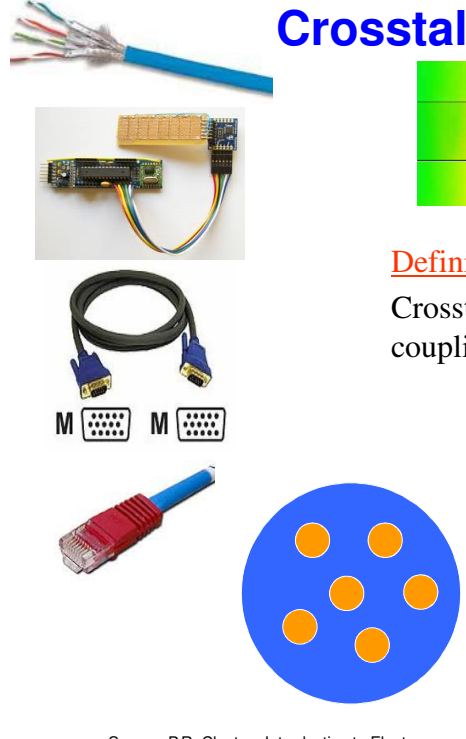
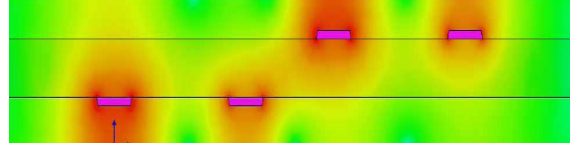
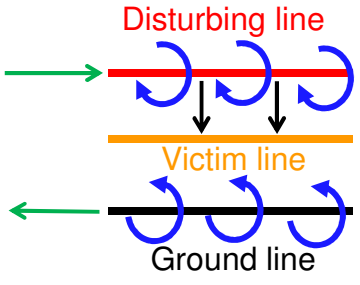


Crosstalk (Chapter 9)

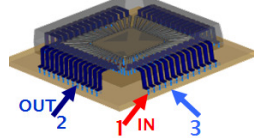
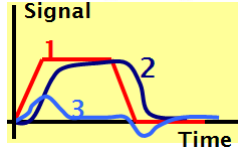
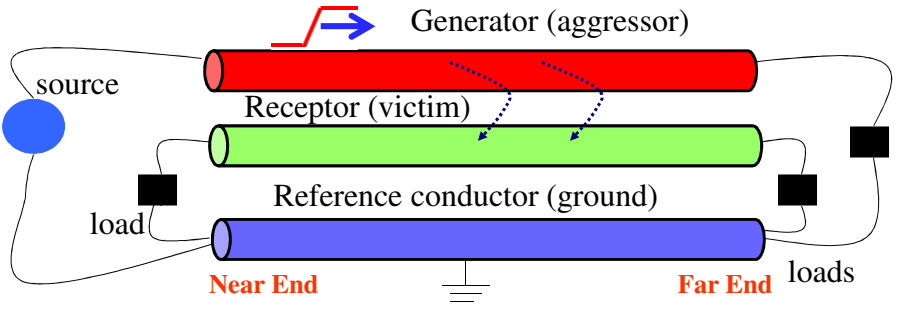
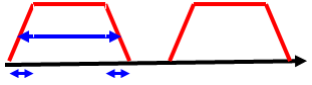
Definition:
Crosstalk is the unintended EM near field coupling between wires and PCB lands.



