y_2^S = 8 + 4 = 12 = y^S$$
$$CS^S = \frac{1}{2}(10-4)(12) = \frac{1}{2}(6)(12) = 36$$
From (1.4), $\pi_1^S = 4(8) - 0.25(8)^2 = 32 - 16 = \$16$
$$\pi_2^S = py_2^S - C(y_2) = (4)(4) - 2(4) = 16 - 8 = \$8$$
So, $PS^S = \pi^S = \pi_1^S + \pi_2^S = \$16 + \$8 = \$24$
$$W^S = CS^S + PS^S = \$36 + \$24 = \$60.$$
$$DWL^S = W^C - W^S = \$64 - \$60 = \$4$$

Thus $y_1^S = 8, y_2^S = 4, P^S = \$4, CS^S = \$36, PS^S = \$24, W^S = \$60, DWL^S = \$4.$

2(a)

Given:

Demand for Pepsi: $q_p = 50 - 2p_p + p_c$; $MC_p = m$

Pepsi's profit function is:
$$\pi_p = p_p q_p - mq_p = p_p(50 - 2p_p + p_c) - m(50 - 2p_p + p_c) = 50p_p - 2p_p^2 + p_p p_c - 50m + 2p_p m - mp_c$$
$$\frac{\delta \pi_p}{\delta p_p} = 50 - 4p_p + p_c + 2m = 0 \Rightarrow 4p_p = 50 + p_c + 2m \Rightarrow p_p = \frac{50 + 2m + p_c}{4}$$
$$\Rightarrow p_p = 12.5 + \frac{m}{2} + \frac{p_c}{4} ....(2.1)$$
which is the best response function for Pepsi.

(i) From(2.1), $\frac{\delta p_p}{\delta m} = \frac{1}{2}$, that is, a \$1 increase in marginal cost yields a \$0.50 increase in price.

(ii) $\frac{\delta p_p}{\delta p_c} = \frac{1}{4}$, that is, a \$1 increase in Coke's price, yields a \$0.25 increase in Pepsi's price.

2(b)

Given:

$$Q = 5L; MP_L = 5; Q = 100 - 5P \Rightarrow P = \frac{100 - Q}{5} = 20 - 0.2Q........(2.2)$$
$$L = 0.2w \Rightarrow w = \frac{L}{0.2} = 5L$$

If the firm is a perfect competitor in the output market and a monopsonist in the labour market, the optimal condition is: $ME = VMP_L$

$$ME = \frac{dE}{dL} = \frac{d(wL)}{dL} = w + L\frac{dw}{dL} = 5L + L(5) = 10L; \text{ because } w = 5L \Rightarrow \frac{dw}{dL} = 5$$

$$VMP_L = PMP_L = (20 - 0.2Q)(5) = \{20 - 0.2(5L)\}(5) = 100 - 5L\,.$$

$$ME = VMP_L \Rightarrow 10L = 100 - 5L \Rightarrow 15L = 100 \Rightarrow L = \frac{100}{15} \approx 6.67; w = 5L = 5(6.67) = 33.35$$

- Thus $L = 6.67; w = 33.35$

Now, $Q = 5L = 5(6.67) = 33.35; P = 20 - 0.2(33.35) = 20 - 6.67 = \$13.33$

$$Profit = \pi = Revenue - Cost = PQ - wL = (13.33)(33.35) - (33.35)(6.67) = (33.35)(13.33 - 6.67)$$
$$= (33.35)(6.66) = \$222.11$$

- Thus $Profit = \pi = \$222.11$

At $L = 6.67, VMP_L = PMP_L = (13.33)(5) = 66.65$

- Deviation = $66.65 - 33.35 = 33.32$