



# Exam Review

## Writing Exams

**State-Graph Construction**

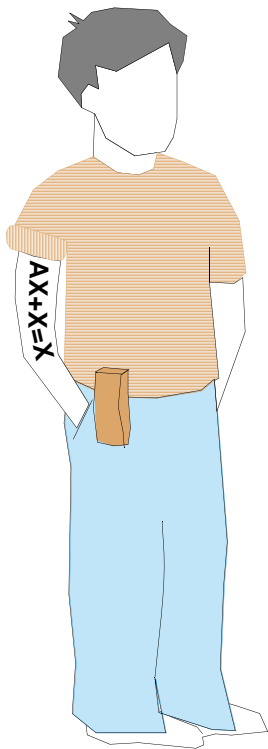
**Small Boolean Problems**

**Multiple Outputs,**

**Synchronous Circuits**

**Asynchronous Circuits**

**Races, Cycles and State Assignment**



1 hr 45 min





## Writing Exams

**Time yourself:** 180 min, 100%  
 150 min + 1/2 hour for cleanup  
 15 min for 10%.  
 3 min for 2%.

**Give Up:** If you can't do it, move on.

**RTFQ:** Read The Foolish Question;  
 read it again DAMN IT.

What was asked for?

- Just the state table?  
 Don't make K-maps.
- Wants a circuit.  
 Then draw it!
- Ask for P of S ( $\Pi$  of  $\Sigma$ )  
 Don't give it  
 or

$$A + BCD + \overline{AD} + Q \quad (\Sigma \text{ of } \Pi)$$

$$(A + BCD)(\overline{AD} + Q)$$





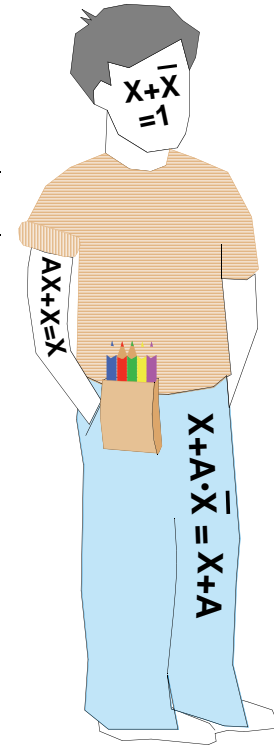
# Boolean Algebra

Exam Ready

Look for simple methods:

Don't do all algebra questions by multiplying out.

$$(X+A+B)(X+A+C)(BC+A)=$$





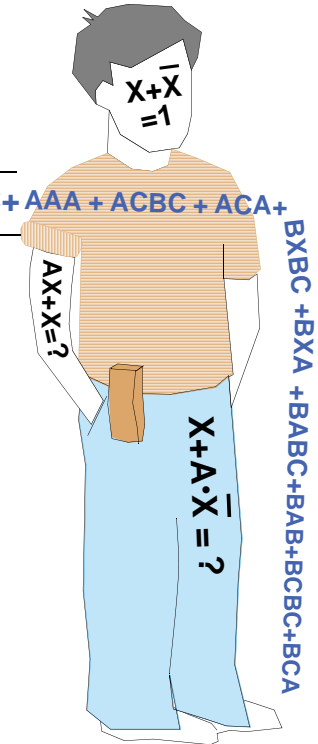
# Boolean Algebra

Exam Unready

Look for simple methods:

Don't do all algebra questions by multiplying out.

$$(X+A+B)(X+A+C)(BC+A) = XXABC + XXA + XABC + XAA + XCBC + XCA + AXBC + AXA + AABC + AAA + ACBC + ACA +$$





# Boolean Algebra

Exam Ready

Look for simple methods:

Don't do all algebra questions by multiplying out.

$$(X+A+B)(X+A+C)(BC+A)=$$

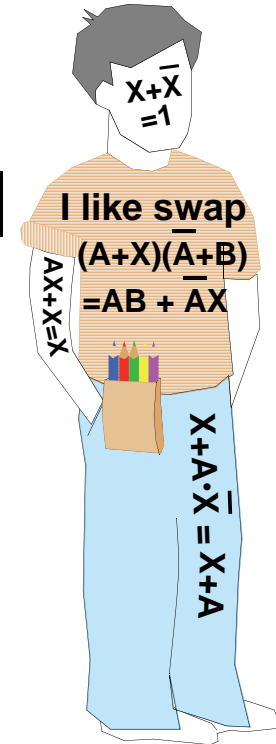
Simplify at each step:

*Right elbow:*  $X + \text{anything} \cdot X = X$

*Left leg:*  $X + \text{anything} \cdot \bar{X} = X + \text{anything}$

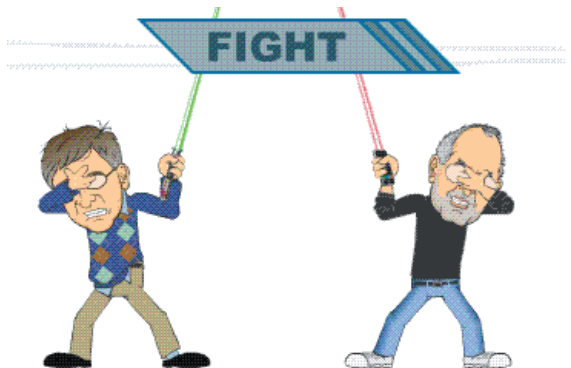
*Nose:*  $X + \bar{X} = 1$

*Shirt:* Swap



Maybe Use Dual

But don't be Duel Happy



Duels don't solve all problems



# Boolean Algebra

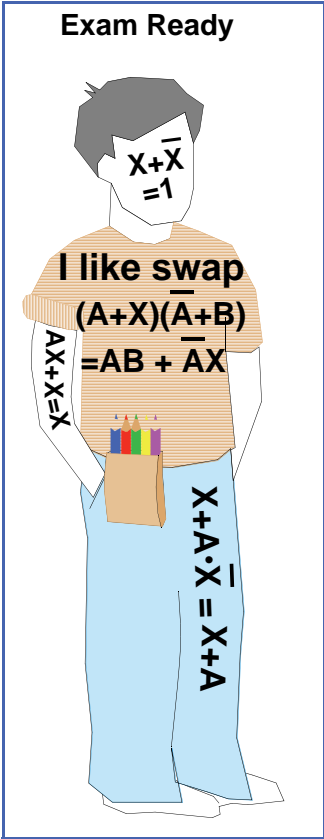
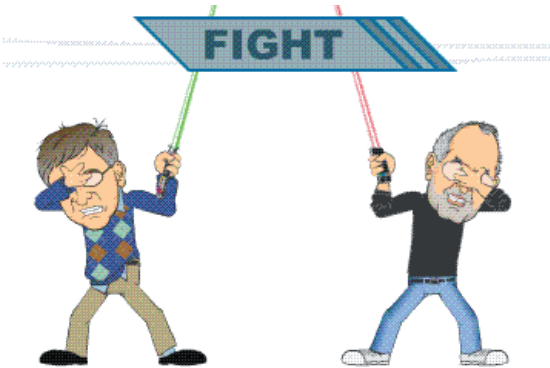
## Dual Usage

- Not for everything

- Use the dual for:.

- (1) Simplifying things with many brackets
  - but look for swap first
- (2) Factoring,
  - but simplify first

Lots of brackets



$$(X+A+B)(X+A+C)(BC+A) =$$

$= (X+A+B)(X+A+C)((BC)+A)$ $F_{dual} = XAB+XAC+(B+C)A$ $= XAB+XAC+BA+CA$ $= BA+CA$ $(F_{dual})_{dual} = F = (B+A)(C+A)$	<p><i>Bracket ALL ANDs</i></p> <p><i>Take dual</i></p> <p><i>Dist 1</i></p> <p><i>BA+anything·BA = BA</i></p> <p><i>CA+anything·CA = CA</i></p>
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# Boolean Algebra

First look for simplifications.

Are there three cm space for the answer?

Do you need thirty? **No! see simplification** =>

**Boolean** ( 1% for stating rules and map names.)

If you use algebra, indicate the rules used at the right side of each line.

1.5%

a) Simplify  $f = BC + BC(\bar{A} + BD)$   
 =

rule:

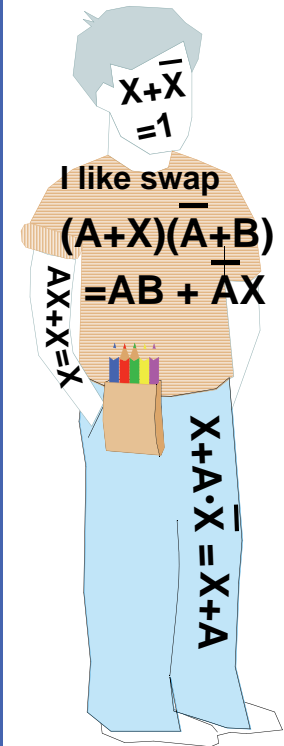
2%

b) Simplify  $g = BC + \bar{B}C(\bar{A} + \bar{D})\bar{A}$   
 =

2%

c) Simplify  $h = (A + B) + \bar{A}\bar{B}\bar{E}\bar{F}\bar{G}$   
 =

Exam Ready





## Common Mistakes

1. Not using  $X + XA = X$  to simplify expressions the **FIRST THING**.
2. Not reducing using  $X + \bar{X}R = X + R$ .

3. Saying  $\overline{a \cdot b}$  is the same as  $\bar{a} \cdot \bar{b}$

$$A \cdot B + \bar{A} \cdot \bar{B} \cdot \text{any} = A \cdot B + \text{any} \quad (\text{S})$$



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4. Saying an expression is equal to its dual.

$$F = A \cdot B \cdot C + (A \cdot C + D) \cdot E$$

$$F = \{A \cdot B \cdot C\} + [\{(A \cdot C) + D\} \cdot E]$$

$$F = \{A + B + C\} \cdot [(\{A + C\} \cdot D) + E] \quad \textit{take dual}$$



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$$\begin{aligned}
 F &= A \cdot B \cdot C + (A \cdot C + D) \cdot E \\
 F &= \{A \cdot B \cdot C\} + [\{(A \cdot C) + D\} \cdot E] \\
 F_{\text{dual}} &= \bar{F} = \{A + B + C\} \cdot [\{(A + C) \cdot D\} + E] \quad \text{take dual}
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*take dual*

5. Saying  ~~$X + 1 = X$~~

$$\text{Simplify } g = (1 + X)(A\bar{X})$$

Everybody knows better than this, but they still do it.



