UBC Physics 301 2014

Homework Assignment 3

Due at class time October 10, 2014

1)

a) Calculate the dipole moment of a spherical shell of radius R whose surface charge density is $\sigma_0(1+\cos\theta)$.

b) What is the dipole moment if the center of the sphere is at Zẑ?

c) What is the dipole moment if the center of the sphere is at Xx̂+Yŷ+Zẑ?

2) A single point charge Q is located at P'(0,0,s). Expand its potential at the field point P in terms of multipoles. The vector **r** that defines the position of P forms an angle θ with the z-axis and r>>s. The distance from Q to P is r'. Disregarding terms of order $\left(\frac{s}{r}\right)^4$ and higher, write our the values of the first three terms in the voltage expansion V_1, V_2, V_3.

3) Show that the energy in the field of an electric dipole of moment p outside a sphere of radius R is $\frac{p^2}{12\pi\varepsilon_0 R^3}$.

4) Coaxial lines comprise two coaxial cylindrical conductors and often serve for transmission of high-frequency circuits. Consider that the outer radius of the inner conductor is 3 mm, and the inner radius of the outer conductor is 5 mm. Calculate the electric energy in the field, per meter, when the voltage difference is 5 V.

5) A capacitor consisting of two concentric spheres is arranged so that the outer sphere can be separated and removed without disturbing the charges on either. The radius of the inner sphere is a and that of the outer sphere is b, and the charges are Q and -Q, respectively

a) If the outer sphere is removed and restored to its original form, find the increase in energy when the two spheres are separated by a large distance.

b) Where does this extra energy come from?

6) A balloon made of a light conducting material could be kept spherical by connecting it to a high voltage supply. The balloon has a diameter of 100 mm and the maximum breakdown field in air is 3 megavolts/m.

a) What is the maximum permissible voltage?

b) What gas pressure, in atmospheres, inside the balloon would have the same effect?

c) How large could the surface mass density (kg/m^2) of the balloon be?

7) Show that $\nabla \bullet \mathbf{E} = 0$ in the dielectric of a coaxial line.
(Hint: apply the divergence theorem to a portion of the dielectric.)

8) High energy electrons bombarding a block of Lucite (a plastic insulator) can penetrate the material and get trapped inside. A sharp rap with a conducting object (e.g. a center punch) can cause the electrons to escape leaving a nice tree-like design where the plastic has broken down.

Take the case of a 0.1 microampere beam bombaring an area of 25 cm^2 of Lucite (ε_r =3.2) which is 12 mm thick, for 1 s, trapping all the electrons 6mm below the surface in a region 2 mm thick. In the following, neglect edge effects and assume a uniform charge density for the trapped electrons. Also, assume both faces of the Lucite are in contact with grounded conducting planes.

a) What is the bound charge density in the charged region?

b) What is the bound charge density at the surface of the Lucite?

c) Show that the potential at the center of the sheet of charge is about 4 kV.

d) What is the energy of the block? Could it explode?