UNIVERSITY OF OTTAWA

Department of Economics	**ECO 2145A**	**Fall 2014**
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1. $P = 600 - 26Q; \qquad TC = 600Q - 40Q^2 + Q^3$

(a) Profit maximizing condition: MR = MC

From the demand equation, total revenue (TR) = $PQ = (600 - 26Q)Q = 600Q - 26Q^2$.

$$\Rightarrow MR = \frac{d(TR)}{dQ} = 600 - (26)2Q = 600 - 52Q.$$

From the total cost function, Marginal cost (MC) = $\frac{d(TC)}{dQ} = 600 - 80Q + 3Q^2$.

MR = MC

$\Rightarrow 600 - 52Q = 600 - 80Q + 3Q^2 \Rightarrow 3Q^2 = 28Q \Rightarrow 3Q = 28 \Rightarrow Q_m = 28/3 \approx 9.33$

From the demand equation, $P_m = 600 - 26(9.33) = 600 - 242.58 = 357.42$

(b) When the monopolist is regulated on an average cost basis, P = AC. From the total cost equation, $AC = TC/Q = (600Q - 40Q^2 + Q^3)/Q = 600 - 40Q + Q^2$.

P = AC $\Rightarrow 600 - 26Q = 600 - 40Q + Q^2 \Rightarrow Q^2 = 14Q \Rightarrow Q = 14$.

$P = 600 - 26(14) = 600 - 364 = 236.$

(c)

$P = MC \Rightarrow 600 - 26Q = 600 - 80Q + 3Q^2 \Rightarrow 3Q^2 = 14Q \Rightarrow 3Q = 54 \Rightarrow Q = 54/3 = 18.$

$P = 600 - 26(18) = 600 - 468 = 132$.

$Loss = -\pi = -(PQ - TC) = -\left[(132)(18) - \left\{600(18) - 40(18)^2 + (18)^3\right\}\right]$

$= -\left[2376 - (10800 - 12960 + 5832)\right] = -(2376 - 3672) = -(1296) = 1296.$

The necessary subsidy to keep the firm in business = \$1296.

2. $P = 100 - \frac{1}{10}Q; \qquad C = 200 + 20Q + \frac{1}{10}Q^2$

Total revenue R $= PQ = 100Q - \frac{1}{10}Q^2$; $MR = \frac{dR}{dQ} = 100 - \frac{1}{5}Q = 100 - 0.2Q$

MC $= \frac{dC}{dQ} = 20 + \frac{2}{10}Q = 20 + 0.2Q$

(a) MR = MC $\Rightarrow 100-0.2Q = 20+0.2Q \Rightarrow 0.4Q = 80 \Rightarrow Q_m = \frac{80}{0.4} = 200.$

$P_m = 100 - \frac{1}{10}(200) = 100-20 = \$80$

(b)

$\pi = R - C - tQ = 100Q - \frac{1}{10}Q^2 - \left(200 + 20Q + \frac{1}{10}Q^2\right) - 8Q \Rightarrow$

$\pi = 100Q - \frac{1}{10}Q^2 - 200 - 20Q - \frac{1}{10}Q^2 - 8Q \Rightarrow 72Q - 0.2Q^2 - 200$ ......(2.1)

$\frac{d\pi}{dQ} = 72 - 0.4Q = 0 \Rightarrow Q = \frac{72}{0.4} = 180;\ P = 100 - \frac{180}{10} = 100 - 18 = \$82.$

From equation (2.1), $\pi = 72(180) - 0.2(180)^2 - 200 = 12960 - 6480 - 200 = \$6280.$

(c) Tax revenue $T = tQ^*, where$ t = tax per unit, and Q^* is monopolist's output.

$\pi = R - C - tQ = 100Q - \frac{1}{10}Q^2 - \left(200 + 20Q + \frac{1}{10}Q^2\right) - tQ = 100Q - \frac{1}{10}Q^2 - 200 - 20Q - \frac{1}{10}Q^2 - tQ$

$= 80Q - 0.2Q^2 - 200 - tQ.$

$\frac{d\pi}{dQ} = 80 - 0.4Q - t = 0 \Rightarrow Q^* = \frac{80-t}{0.4}.$

Tax revenue, $T = tQ^* = t\left(\frac{80-t}{0.4}\right) = \frac{80t - t^2}{0.4}$.......(2.2).

To obtain the optimal level of tax per unit, we maximize T with respect to t. From (2.2),

$\frac{dT}{dt} = \frac{1}{0.4}(80-2t) = 0 \Rightarrow 80 - 2t = 0 \Rightarrow t^* = \frac{80}{2} = \$40$ per unit.