## Common Mistakes

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$$\text{Simplify } g = (1 + X)(\bar{A}\bar{X}) = (1)(\bar{A}\bar{X}) = \bar{A}\bar{X}$$

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6. When you take dual, or general Demorgan, do not put in the brackets in your head.

$$F = (AB + C) + DE \implies F_{\text{dual}} = (A+BC)(D+E) ?$$



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$$F = (AB + C) + DE \implies F_{\text{dual}} = \overline{(A+BC)(D+E)}$$

$$F = (AB + C) + DE = ((AB) + C) + (DE) \implies F_{\text{dual}} ((A+B)C)(D+E) \quad (D2)$$



## Common Mistakes

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7. Not knowing D2..  $X + AB = (X + A)(X + B)$

$$\text{Factor } h = \bar{A} + BC\bar{E}$$



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7. Not knowing D2.  $X + AB = (X + A)(X + B)$

$$\begin{aligned}
 \text{Factor } h &= \bar{A} + B\bar{C}\bar{E} & h &= (\bar{A} + B)(\bar{A} + \bar{C}\bar{E}) \\
 & & &= (\bar{A} + B)(\bar{A} + \bar{C})(\bar{A} + \bar{E})
 \end{aligned}$$



# Boolean Algebra Practice

1. Simplify  $(A + C + \overline{C} \cdot \overline{A})$

2. Simplify  $\overline{CD} + CE$

3. Take the dual of  $F = (A + QB)(X + Y) + \overline{Z}$

4. Factor  $X + B\overline{C}$

5. Factor  $X + ABC\overline{C}$

6. Find the dual of Simply  $G = (A + 1)(B + C) + \overline{D}$

7. Factor  $AB + CD$

8. Construct with MUXs  
Extra hardware lowers mark

The diagram shows a 2-to-1 multiplexer. The top-left input is labeled 'A' and the top-right input is labeled 'A-bar'. The output is labeled with the expression  $AC + \overline{A}D + DCB$ . Two lines extend from the bottom of the multiplexer symbol, representing the data outputs.



# Boolean Algebra Practice

1. Simplify  $(A + C + \overline{C} \cdot \overline{A})$

$(A + C + \overline{A}) = (A + \overline{A} + C) = 1$  rule  $C + \overline{C}A = C + A$   
rule  $1 + A = 1$

2. Simplify  $\overline{C}D + CE$

$\overline{C} + \overline{D} + CE = \overline{C} + \overline{D} + E$  rule DeMorg  
rule  $C + \overline{C}E = C + E$

3. Take the dual of  $F = (A + QB)(X + Y) + \overline{Z}$

$F = \{(A + (QB))(X + Y)\} + \overline{Z}$  bracket ANDs  
 $F_{dual} = \{(A(Q+B)) + (XY)\} \overline{Z}$

4. Factor  $X + B\overline{C}$

$(X + B)(X + \overline{C})$  rule D2

5. Factor  $X + ABC\overline{C}$

$(X + AB)(X + \overline{C})$  rule D2  
 $= (X + A)(X + B)(X + \overline{C})$  rule D2

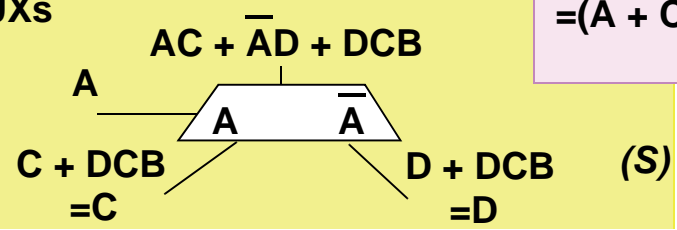
6. Find the dual of Simply  $G = (A + 1)(B + C) + \overline{D}$

$\{(A + 1)(B + C)\} + \overline{D}$  bracket ANDs  
 $G_{dual} = \{(A0) + (BC)\} \overline{D}$  dual  $1 \leftrightarrow 0$   
 $= BCD\overline{D}$  rules  $A0=0; x+0=x$

7. Factor  $AB + CD$

$(AB + C)(AB + D)$  rule D2  
 $= (A + C)(B + C)(A + D)(B + D)$  rule D2

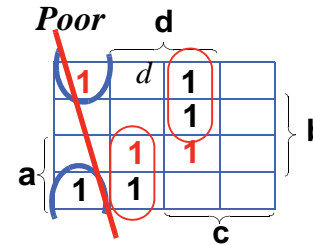
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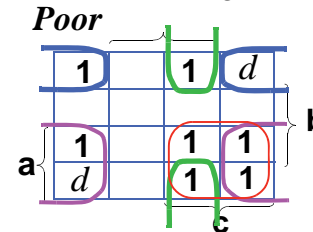


## K-Maps; Common Errors

- **Check Your Map Entries**  
One variable in the wrong square, you're toast!
  
- **Check for Wrap Around,**  
and Wider Wrap Around
  
- **Using algebra after map simplification.**  
Usually no help:  
A K-map gives the simplest  $\Sigma$  of  $\Pi$  circuit.

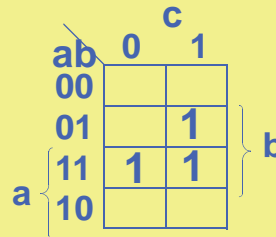


*What is poor?*



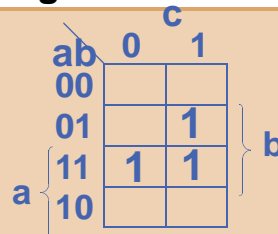
*You find two better loops*

Simplify  $ab + \bar{a}bc$   
to the simplest  $\Sigma$  of  $\Pi$  circuit



**A bit of factoring might lower gate count.**

Simplify  $ab + \bar{a}bc$   
to the simplest circuit  
Minimum gates  
and letters.

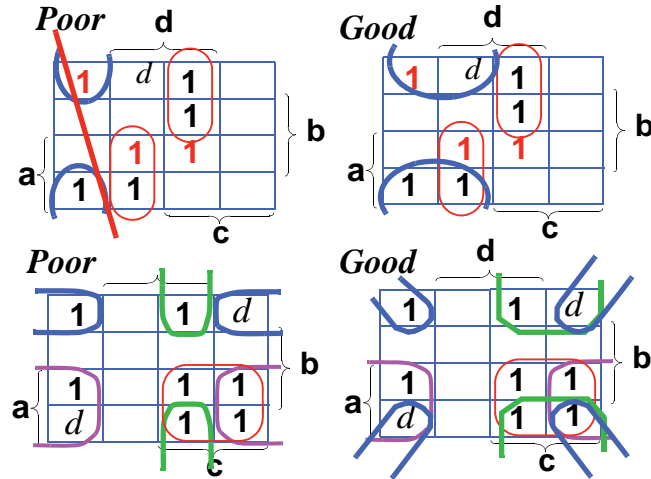


$ab + ac$



## K-Maps; Common Errors

- **Check Your Map Entries**  
One variable in the wrong square, you're toast!
- **Check for Wrap Around, and Wider Wrap Around**
- **Using algebra after map simplification.**  
Usually no help:  
A K-map gives the simplest  $\Sigma$  of  $\Pi$  circuit. (If you loop correctly)



**Simplify  $ab + \bar{a}bc$  to the simplest  $\Sigma$  of  $\Pi$  circuit**

ab	0	1
00		
01		1
11	1	1
10		

**$ab + bc$**       **3 gates**  
**4 letters**

**$ab + bc = (ab + b)(ab + c)$     (D2)**  
 **$= (b)(ab + c)$                     (S)**  
 **$= (ab + bc)$                          (D1)**  
**So what**

**A bit of factoring sometimes lowers gate count.**

**Simplify  $ab + \bar{a}bc$  to the simplest circuit, Minimum gates and letters.**

ab	0	1
00		
01		1
11	1	1
10		

**Use D1**

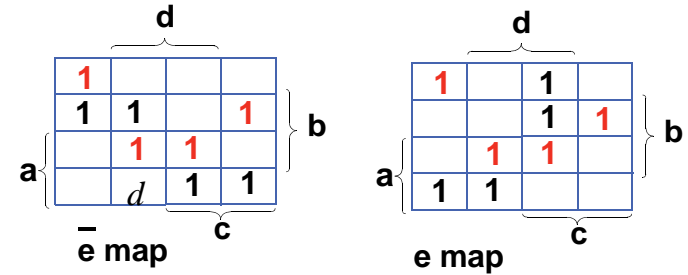
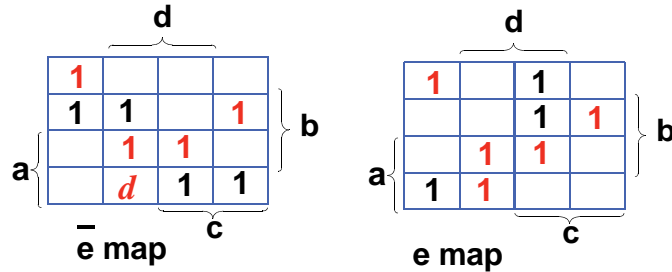
**$ab + ac = a(b + c)$**       **2 gates**  
**3 letters**



## K-Maps; Common Errors

- Do not confuse **5-variable maps** and **dual-output maps**.

