UNIVERSITY OF OTTAWA

Department of Economics	**ECO 2145A**	**Fall 2014**
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1. Given:

Demand in market1: $P_1 = 60 - 15Q_1$; demand in market 2: $P_2 = 30 - 12Q_2$

Total cost $C = 27 + 12Q$ (where $Q = Q_1 + Q_2$) $\Rightarrow MC = \frac{dC}{dQ} = 12$

(a) Price discrimination

$$\pi = R_1(Q_1) + R_2(Q_2) - C = P_1Q_1 + P_2Q_2 - C = (60 - 15Q_1)Q_1 + (30 - 12Q_2)Q_2 - 27 - 12(Q_1 + Q_2)$$
$$= 48Q_1 - 15Q_1^2 + 18Q_2 - 12Q_2^2 - 27$$

$$\frac{\delta\pi}{\delta Q_1} = 48 - 30Q_1 = 0 \Rightarrow Q_1^* = \frac{48}{30} \approx 1.6;$$

$$\frac{\delta\pi}{\delta Q_2} = 18 - 24Q_2 = 0 \Rightarrow Q_2^* = \frac{18}{24} \approx 0.75$$

From demand equation in market 1, $P_1^* = 60 - 15(1.6) = 60 - 24 = \$36$

From demand equation in market 2, $P_2^* = 30 - 12(0.75) = 30 - 9 = \$21$

$$\pi = P_1^*Q_1^* + P_2^*Q_2^* - C = (36)(1.6) + (21)(0.75) - 27 - 12(1.6 + 0.75) = 57.6 + 15.75 - 27 - 28.2 = \$18.15$$

(c) If discrimination is not possible, then the optimal condition is: $MR_G = MC$, where MR_G =horizontal sum of MR_1 and MR_2

For horizontal sum, replace P_1 and P_2 by P, that is, $P_1 = P_2 = P$

From market 1, $P_1 = 60 - 15Q_1 \Rightarrow P = 60 - 15Q_1 \Rightarrow Q_1 = \frac{-P + 60}{15}$

From market 2, $P_2 = 30 - 12Q_2 \Rightarrow P = 30 - 12Q_2 \Rightarrow Q_2 = \frac{30 - P}{12}$

Horizontal sum of market demands

$$= Q_G = Q_1 + Q_2 = \frac{-P + 60}{15} + \frac{30 - P}{12} = \frac{-9P + 390}{60} \Rightarrow 60Q_G = -9P + 390 \Rightarrow P = \frac{130}{3} - \left(\frac{20}{3}\right)Q_G \ldots(1.1)$$

From (1.1), $MR_G = \frac{130}{3} - 2\left(\frac{20}{3}\right)Q_G = \frac{130}{3} - \frac{40}{3}Q_G$

$$MR_G = MC \Rightarrow \frac{130}{3} - \frac{40}{3}Q_G = 12 \Rightarrow Q_G^* = \frac{94}{40} \approx 2.35$$

$$\pi^* = P^*Q_G^* - C = (27.66)(2.35) - [27 + 12(2.35)] = \$9.8$$

2. Demand:

$$P = 25 + 0.001Y - 0.25Q \Rightarrow Q = \frac{25 + 0.001Y - P}{0.25} = 100 + 0.004Y - 0.4P \text{........}(2.1)$$

(a) For the rich, $Q_R = [100 + 0.004(15000) - 4P]x600 = 96000 - 2400P$

For the poor, $Q_P = [100 + 0.004(1000) - 4P]x400 = 56000 - 1600P$

For the aggregate $Q_A = Q_R + Q_P = 152000 - 4000P \text{..................................}(2.2)$

(b) From (2.2), $P = 38 - 0.00025Q_A \text{...................}(2.3)$

For <u>no discrimination</u>, $MR_A = MC$

From (2.3), $MR_A = 38 - 2(0.00025)Q_A = 38 - 0.0005Q_A$. From the cost function,

$$MC = \frac{\delta C}{\delta Q_A} = 8$$

$$MR_A = MC \Rightarrow 38 - 0.0005Q_A = 8 \Rightarrow Q_A^* = \frac{30}{0.0005} = 60000$$

