UNIVERSITY OF OTTAWA

Department of Economics **ECO 2145A** **Fall 2014**

Dr. M. Rafiquzzaman **Test # 1** **Solution**

Multiple choice:

#1 A
#2 D
#3 B
#4 D
#5 A

Problem part:

1. $P = 100 - \frac{1}{10}Q; \qquad C = 200 + 20Q + \frac{1}{10}Q^2$

Total revenue R $= PQ = 100Q - \frac{1}{10}Q^2$; $MR = \frac{dR}{dQ} = 100 - \frac{1}{5}Q = 100 - 0.2Q$

MC $= \frac{dC}{dQ} = 20 + \frac{2}{10}Q = 20 + 0.2Q$

(a) MR = MC $\Rightarrow 100 - 0.2Q = 20 + 0.2Q \Rightarrow 0.4Q = 80 \Rightarrow Q_m = \frac{80}{0.4} = 200.$

$P_m = 100 - \frac{1}{10}(200) = 100 - 20 = \$80$

(b)

$$\pi = R - C - tQ = 100Q - \frac{1}{10}Q^2 - \left(200 + 20Q + \frac{1}{10}Q^2\right) - 8Q \Rightarrow$$

$$\pi = 100Q - \frac{1}{10}Q^2 - 200 - 20Q - \frac{1}{10}Q^2 - 8Q \Rightarrow 72Q - 0.2Q^2 - 200 ......(2.1)$$

$$\frac{d\pi}{dQ} = 72 - 0.4Q = 0 \Rightarrow Q = \frac{72}{0.4} = 180; P = 100 - \frac{180}{10} = 100 - 18 = \$82.$$

From equation (2.1), $\pi = 72(180) - 0.2(180)^2 - 200 = 12960 - 6480 - 200 = \$6280.$

(c) Tax revenue $T = tQ^*, where$ t = tax per unit, and Q^* is monopolist's output.

$$\pi = R - C - tQ = 100Q - \frac{1}{10}Q^2 - \left(200 + 20Q + \frac{1}{10}Q^2\right) - tQ = 100Q - \frac{1}{10}Q^2 - 200 - 20Q - \frac{1}{10}Q^2 - tQ$$

$$= 80Q - 0.2Q^2 - 200 - tQ.$$

$$\frac{d\pi}{dQ} = 80 - 0.4Q - t = 0 \Rightarrow Q^* = \frac{80 - t}{0.4}.$$

Tax revenue, $T = tQ^* = t\left(\frac{80 - t}{0.4}\right) = \frac{80t - t^2}{0.4}$.......(2.2).

To obtain the optimal level of tax per unit, we maximize T with respect to t. From (2.2),

$\frac{dT}{dt} = \frac{1}{0.4}(80 - 2t) = 0 \Rightarrow 80 - 2t = 0 \Rightarrow t^* = \frac{80}{2} = \$40$ per unit.

2(a)

Given $p = 90 - Q, MC = AC = m = \$30$.

$MR = 90 - 2Q$.

Profit maximization condition: $MR = MC \Rightarrow 90 - 2Q = 30 \Rightarrow 2Q = 60 \Rightarrow Q = \frac{60}{2} = 30$

From the demand equation $p = 90 - 30 = \$60$

Thus Q=30, p=$60.

2(b)

$\pi = p_1Q_1 + p_2(Q_2 - Q_1) - mQ_2 = (90 - Q_1)Q_1 + (90 - Q_2)(Q_2 - Q_1) - 30Q_2$

$= 90Q_1 - Q_1^2 + 90Q_2 - 90Q_1 - Q_2^2 + Q_1Q_2 - 30Q_2 = -Q_1^2 + 60Q_2 - Q_2^2 + Q_1Q_2$

$\frac{d\pi}{dQ_1} = -2Q_1 + Q_2 = 0 \qquad \ldots..(2.1)$

$\frac{d\pi}{dQ_2} = 60 - 2Q_2 + Q_1 = 0 \quad \ldots\ldots(2.2)$

Solving (2.1) and (2.2) simultaneously, we get the profit maximizing level of Q_1 and Q_2 .

From (2.1), $Q_2 = 2Q_1$. Substituting this into (2.2), we get

$60 - (2Q_1)2 + Q_1 = 0 \Rightarrow 60 - 4Q_1 + Q_1 = 0 \Rightarrow 3Q_1 = 60 \Rightarrow Q_1 = \frac{60}{3} = 20$.

From (2.2), $Q_2 = 2Q_1 = 2(20) = 40$.

Thus, $Q_1 = 20, Q_2 = 40$

Thus, $p_1 = \$70,\ p_2 = \$50$

That is, the monopoly charges a price of \$70 for any quantity between 1 and 20 -- the first block – and \$50 on any units beyond 20 – the second block.

Please see Figure 2.1 for #2 of Test 1 in the Website just below the Solutions of Test #1.

From the first block (i.e., $Q_1 = 20$), CS = A= $\frac{1}{2}$ (90-70)(20) = 200

From the 2nd block, CS = C = $\frac{1}{2}$ (70-50)(40-20) = 200

Total CS = 200 + 200 =400

From the first block, π_1= B = (70 – 30) (20) = 800
From the 2nd block, π_2= L = (50 – 30) (40 - 20) = 400

Then PS = $\pi_1 + \pi_2$ = 800 + 400 = 1,200
Welfare = CS + PS = 400 + 1200 =1,600

Deadweight loss calculation

In case of perfect competition: p=MC ⇒ 90 – Q = 30 ⇒ $Q = 60, p = 30.$

CS under competition = A + B + C + L + M = $\frac{1}{2}$ (90 -30) (60) = 1800
PS under competition = 0
Welfare under competition = CS + PS = 1800 + 0 = 1800
DWL = Welfare under competition – Welfare under block pricing = 1800 – 1600 = 200, which is area A.
Area A = $\frac{1}{2}$ (50 – 30) (60 – 40) = 200

Thus $Q_1 = 20, Q_2 = 40, p_1 = 70, p_2 = 50,$ CS = 400, PS = 1200, Welfare = 1600, DWL = 200.