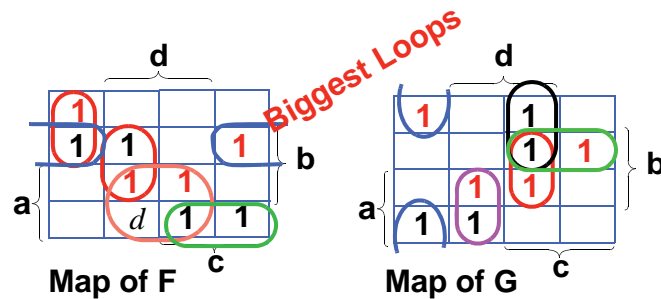
**5-variable** Bigger is always Better,  
 Enlarge loops to use both maps.



### Multiple output

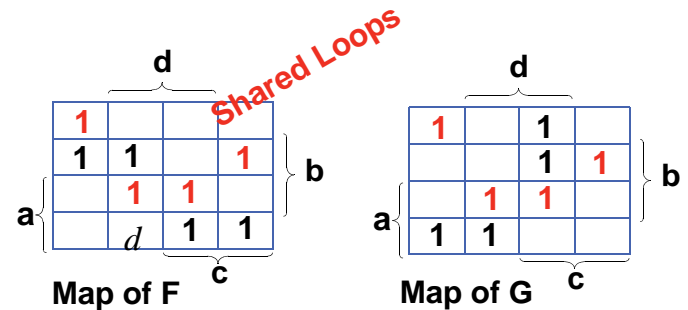
Share loops, saves gates  
 Smaller is Sometimes Simpler.

- sharing lowers gate count.
  - sharing increase may letter count.
- } Balance these



**12 gates, none shared**

28 letters



**9 gates, 3 shared**

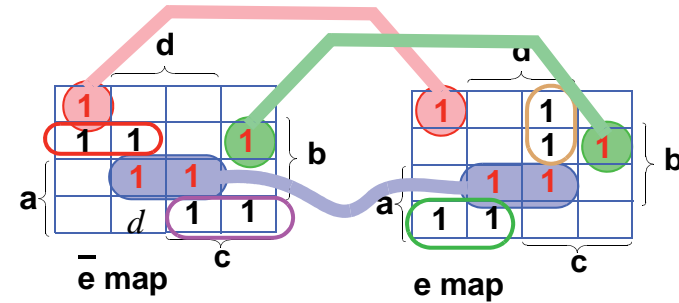
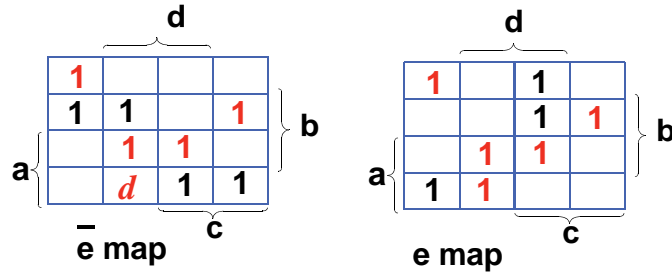
25 letters



## K-Maps; Common Errors

- Do not confuse **5-variable maps** and **dual-output maps**.

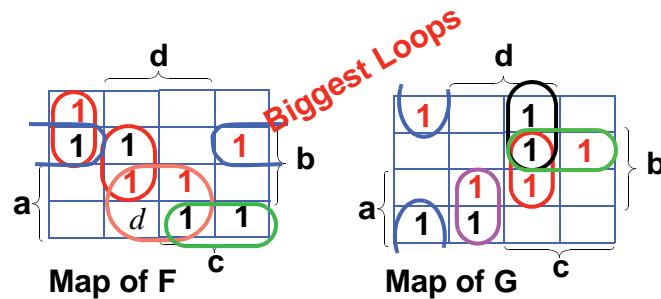
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### Multiple output

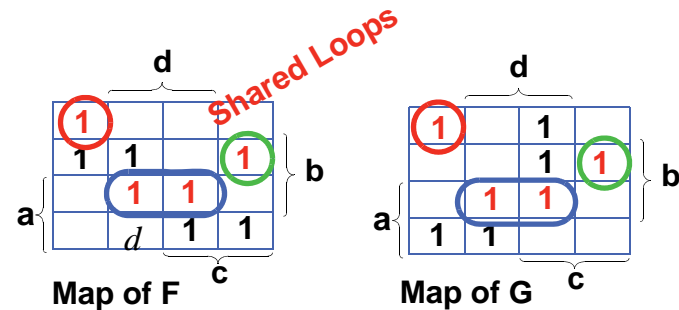
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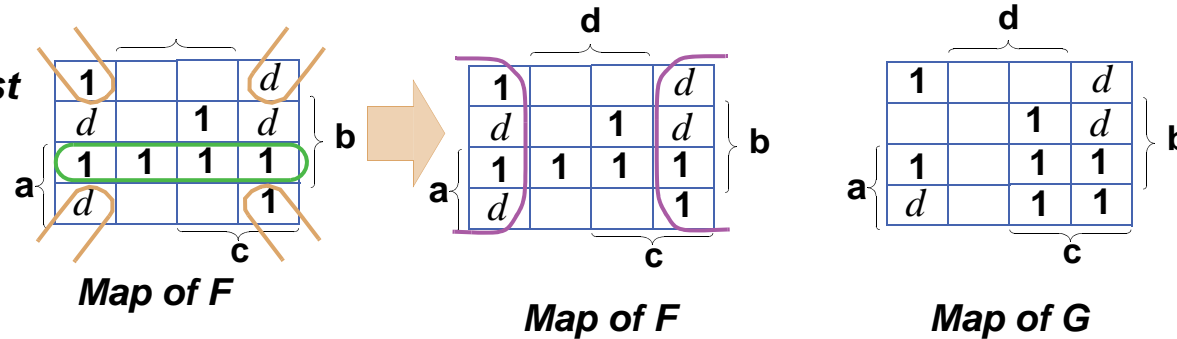
25 letters



## K-Maps; Common Errors

### • Multiple Output Mapping Rules

**Do Half-Maps First  
(Except for PLAs)**



**Then do lone "1"s**

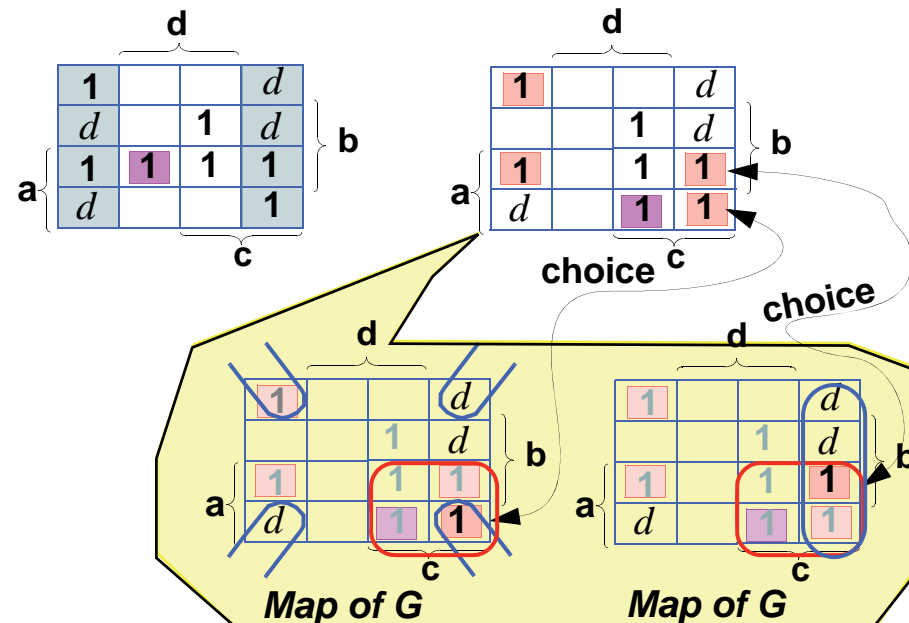
\* **No friend** 1

\* **Only friends gone to dark side** 1

\* **No friends looping**

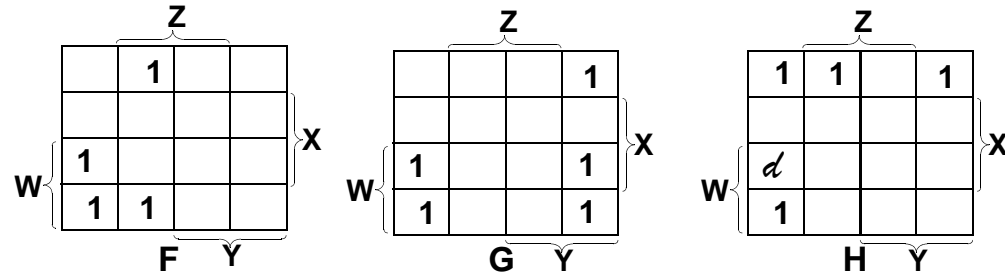
*May give a choice.*

*Do squares with no choice first*



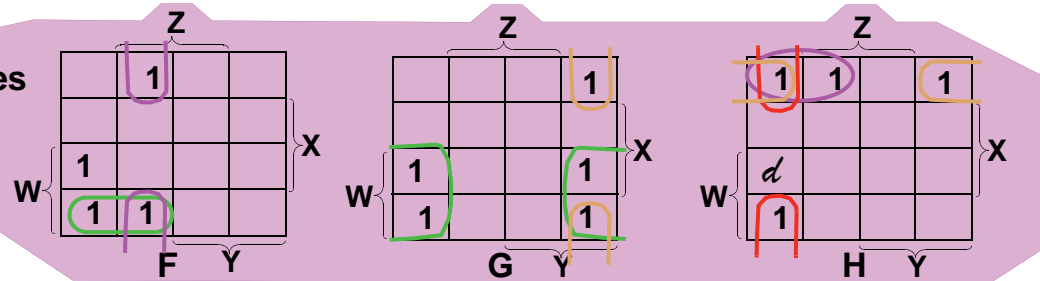


Find the minimum  $\Sigma$  of  $\Pi$



Poor Method

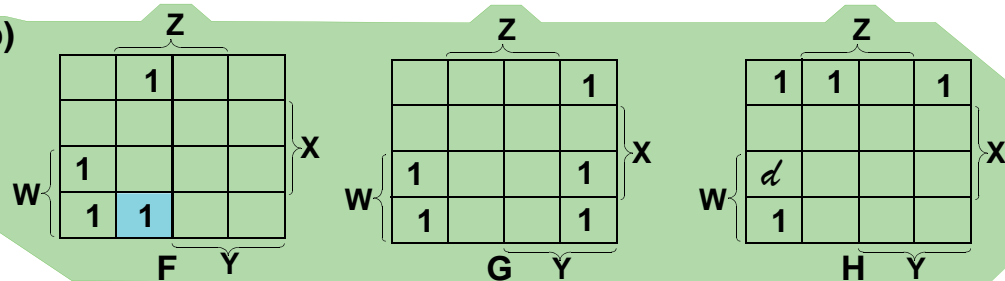
Largest circles  
7 AND terms



Better Method

(1) Circle half maps (none)