From (2.3), $P^* = 38 - 0.0005(60000) = \$23$

Therefore, $Q_R^* = 96000 - 2400(23) = 40{,}800;$ $Q_P^* = 56000 - 1600(23) = 19{,}200$

(c) For <u>price discrimination</u>, profit maximizing conditions are: $MR_R = MC$ and $MR_P = MC$

From part (a),

$$Q_R = 96000 - 2400P \Rightarrow P = 40 - \left(\frac{1}{2400}\right)Q_R \Rightarrow MR_R = 40 - 2\left(\frac{1}{2400}\right)Q_R = 40 - \left(\frac{1}{1200}\right)Q_R$$

$$MR_R = MC \Rightarrow 40 - \left(\frac{1}{1200}\right)Q_R = 8 \Rightarrow Q_R^* = (32)(1200) = 38{,}400.$$

$$P_R^* = 40 - \left(\frac{1}{2400}\right)(38400) = \$24$$

Also from part (a),

$$Q_P = 56000 - 1600P \Rightarrow P = \left(\frac{1}{1600}\right)(56000 - Q_P) = 35 - \left(\frac{1}{1600}\right)Q_P \Rightarrow MR_P = 35 - 2\left(\frac{1}{1600}\right)Q_p$$

$$So, MR_P = 35 - \left(\frac{1}{800}\right)Q_P$$

$$MR_P = MC \Rightarrow 35 - \left(\frac{1}{800}\right)Q_P = 8 \Rightarrow Q_P^* = (27)(800) = 21{,}600$$

$$P_R^* = 35 - \left(\frac{1}{1600}\right)(21600) = 35 - 13.5 = \$17.5$$

3. Given: $P_1 = 100 - 2Q_1$; $P_2 = 80 - Q_2$; $TC = 20 + 4(Q_1 + Q_2)$

(a) $\pi = R_1 + R_2 - TC$, where R_1 is the total revenue from Market1and R_2 is the total revenue from Market2. Then,

$$\pi = P_1Q_1 + P_2Q_2 - TC = (100 - 2Q_1)Q_1 + (80 - Q_2)Q_2 - [20 + 4(Q_1 + Q_2)]$$
$$= 96Q_1 - 2Q_1^2 + 76Q_2 - Q_2^2 + 20.............(3.1)$$

(b)

$$\frac{\delta\pi}{\delta Q_1} = 96 - 4Q_1 = 0 \Rightarrow Q_1^* = \frac{96}{4} = 24;\ P_1^* = 100 - 2(24) = \$52$$
$$\frac{\delta\pi}{\delta Q} = 76 - 2Q_2 = 0 \Rightarrow Q_2^* = \frac{76}{2} = 38;\ P_2^* = 80 - 38 = \$42$$

(c)

$MR_1 = 100 - 2(2Q_1) = 100 - 4Q_1$. Then, $MR_1\big|_{Q_1^*=24} = 100 - 4(24) = \$4$

$MR_2 = 80 - (2Q_2)$. Then, $MR_1\big|_{Q_2^*=38} = 80 - 2(38) = \$4$

(d) $\pi^* = R_1 + R_2 - TC = \$52(24) + \$42(38) - \$[20 + (24 + 38)] = \$2576$

(e) If no discrimination is permissible, we need to work with the horizontal sum of the two market demand curves.