3. Given:
Demand in market1: $P_1 = 15 - Q_1$; demand in market 2: $P_2 = 25 - 2Q_2$

Total cost $C = 5 + 3Q$ (where $Q = Q_1 + Q_2$) $\Rightarrow MC = \frac{dC}{dQ} = 3$

(a) Price discrimination

$$\pi = R_1(Q_1) + R_2(Q_2) - C = P_1Q_1 + P_2Q_2 - C = (15 - Q_1)Q_1 + (25 - Q_2)Q_2 - 5 - 3(Q_1 + Q_2)$$
$$= 12Q_1 - Q_1^2 + 22Q_2 - 2Q_2^2 - 5$$

$$\frac{\delta\pi}{\delta Q_1} = 12 - 2Q_1 = 0 \Rightarrow Q_1^* = \frac{12}{2} = 6;$$
$$\frac{\delta\pi}{\delta Q_2} = 22 - 4Q_2 = 0 \Rightarrow Q_2^* = \frac{22}{4} = 5.5$$

From demand equation in market 1, $P_1^* = 15 - 6 = \$9$

From demand equation in market 2, $P_2^* = 25 - 2(5.5) = \$14$

$$\pi = P_1^*Q_1^* + P_2^*Q_2^* - C = (9)(6) + (14)(5.5) - 5 - 3(6 + 5.5) = 54 + 77 - 5 - 34.5 = \$91.5$$

Deadweight loss calculation:

In case of competition, $MC = 3$.

In market 1, $MC = P_1 \Rightarrow 15 - Q_1 = 3 \Rightarrow Q_1^C = 12$; $P_1^C = 3$

In market 2, $MC = P_2 \Rightarrow 25 - 2Q_2 = 3 \Rightarrow Q_2^C = 11$; $P_2^c = 3$

The deadweight loss due to monopoly is: $DWL = \frac{1}{2}(P_m - P_C)(Q_c - Q_m)$. Using this,

$$DWL_1 = \frac{1}{2}(9 - 3)(12 - 6) = \frac{1}{2}(6)(6) = \$18$$
$$DWL_2 = \frac{1}{2}(14 - 3)(11 - 5.5) = \frac{1}{2}(11)(5.5) = \$30.25$$
$$DWL = DWL_1 + DWL_2 = 18 + 30.25 = \$48.25$$

Thus in case of discrimination:

$P_1^m = \$9$; $P_2^m = \$14$; $Q_1^m = 6$; $Q_2^m = 5.5$; $DWL_1 = \$18$; $DWL_2 = \$30.25$;

$DWL = \$48.25$; $\pi^* = \$91.5$

(b) If discrimination is not possible, then the optimal condition is: $MR_G = MC$, where MR_G =horizontal sum of MR_1 and MR_2

For horizontal sum, replace P_1 and P_2 by P, that is, $P_1 = P_2 = P$

From market 1, $P_1 = 15 - Q_1 \Rightarrow P = 15 - Q_1 \Rightarrow Q_1 = 15 - P$

From market 2, $P_2 = 25 - 2Q_2 \Rightarrow P = 25 - 2Q_2 \Rightarrow Q_2 = 12.5 - \frac{1}{2}P$

Horizontal sum of market demands

$$= Q_G = Q_1 + Q_2 = 15 - P + 12.5 - \frac{1}{2}P = 27.5 - \frac{3}{2}P \Rightarrow 2Q_G = 55 - 3P \Rightarrow P = \frac{55}{3} - \frac{2}{3}Q_G \ldots(3.1)$$

From (3.1), $MR_G = \frac{55}{3} - \frac{4}{3}Q_G$

$$MR_G = MC \Rightarrow \frac{55}{3} - \frac{4}{3}Q_G = 3 \Rightarrow 55 - 4Q_G = 9 \Rightarrow Q_G^* = \frac{46}{4} = 11.5;$$

$$P^* = \frac{55}{3} - \frac{2}{3}(11.5) = 18.33 - 7.67 = \$10.63$$

$$\pi^* = P^* Q_G^* - C = (10.63)(11.5) - [5 + 3(11.5)] = 122.25 - 5 - 34.5 = \$82.75$$

From market 1, $Q_1^* = 15 - 10.63 = 4.37$

From marker 2, $Q_2^* = 12.5 - \frac{1}{2}(10.63) = 7.18$

Deadweight loss calculation:
In case of competition,

From market 1, $P_1 = MC \Rightarrow 15 - Q_1 = 3 \Rightarrow Q_1^C = 12$, $P_1^C = 3$

From marker 2, $P_2 = MC \Rightarrow 25 - 2Q_2 = 3 \Rightarrow Q_2^C = 11$

$$DWL_1 = \frac{1}{2}(10.63 - 3)(12 - 4.37) = \frac{1}{2}(7.63)(7.63) = \$29.11$$

$$DWL_2 = \frac{1}{2}(10.63 - 3)(11 - 7.8) = \frac{1}{2}(7.63)(3.82) = \$14.57$$

$$DWL = DWL_1 + DWL_2 = \$29.11 + \$14.57 = \$43.68$$

Thus in case of no discrimination,

$P_1^m = \$10.63$; $P_2^m = \$10.63$; $Q_1^m = 4.37$;
$Q_2^m = 7.18$; $Q^* = 11.5$; $\pi^* = \$82.75$; $DWL_1 = \$29.11$; $DWL_2 = \$14.57$; $DWL = \$43.68$