(2) No Friends, ("1"s on only one map)



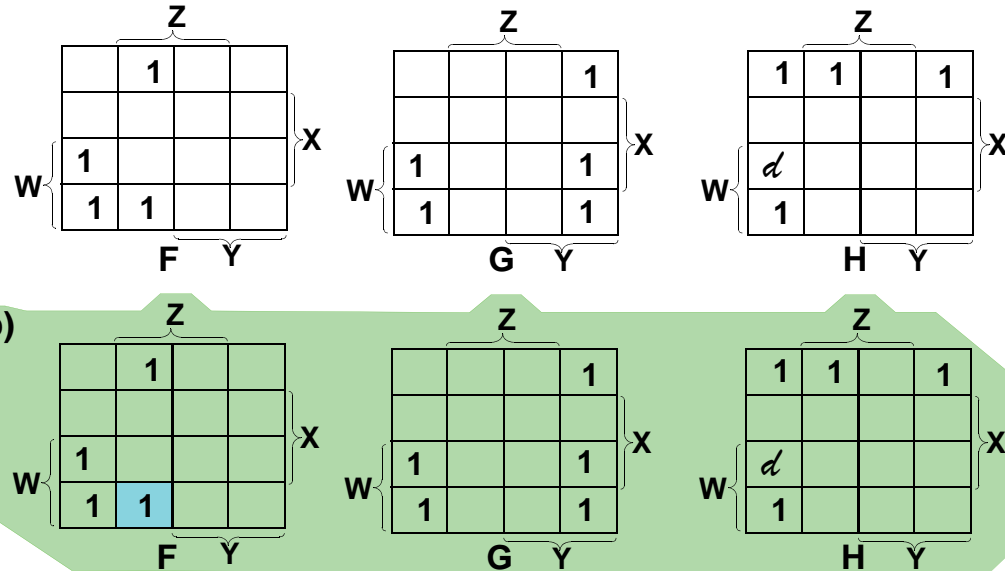


Find the minimum  $\Sigma$  of  $\Pi$

Better Method

(1) Circle half maps (none)

(2) No Friends, ("1"s on only one map)

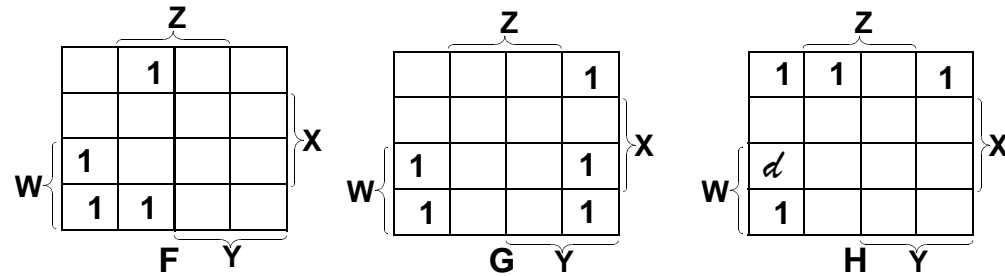




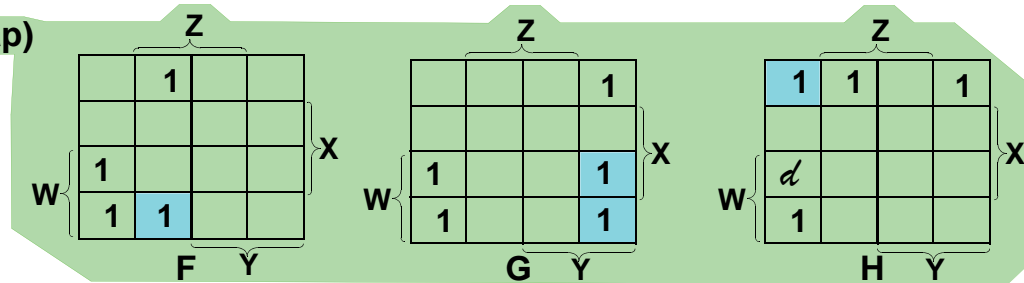
# Find the minimum $\Sigma$ of $\Pi$

## Better Method

- (1) Circle half maps (none)
- (2) No Friends, ("1"s on only one map)

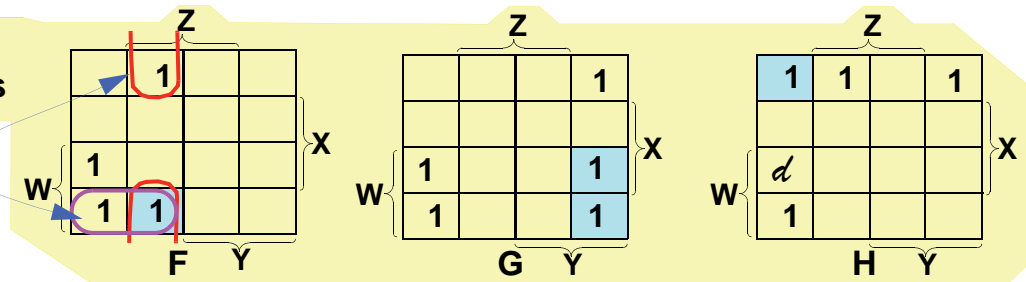


Loop Friendless "1"s, they will never share



Unfortunately you have choices; several ways to loop some squares

Choices



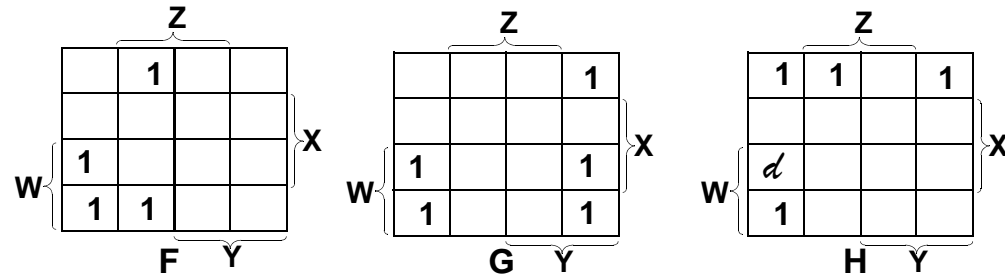


Sketch the Output

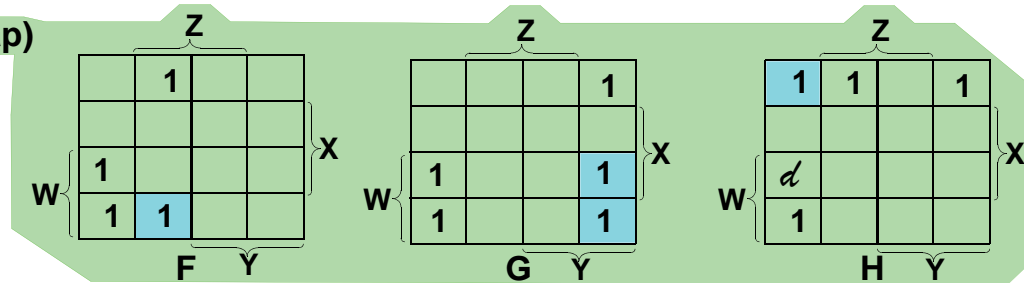
# Find the minimum $\Sigma$ of $\Pi$

## Better Method

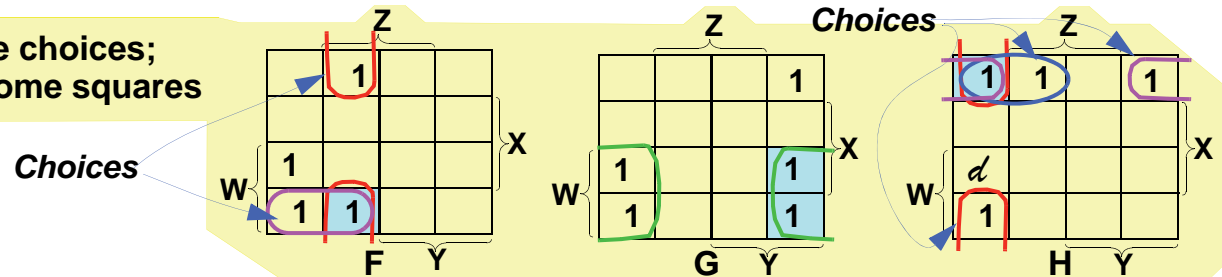
- (1) Circle half maps (none)
- (2) No Friends, ("1"s on only one map)