Replace P_1 by $P \Rightarrow P = 100 - 2Q_1 \Rightarrow Q_1 = 50 - \frac{1}{2}P$

Replace P_2 by $P \Rightarrow P = 80 - Q_2 \Rightarrow Q_2 = 80 - P$

Market demand is:

$$Q = Q_1 + Q_2 = 50 - \frac{1}{2}P + 80 - P = 130 - \frac{3}{2}P \Rightarrow 2Q = 260 - 3P \Rightarrow P = \frac{260}{3} - \frac{2}{3}Q.....(3.1)$$

$MR_T = \frac{260}{3} - 2\left(\frac{2}{3}Q\right) = \frac{260}{3} - \frac{4}{3}Q$, where MR_T = marginal revenue corresponding to the horizontal sum of the two demand curves. Recall

$$TC = 20 + 4(Q_1 + Q_2) = 20 + 4Q \Rightarrow MC = \frac{\delta(TC)}{\delta Q} = 4$$

$$MR_T = MC \Rightarrow \frac{260}{3} - \frac{4}{3}Q = 4 \Rightarrow 260 - 4Q = 12 \Rightarrow Q_T^* = \frac{248}{4} = 62$$

From (3.1), $P^* = \frac{260}{3} - \frac{2}{3}(62) \approx \$45.33$

$$Q_1^* = 50 - \frac{1}{2}(45.33) \approx 27.33;\ Q_2^* = 80 - 45.33 = 34.67.$$

Note that $Q_T^* = Q_1^* + Q_2^* = 27.33 + 34.67 = 62$

$\pi^* =$ Total market revenue – total cost $= \$45.33(62) - [20 + 4(62)] = \$2542.46$

(f) Elasticity in market 1 $= \varepsilon_1 = \left(\frac{\delta Q_1}{\delta P_1}\right)\left(\frac{P_1}{Q_1}\right) = \left(-\frac{1}{2}\right)\left(\frac{45.33}{27.33}\right) = -0.83$

Elasticity in market 2 $= \varepsilon_2 = \left(\frac{\delta Q_2}{\delta P_2}\right)\left(\frac{P_2}{Q_2}\right) = (-1)\left(\frac{45.33}{34.67}\right) = -1.3$

This confirms that the more inelastic the demand, the higher the price should be charged. Accordingly, in part (b), $P_1^* = \$52$ and $P_2^* = \$42$.

2.1 With perfect price discrimination, the firm sells each unit at the maximum amount any customer is willing to pay for it, so prices differ across customers, and a given customer may pay more for some units than for others. By selling each unit of its output to the customer who values it the most at the maximum price that person is willing to pay, the perfectly price-discriminating monopoly captures all possible consumer surplus. Maximum potential consumer surplus (*CS*) is equal to the area under the linear demand curve and above the marginal cost. This is the area of a triangle with a base equal to the quantity demanded, which is 60 units (from 30 = 90 – *Q*), and a height equal to the difference in where the demand curve hits the vertical, price axis (90) and the $30 marginal cost, which is 60:

$$CS = 0.5(60)(60);$$
$$CS = \$1{,}800.$$

This becomes the monopoly's profit.

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If the firm charges a single price, then the firm maximizes profit by setting marginal cost ($30) equal to marginal revenue. The marginal revenue curve is the same as the demand curve but with twice the slope:

$$MR = 90 - 2q.$$

Setting marginal cost equal to marginal revenue, the profit-maximizing quantity is

$$q = 30 \text{ units}.$$

The profit-maximizing price is that where consumers demand this quantity:

$$p = \$60.$$

Profit equals the difference in price ($60) and marginal cost ($30) multiplied by the units produced (30):

$$\pi = (60 - 30)(30) = \$900.$$

Consumer surplus is equal to the area under the linear demand curve and above the price. This is the area of a triangle with a base equal to the quantity demanded, which is 30 units, and a height equal to the difference in where the demand curve hits the vertical, price axis (90) and the $60 price:

$$CS = \$450.$$

Welfare equals the monopoly's producer surplus (which equals profit since there are no fixed costs) plus consumer surplus:

$$\text{Welfare} = 900 + 450 = 1{,}350.$$

Deadweight loss equals the area under the demand curve and above the marginal cost curve for units not produced. This is equal to the area of a triangle with a base equal to the difference in the profit-maximizing quantity (30) and the efficient quantity where demand equals marginal cost (60, from $30 = 90 - Q$) multiplied by a height equal to the difference in the profit-maximizing price ($60) and marginal cost ($30):

DWL = 0.5(30)(30) = $450.