3. $P = 4 - 0.001Q \Rightarrow Total\,Revenue(R) = PQ = 4Q - 0.001Q^2 \Rightarrow MR = \frac{dR}{dQ} = 4 - 0.002Q$

MC = 0. [Note: $LTC = (LAC)Q = 3200 \Rightarrow MC = \frac{d(LTC)}{dQ} = 0$].

(a) $MR = MC \Rightarrow 4 - 0.002Q = 0 \Rightarrow Q_m = \frac{4}{0.002} = 2000;$ $P_m = 4 - 0.001(2000) = \$2$ per call.

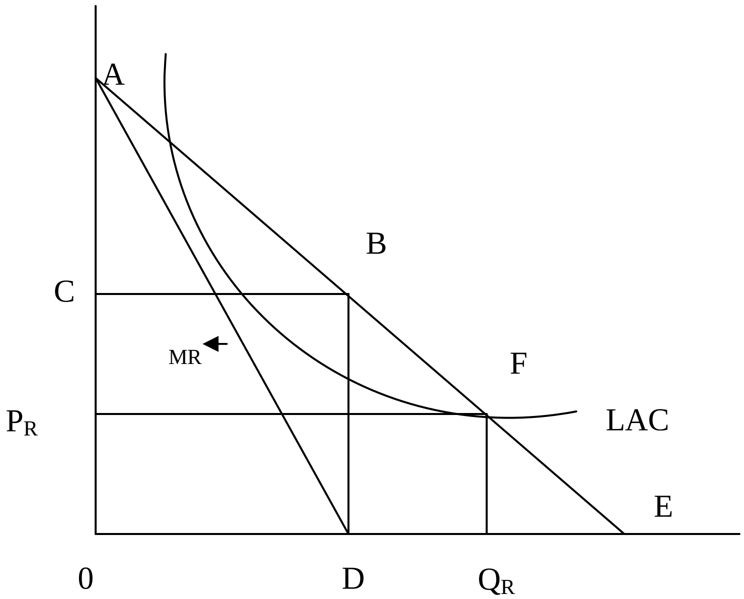
(b) Under competition, $P = MC \Rightarrow 4 - 0.00Q = 0 \Rightarrow Q_C = \frac{4}{0.001} = 4000; P_C = 0$

Consumer surplus (CS) under competition is: $(CS)^C = areaAOE = \frac{1}{2}(4)(4000) = \$8000.$

Consumer surplus under monopoly is: $(CS)^m = areaABC = \frac{1}{2}(4-2)(2000) = \$2000.$

Loss of consumers' surplus $= (CS)^C - (CS)^m = \$8000 - \$2000 = \$6000.$

(c) It is constrained to breakeven $\Rightarrow P = LAC \Rightarrow optimalprice = P_R$, and optimal output $= Q_R$. In the diagram, CS under regulation = AFP_R. CS under competition = AOE .
Loss in CS = $AOE - AFP_R = P_R OEF$.



<u>Solution of Textbook problems</u>

1.8 The firm's profit function π is (where R is total revenue and C is total cost):

$$\pi = R(Q) - C(Q)$$

$$\pi = pQ - [25 + Q + 0.5Q^2]$$

$$\pi = [13 - Q]Q - [25 + Q + 0.5Q^2].$$

Maximizing this with respect to Q to find the profit – maximizing quantity:

$$\frac{\partial \pi}{\partial Q} = 13 - Q - Q - 1 - Q$$

$$13 - Q - Q - 1 - Q = 0$$

$$12 - 3Q = 0$$

$$Q = 4 \text{ units.}$$

Using the demand equation, the profit – maximizing price is

$$p = 13 - Q$$

$$p = 13 - (4)$$

$$p = 9.$$

Profit per unit is the $9 price minus the average total cost of production. The average total cost (ATC) of production is

$$\text{ATC} = \frac{C}{Q}$$

$$\text{ATC} = \frac{25 + Q + 0.5Q^2}{Q}$$

$$\text{ATC} = (25/Q) + 1 + 0.5Q.$$

When producing four units of output, this cost is

$$\text{ATC} = (25/4) + 1 + 0.5(4)$$

$$\text{ATC} = 6.25 + 1 + 2$$

$$\text{ATC} = \$9.25.$$

Therefore, the profit margin is –$0.25, from the $9 price minus the $9.25 average total cost of production.

Total profit is –$1, from the profit margin (–$0.25) multiplied by the profit-maximizing quantity (4 units).

If the monopoly shuts down, then its losses equal fixed costs ($25). If fixed costs are greater than the monopoly's losses, then the monopoly should operate in the short run. Since profit is –$1 and fixed costs are $25, the monopoly should operate.