Loop Friendless "1"s, they will never share



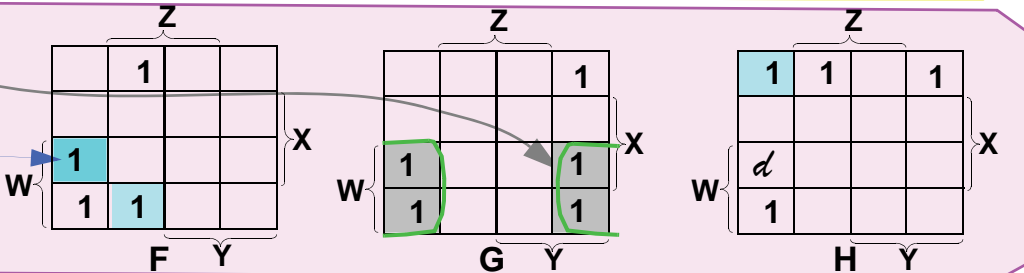
Unfortunately you have choices; several ways to loop some squares



## First

Loop square with no choices

Leaves new lone 1  
With one choice  
Loop it





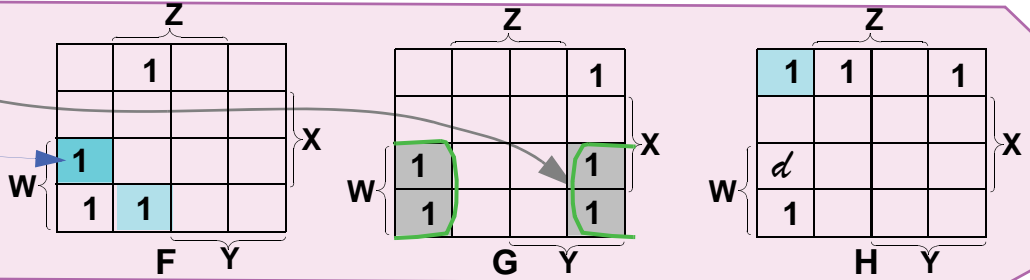
# Common Mistakes: ■

# Common Mistakes

(Repeat from last slide)

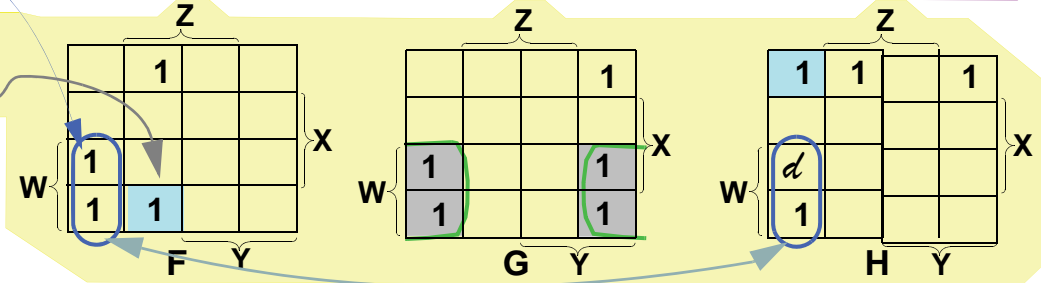
Loop square with no choices

Leaves new lone 1  
No choice  
So loop it



New loop removes choice

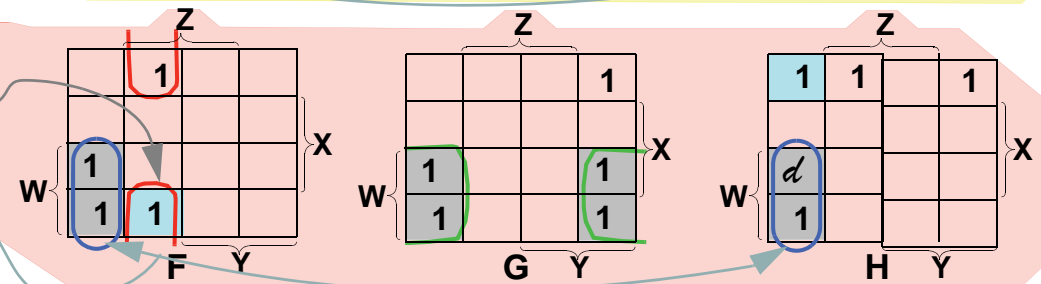
No more choice



New loop removes choice

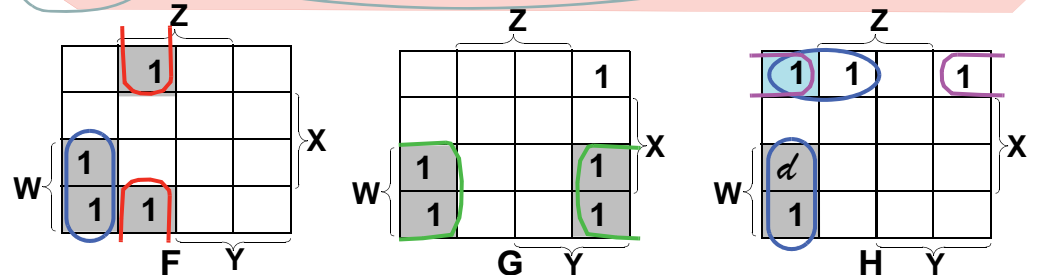
No more choice

Loop can be shared



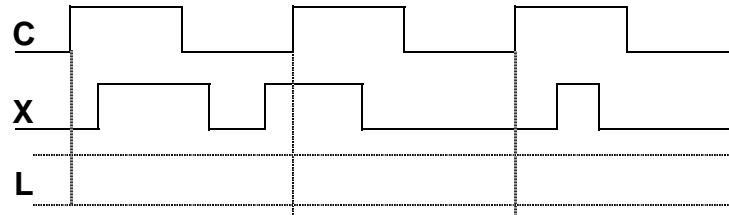
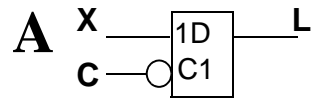
## You Finish it

5 ANDS

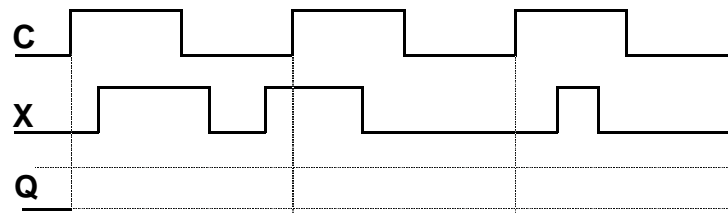
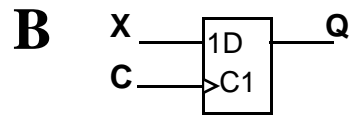




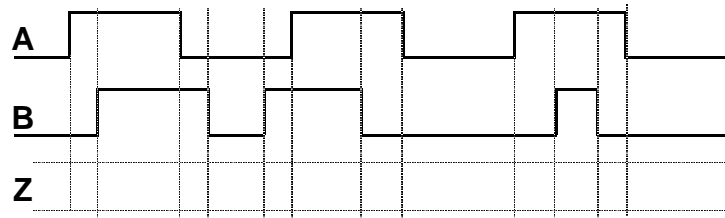
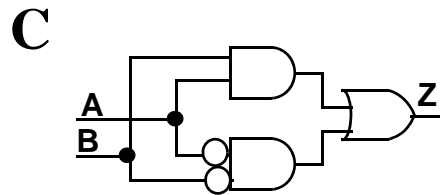
### Example.1-32a Sketch the Output Waveform



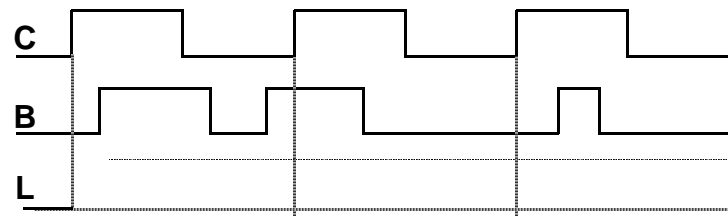
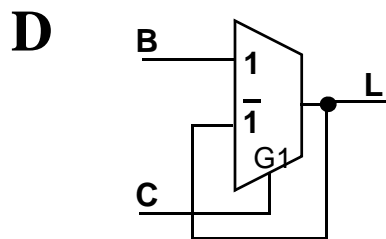
*D Latch (inverted clock)  
Transparent when?  
Latched when?*



*D Flip-Flop  
\_\_\_\_\_ edge triggered*



*Equivalent to  
\_\_\_\_\_ gate*

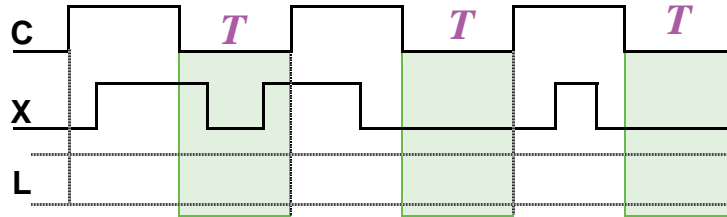
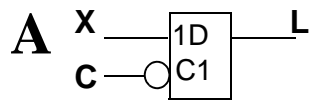


*D Latch  
Transparent when \_\_\_\_  
Latched when \_\_\_\_*

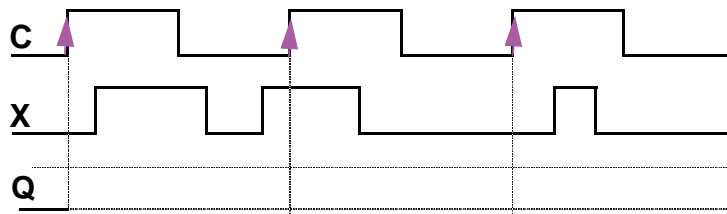
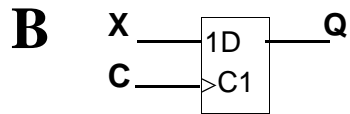
**Latches etc from Dec '96.**