3.9 Using Equation 12.2,

$$p_{US}(1 - 1/2) = 10 = p_J(1 - 1/5)$$

$p_{US} = \$20$, $p_J = \$12.50$.

4.2 Setting marginal revenue equal to marginal cost yields $Q^* = 30$, $p^* = 60$. Profit is $900, consumer surplus is $450, welfare is $1,350($PS + CS$), and deadweight loss is $450.

4.3 For simplicity, assume the demand function is linear and its equation is $Q = a - bP$, where a and b are positive numbers and $a > m$. The nonlinear price discriminating utility's profit is:

$$\pi = P_1Q_1 + P_2(Q_2 - Q_1) + P_3(Q_3 - Q_2) - mQ_3$$

$$\pi = P_1(a - bP_1) + P_2[(a - bP_2) - (a - bP_1)] + P_3[(a - bP_3) - (a - bP_2)] - m(a - bP_3)$$

$$\pi = P_1(a - bP_1) + bP_2[(P_1 - P_2)] + bP_3[P_2 - P_3)] - m(a - bP_3).$$

F.O.C:

$$\partial p/\partial P_1 = a - 2bP_1 + bP_2$$

$$= 0,\quad \partial p/\partial P_2$$

$$= bP_1 - 2bP_2 + bP_3$$

$$= 0,\quad \partial p/\partial P_3$$

$$= bP_2 - 2bP_3 + mb = 0.$$

Solving these three equations we get:

$$P_1 = (3a/4b) + m/4,$$

$$P_2 = (a/2b) + m/2,$$

$$P_3 = (a/4b) + 3m/4.$$

5.3 A two-part price is where a firm charges a consumer a lump-sum fee for the right to buy as many units of the good as the consumer wants at a specified price. The optimal two-part price for the Club is to charge Joe a per-unit price equal to marginal cost ($20) and then set the lump-sum fee equal to Joe's consumer surplus. Joe's consumer surplus (*CS*) is equal to the area under his linear demand curve and above the per-round price. This is the area of a triangle with a base equal to the quantity demanded, which is 50 units and a height equal to the difference in where the demand curve hits the vertical, price axis and the $20 price, which is 100:

$$CS = 0.5(50)(100)$$
$$CS = \$2{,}500.$$

If the firm charges a single price, then the firm maximizes profit by setting marginal cost ($20) equal to marginal revenue. The marginal revenue curve is the same as the demand curve but with twice the slope:

$$MR = 120 - 4q.$$

Setting marginal cost equal to marginal revenue, the profit-maximizing quantity is

$$MC = MR$$
$$20 = 120 - 4q$$
$$4q = 100$$
$$q = 25 \text{ units.}$$

The profit-maximizing price is that where consumers demand this quantity:

$$p = 120 - 2q$$
$$p = 120 - 2(25)$$
$$p = 120 - 50$$
$$p = \$70.$$

Profit equals the difference in price ($70) and marginal cost ($20) multiplied by the units produced (25):

$$\Pi = \$1{,}250.$$

Thus, the difference in profit from using the two-part tariff is $1,250 (from profit of $2,500 with the two-part tariff minus profit of $1,250 from charging a single price).

7.2 Assume one unit of advertising costs $1. The profit function is

$$p = (100 - Q + A^{1/2})Q - 10Q - A.$$

The resulting first-order conditions are

$$\partial p/\partial A = (Q/2)A^{-1/2} - 1 = 0$$

$$\partial p/\partial Q = 100 - 2Q + A^{1/2} - 10 = 0$$

$$A^* = 900$$

$$Q^* = 60$$

$$p^* = 70.$$