1.10 The revenue function is: $R(Q) = p \cdot Q = (10 - \frac{Q}{1000})Q;$ then the F.O.C. is:

$$\frac{dR(Q)}{dQ} = 10 - \frac{Q}{500} = 0 \quad \Rightarrow \quad Q^* = 5000 \quad \Rightarrow \quad p^* = 10 - \frac{Q^*}{1000} = 5.$$

The revenue-maximizing price is $5.

The sum of the cable car revenues and the economic impact is:

$$S(Q) = R(Q) + EI(Q) = \left(10 - \frac{Q}{1000}\right)Q + 4Q = \left(14 - \frac{Q}{1000}\right)Q.$$

The F.O.C. is:

$$\frac{dS(Q)}{dQ} = 14 - \frac{Q}{500} = 0 \quad \Rightarrow \quad Q^* = 7000 \quad \Rightarrow \quad p^* = 10 - \frac{Q^*}{1000} = 3.$$

The optimal price is $3.

2.4 Given that Apple's marginal cost was constant, its average variable cost equaled its marginal cost, \$200. Its average fixed cost was its fixed cost divided by the quantity produced, $736/Q$. Thus, its average cost was $AC = 200 + 736/Q$. Because the inverse demand function was $p = 600 - 25Q$, Apple's revenue function was $R = 600Q - 25Q^2$, so $MR = dR/dQ = 600 - 50Q$. Apple maximized its profit where $MR = 600 - 50Q = 200 = MC$. Solving this equation for the profit-maximizing output, we find that $Q = 8$ million units. By substituting this quantity into the inverse demand equation, we determine that the profit-maximizing price was $p =$ \$400 per unit, as the figure shows. The firm's profit was $\pi = (p - AC)Q = [400 - (200 + 736/8)]8 =$ \$864 million. Apple's Lerner Index was $(p - MC)/p = [400 - 200]/400 = \frac{1}{2}$.

According to Equation 11.11, a profit-maximizing monopoly operates where $(p - MC)/p = -1/\varepsilon$. Combining that equation with the Lerner Index from the previous step, we learn that $\frac{1}{2} = -1/\varepsilon$, or $\varepsilon = -2$.

3.2 A monopoly will maximize profits by picking a price, p, such that

$$p = \frac{MC}{\left(1 + \frac{1}{\varepsilon}\right)}.$$

If marginal cost equals m, then

$$p_1 = \frac{M}{\left(1 + \frac{1}{\varepsilon}\right)}.$$

The per-unit tax raises marginal cost by \$1, so

$$p_2 = \frac{M + 1}{\left(1 + \frac{1}{\varepsilon}\right)}.$$

In turn,

$$p_2 - p_1 = \frac{M + 1}{\left(1 + \frac{1}{\varepsilon}\right)} - \frac{M}{\left(1 + \frac{1}{\varepsilon}\right)}$$

$$p_2 - p_1 = \frac{1}{\left(1 + \frac{1}{\varepsilon}\right)}.$$

If ε equals –2, then the change in price is

$$p_2 - p_1 = \frac{1}{\left(1 + \frac{1}{-2}\right)} = \$2.00.$$

7.5 To get the marginal expenditure curve, multiply the supply curve by Q, then take the derivative with respect to Q. $ME = 10 + 2Q$. The monopsony determines the amount of the factor to purchase by setting marginal expenditure equal to demand:

$$ME_L = D$$
$$10 + 2Q = 50 - Q$$
$$3Q = 40$$
$$Q = 40/3.$$

Substituting this quantity into the supply equation,

$$P = 10 + Q$$
$$P = 10 + 40/3$$
$$P = 70/3.$$

The competitive equilibrium is $Q = 20$, $p = 30$, obtained by setting supply equal to demand ($10 + Q = 50 - Q$).

A question on monopoly and tax

Assume that a monopolist faces an inverse demand function $p = p(Q)$ and a cost function $C(Q)$.

(a) Derive the necessary (first order) and sufficient (second order) conditions for the monopolist's profit maximizing level of output.
(b) Now assume that the government imposes a specific tax of τ dollars per unit on the monopoly. Derive the new necessary and sufficient conditions in case of this specific tax.
(c) Compare the outcomes of the equilibriums in (a) and (b).
(d) What happens to the level of output and price if specific tax rises (show your derivations).
(e) Show diagrammatically the welfare under monopoly before and after the specific tax. In this case show the deadweight loss due to the specific tax (you may assume a linear demand function and an upward sloping linear marginal cost curve).
(f) Review the graph and the example on page 380 (text book).