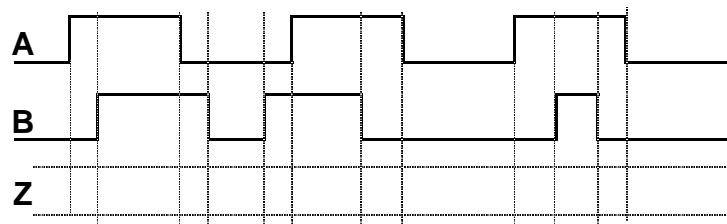
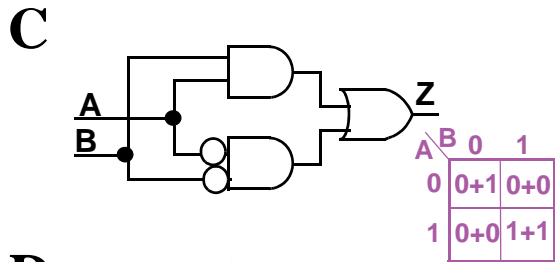
### Example.1-32b Sketch the Output Waveforms



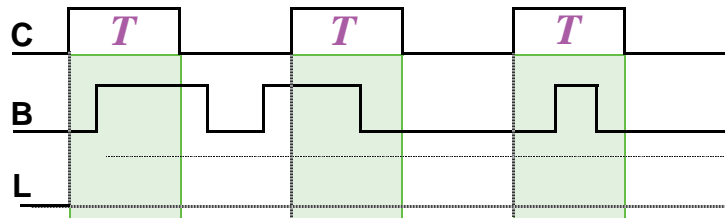
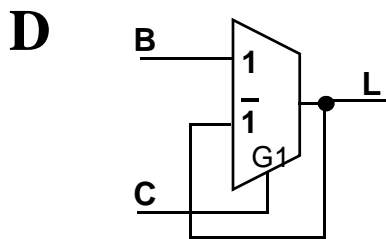
*D Latch (inverted clock)  
Transparent when C low  
Latched when C high*



*D Flip-Flop  
Rising edge triggered*



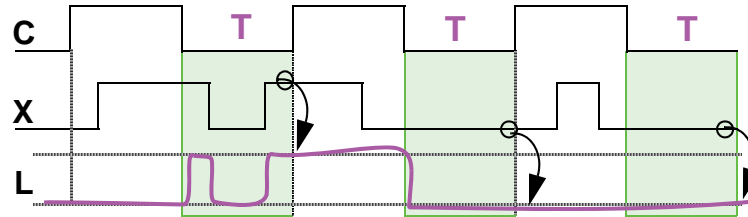
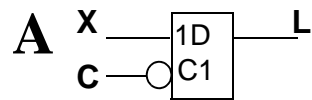
*XNOR gate*



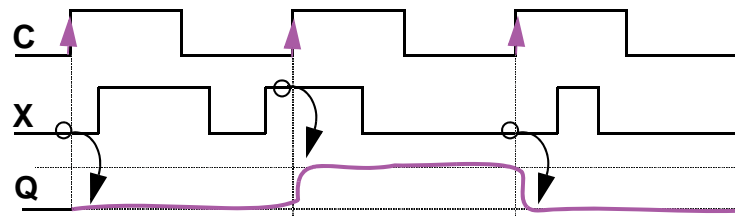
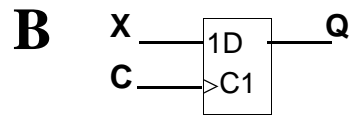
*D Latch  
Transparent when C=1  
Latched when C=0*



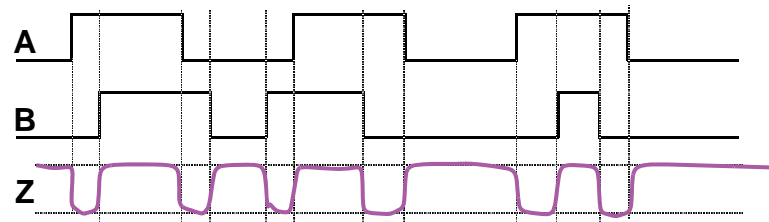
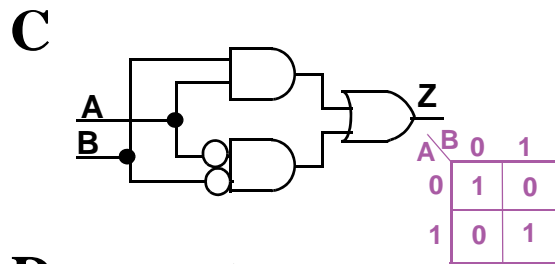
### Example.1-32c Sketch the Output Waveforms



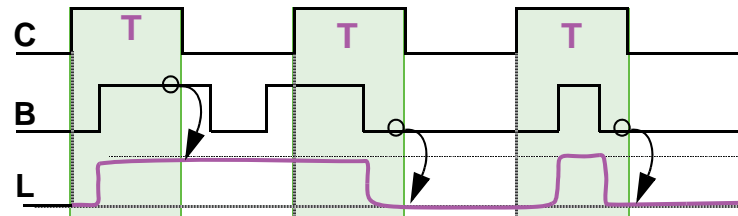
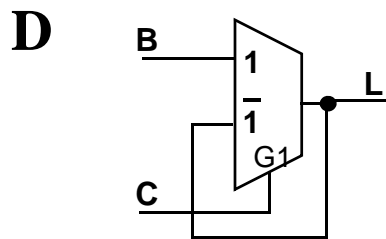
*D Latch (inverted clock)  
Transparent when clock low  
Latched when high*



*D Flip-Flop  
Rising edge triggered*



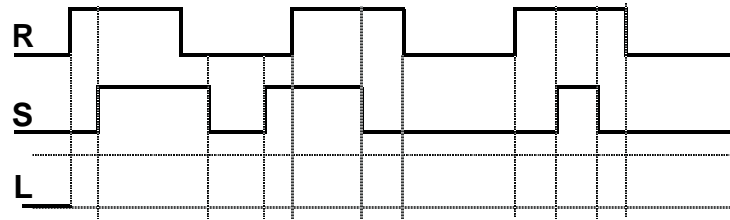
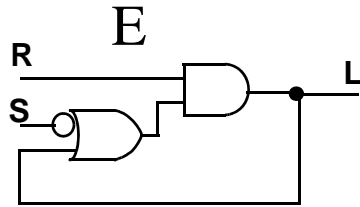
*XNOR gate*



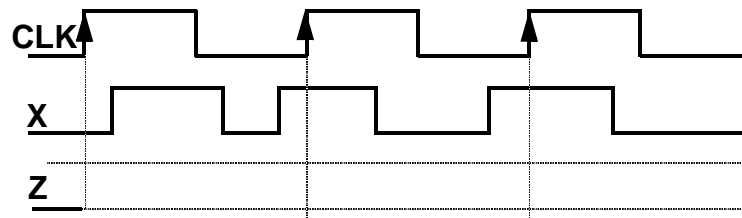
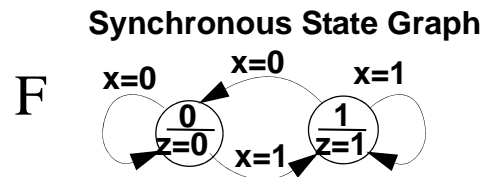
*D Latch  
Transparent when C=1  
Latched when C=0*



### Example.1-33a Sketch the Output Waveforms

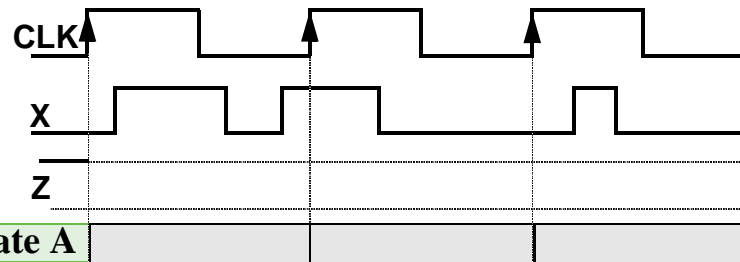
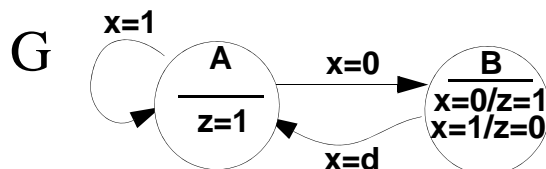


*Reset(L)-Set(L) Latch*  
*Reset*  $L = 0$  when  $R=0$   
*Set*  $L = 1$  when  $RS=10$   
*Store output when*  $SR=11$



*State=Z*

### Synchronous State Graph



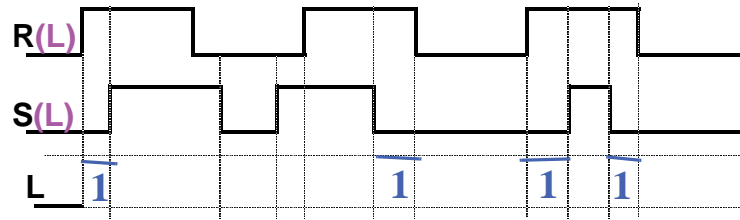
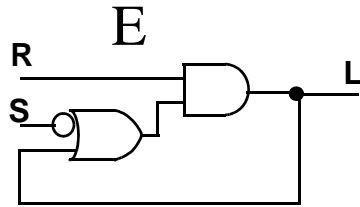
State A

Show state as well as Z

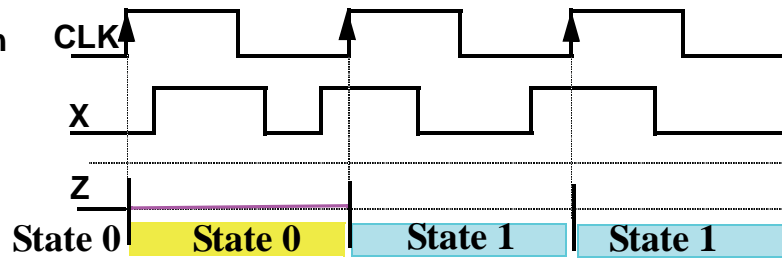
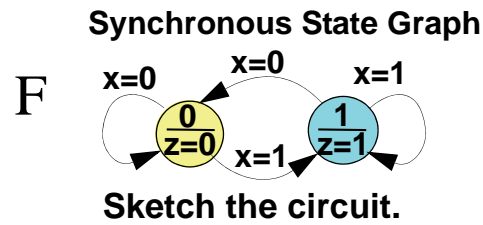
*State A*  
*State B*



