In-class Exercise For Section 1.4

Exercise 1

Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 4 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$. Find Ax

$$Ax = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} = 1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} * - 4\begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ -20 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 4 + 4 \\ 2 + 6 - 20 \end{bmatrix} = \begin{bmatrix} 9 \\ -12 \end{bmatrix}$$

Exercise 2

If $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix}$, solve $Ax = b$

$\because Ax = A$

$$\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_1 \\ x_1 \end{bmatrix} + \begin{bmatrix} 2x_2 \\ 2x_2 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ 2x_1 + 2x_2 \\ x_1 + x_2 + x_3 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 2 & 2 & 0 & 4 \\ 1 & 1 & 1 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 2 & 2 & 0 & 4 \\ 1 & 2 & 0 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & -2 & -2 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$R_1 \Leftrightarrow R_3$ $\quad$ $R_2 \leftarrow R_2 - 2R_1$, $R_3 \leftarrow R_3 - R_1$ $\quad$ $R_2 \leftarrow -\frac{1}{2}(R_2)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1 - R_2$, $R_3 \leftarrow R_3 + R_2$ $\quad$ $R_2 \Leftrightarrow R_1$ $\quad$ $R_1 \leftarrow R_1 - R_2$

Therefore, $\begin{cases} x_1 = 1 \\ x_2 = 1 \\ x_3 = 1 \end{cases}$. Hence. We have a unique solution $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Exercise 3

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 7 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$, solve $Ax = b$.

$\because Ax = b$

$\therefore \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 3 & 4 & 7 & 2 \\ 3 & 4 & 5 & 3 \\ 1 & 2 & 3 & 1 \end{array}\right] \Rightarrow$

The Form of Final Answer is

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 1 \end{array}\right]$

"$*$" means it could be any number.

$0 \neq 1$ Hence. $Ax = b$ has No solution

Exercise 4

$A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. For what values b_1. b_2 and b_3 does the equation $Ax = b$ have a solution

$$\left[\begin{array}{ccc|c} 1 & -3 & -4 & b_1 \\ -3 & 2 & 6 & b_2 \\ 5 & -1 & -8 & b_3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & b_2 + 3b_1 \\ 0 & 14 & 12 & b_3 - 5b_1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & b_2 + 3b_1 \\ 0 & 0 & 0 & b_1 + 2b_2 + b_3 \end{array}\right]$$

$R_2 \leftarrow R_2 + 3R_1$ $\qquad$ $R_3 \leftarrow R_3 + 2R_2$

$R_3 \leftarrow R_3 - 5R_1$

$\therefore$ For $Ax = b$ to have a solution. $b_1 + 2b_2 + b_3 = 0$

Exercise 5 $\quad$ Do the columns of the following matrix span R^4?

$$A = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & & -2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

$R_3 \leftarrow R_3 - R_1$ $\qquad$ $R_3 \leftarrow R_3 + R_2$

$R_4 \leftarrow R_4 + 2R_1$ $\qquad$ $R_4 \leftarrow R_4$

No Solution.

Since A does not has a pivot position in every row

In other way, A does not span R^4.