**Example.1-33b Sketch the Output Waveforms**

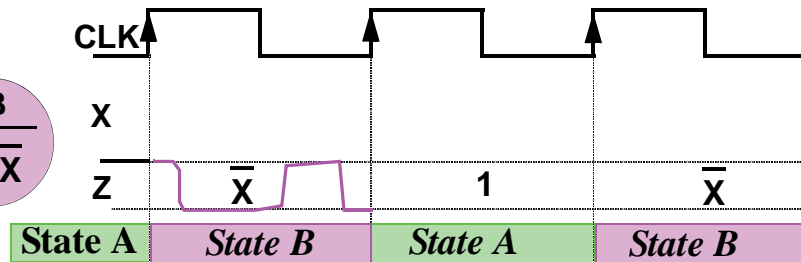
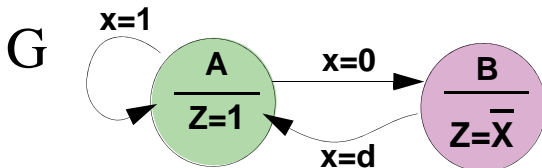


*Reset(L)-Set(L) Latch*  
*Set L = 1 when RS=10*  
*Reset L = 0 when R=0*  
*Store output when SR=11*  
*Set dominant*



*State=Z*  
*State only changes on the clock edge*

**G** Synchronous State Graph

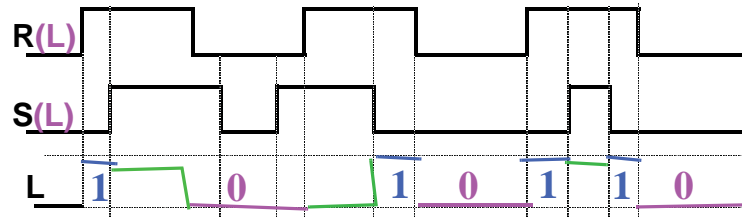
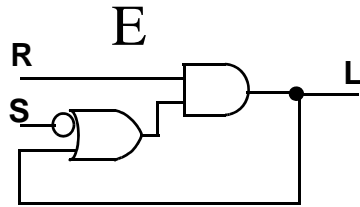


Show state as well as Z

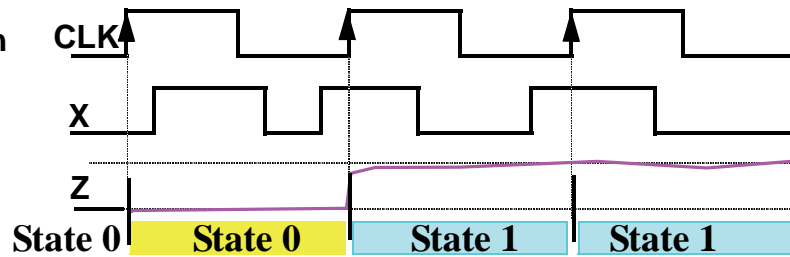
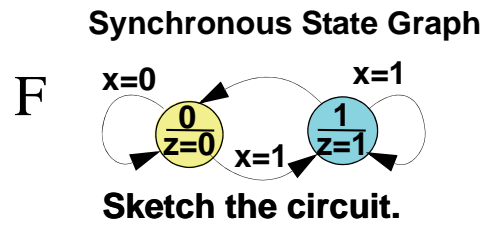
*State A*  
*State B*



**Example.1-33c Sketch the Output Waveforms**

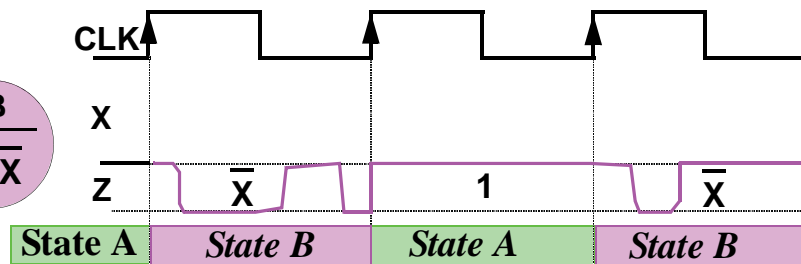
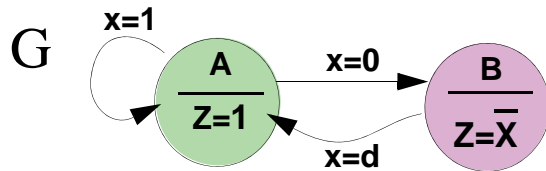


*Reset(L)-Set(L) Latch*  
*Set L = 1 when RS=10*  
*Reset L = 0 when R=0*  
*Store output when SR=11*  
*Set dominant*



*State=Z*  
*State only changes on the clock edge*

**Synchronous State Graph**



**Show state as well as Z**

*State A*  
*State B*



# Races and Cycles

## Race

1 input change  
changes 2 state variable (or more)



### Stable State

Next state = State

Always circle stable states (10)

When looking for races

Start at stable states

State A B	Next State a <sup>+</sup> b <sup>+</sup>	
	s=0	s=1
00	00	11
01	00	01
11	10	11
10	01	10

Non critical

## Critical Race

Race cause final state to be wrong

## Non Critical Race

Race ends in correct final state

## Race between transient states

Race cause final state to be wrong

## Who cares Race

Race on never used transition

## Cycle

Oscillation

State A B	Next State a <sup>+</sup> b <sup>+</sup>	
	s=0	s=1
00	00	01
01	10	10
11	10	11
10	11	10

Cycle



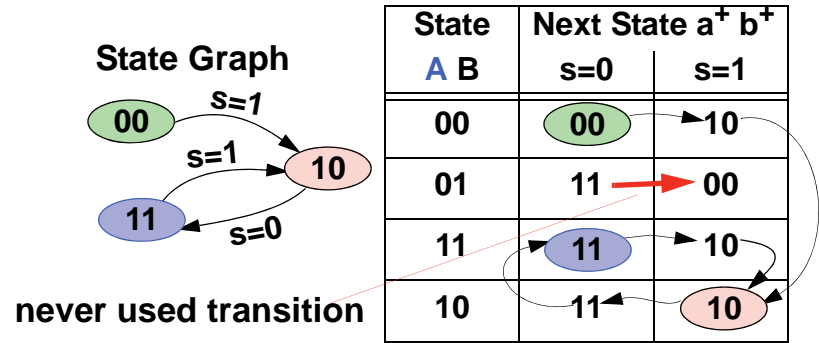
# Races

## Who cares Race

Race on never used transition

Must start looking from a stable state

Never find this race

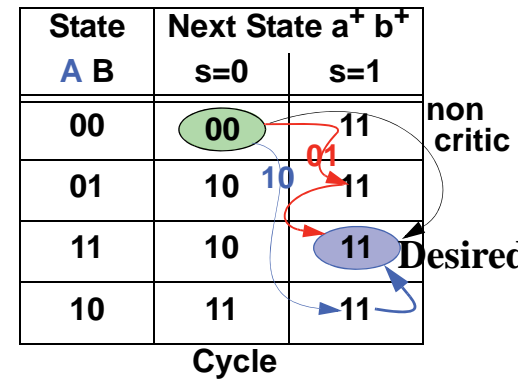


## Non Critical Race

Race ends in correct final state

First change 00->11 or 00->01 or 00->10

Race ends in correct final state

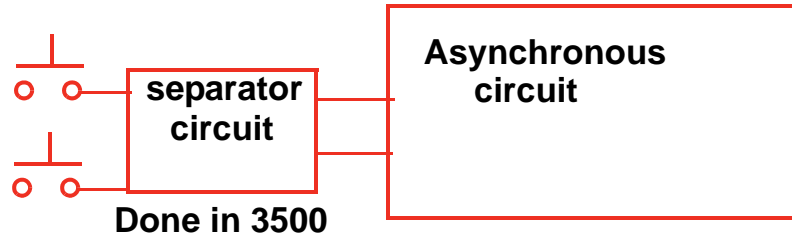




# Races In Asynchronous State Tables

## .Double Input Changes Are Not Allowed

If two inputs change at once, build an input circuit to separate them



### Allowed input changes

starting from input 00

state	next state $g^+ h^+$			
GH	xy=00	xy=01	xy=11	xy=10
00	00	01	11	10
01				
11				
10				

Diagram showing state transitions from 00 to 01, 11, and 10. A green circle highlights the 00 state. Red arrows indicate transitions to 01, 11, and 10. A green arrow shows a transition from 00 to 01.

starting from input 01

state	next state $g^+ h^+$			
GH	xy=00	xy=01	xy=11	xy=10
00				
01		01		
11				
10				

Diagram showing state transitions from 01 to 01, 11, and 10. A purple circle highlights the 01 state. Red arrows indicate transitions to 01, 11, and 10.

starting from input 11

state	next state $g^+ h^+$			
GH	xy=00	xy=01	xy=11	xy=10
00				
01				
11			11	10
10				

Diagram showing state transitions from 11 to 11 and 10. A blue circle highlights the 11 state. Blue arrows indicate transitions to 11 and 10.

starting from input 10

state	next state $g^+ h^+$			
GH	xy=00	xy=01	xy=11	xy=10
00				
01				
11				11
10				

Diagram showing state transitions from 10 to 11 and 10. A red circle highlights the 11 state. Red arrows indicate transitions to 11 and 10.

### When Checking For Races

- 1) Always start at a stable state
- 2) Check both sides
- 3) Remember wrap around



## Find Races In Asynchronous State Tables

### Example.1-1 .Find Races and Cycles

#### a) State Tables to be checked

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab = 01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

Double input changes never happen.



## Find Races In Asynchronous State Tables

### .Find Races and Cycles

#### a) State Tables to be checked

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab = 01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

#### b) Circle stable states

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab = 01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

Double input changes never happen.



# Find Races In Asynchronous State Tables

## Example.1-1 .Find Races

a) State Tables for poor design

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

b) Circle stable states

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

c) Start at a stable state  
 Check next state on both sides  
 Do F<sup>+</sup> and G<sup>+</sup> both change?

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10



# Find Races In Asynchronous State Tables

## Example.1-1 .Find Races

a) State Tables for poor design

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

b) Circle stable states

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

c) Starting at stable states  
Change one input, either a or b  
Check for F<sup>+</sup> and G<sup>+</sup> both changing.

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

d) Check Race 1

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	01	11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

**Race 1**  
Starts at  
FG=00, ab=10  
a changes 1->0  
May end in  
FG=11, ab=00  
FG=10, ab=00



# Find Races In Asynchronous State Tables

## Example.1-1 .Find Races

### a) State Tables for poor design

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

### b) Circle stable states

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

### c) Starting at stable states Change one input, either a or b Check for F<sup>+</sup> and G<sup>+</sup> both changing.

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab =01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

Fixed thanks to Priti

### e) Check Race 2

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	00	ab =01	ab = 11	10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

**Race 2**  
Starts at FG=01, ab=11  
a changes 1->0  
Will end in FG=11, ab=01  
Noncritical race

Races that don't start in a stable state are "who cares" unless part of a larger path.



# Find Races In Asynchronous State Tables (Cont)

## Example.1-1 .Find Races Continued

b) Circle stable states

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab = 01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

f) If the first step is not a race.  
Check for both F<sup>+</sup> and G<sup>+</sup> changing  
as transitions go down the column.

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	ab = 00	ab = 01	ab = 11	ab = 10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

g) Check Race 3

Present State FG	Next State F <sup>+</sup> G <sup>+</sup> For inputs:			
	00	ab = 01	ab = 11	10
FG=00	11	01	00	00
FG=01	11	10	01	01
FG=11	11	11	11	01
FG=10	10	11	00	10

Race 3

Starts at  
FG=00, ab=11  
a changes 1->0

Race involves  
FG=01, ab=01  
FG=10, ab=01  
FG=11, ab=01

Will end in  
FG=11, ab=01

Noncritical race between transient states



# Finding Races In Asynchronous State Tables

## Example.1-2 Find Races

a) Find races

XY	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
	00	01	11	10
00	00	01	10	00
01	01	11	00	10
11	00	00	01	11
10	00	10	11	10

b) Circle stable states

XY	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
	00	01	11	10
00	00	01	10	00
01	01	11	00	10
11	00	00	01	11
10	00	10	11	10

c) Starting from stable states look for double state changes

XY	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
	00	01	11	10
00	00	01	10	00
01	01	11	00	10
11	00	00	01	11
10	00	10	11	10

No race: *i) Not allowed input change*  
*ii) Doesn't start in stable state*

d) Check 1<sup>st</sup> race

XY	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
	00	01	11	10
00	00	01	10	00
01	01	11	00	10
11	00	00	01	11
10	00	10	11	10

Critical.

Can end in three different different states

Start state 01 (ab=00)  
 End: one of state 00 (ab=10)  
 state 11 (ab=10)  
 state 10 (ab=10)

e) Check 2<sup>nd</sup> race

XY	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
	00	01	11	10
00	00	01	10	00
01	01	11	00	10
11	00	00	01	11
10	00	10	11	10

Critical.

Can end in two different states

Start state 11 (ab=10)  
 End: one of state 00 (ab=00)  
 state 01 (ab=00)

Remember double input changes are not allowed.

Races that don't start in a stable state are "who cares" unless part of a larger path.



# Finding Cycles

## Example.1-3 Find a Cycle

One which oscillates until an input changes..

### a) Find a true cycle

	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
XY	00	01	11	10
00	00	01	10	00
01	01	10	00	10
11	00	00	01	11
10	00	10	11	10

### b) Circle stable states

	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
XY	00	01	11	10
00	00	01	10	00
01	01	10	00	10
11	00	00	01	11
10	00	10	11	10

Starting at stable state  
check both sides

### c) From stable state

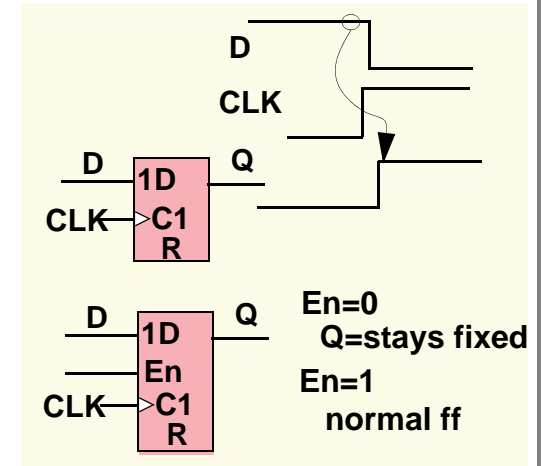
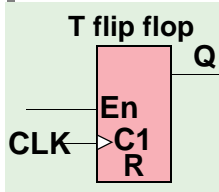
	x <sup>+</sup> y <sup>+</sup> for inputs ab:			
XY	00	01	11	10
00	00	01	10	00
01	01	10	00	10
11	00	00	01	11
10	00	10	11	10

Cycle  
No stable state  
Oscillates until  
input changes.



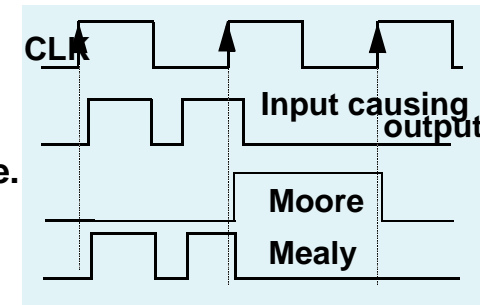
### Synchronous Machines

- **D-Flip flops**  
 Sample D input just before the clock..  
 Transfer D to output just after the clock.  
 Q never changes except at clock edges
- **T-flip flops**  
 Toggle Q after every clock edge if  
 enabled (provided it has an enable)



0 00  
 0 01  
 0 11  
 0 10  
 1 00  
 1 01  
 1 11  
 1 10

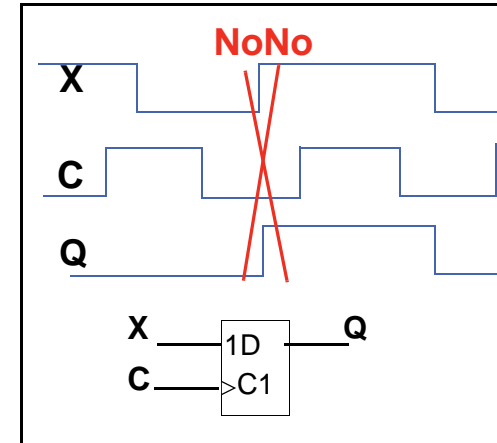
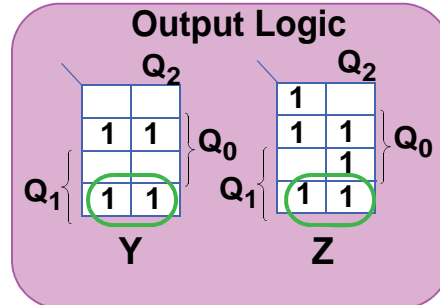
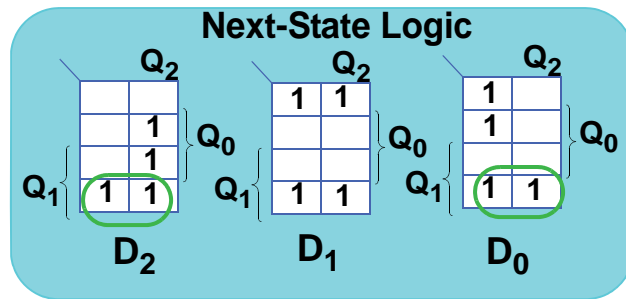
- **Put state tables in K-map order**
- **To tell Moore from Mealy in word problems**
  1. **Moore Outputs**
    - Outputs will appear after the next active clock edge.
    - Outputs will last a full clock cycle.
    - Glitch free outputs.
  2. **Mealy Outputs**
    - Outputs will appear after the input changes





## Synchronous Machines

- Change states only at an active clock edge.  
Watch on timing diagrams.
- FSMs are multiple output machines.  
Watch for shared gates when you loop maps.



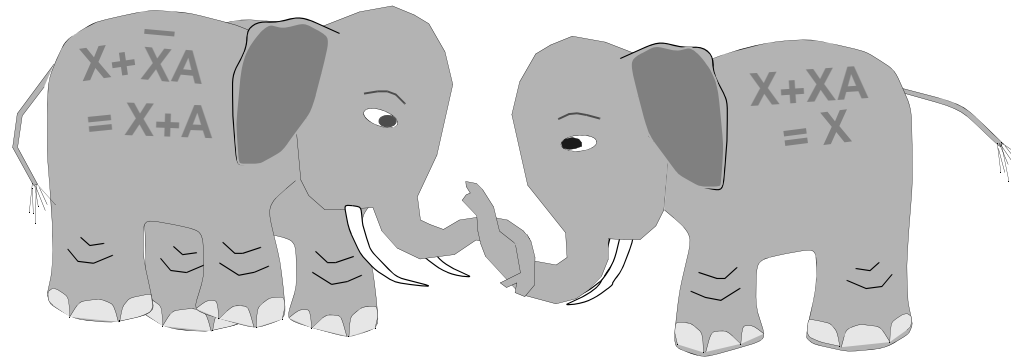
## Key Points

- Read the question! All of it!
- Always check for  $Ax + \bar{A} = A$  Use this to simplify anywhere.
- Always reduce  $A + \bar{A}x = A + x$  Use anywhere except for hazards

Too many people say  ~~$x + 1 = x$~~



# Good Luck on Your Exams



# Have a Good Summer