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

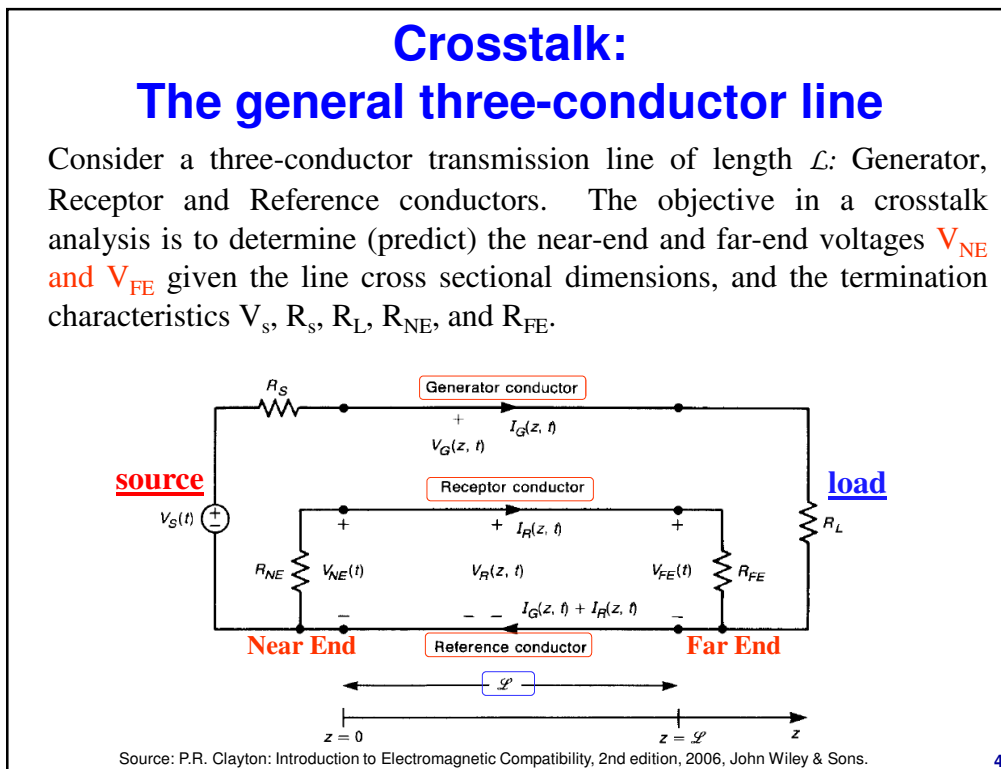
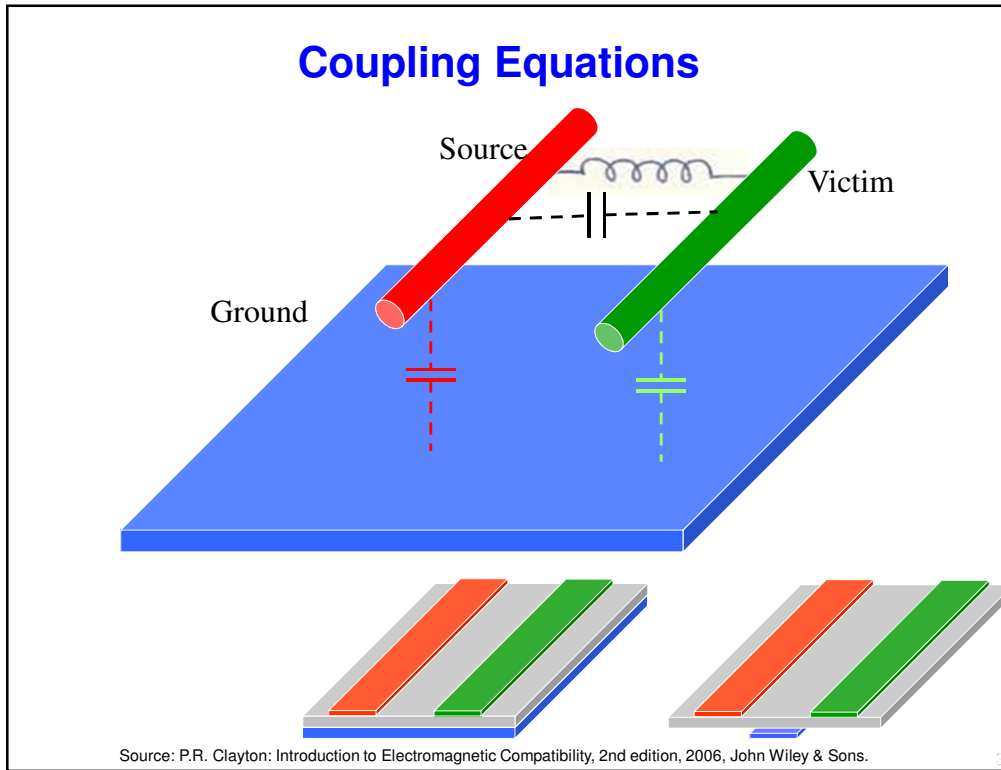
Crosstalk (Chapter 9)

Definition:
A minimum of three conductors must exist to have a crosstalk in a system:
Generator (G), Receptor (R), Reference (0).

- Crosstalk occurs during Rising/Falling Edges
- Sources: Packages, connectors, vias,...

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.



Some typical three-conductor transmission lines

Wire-type line cross sections whose reference conductors (0) are :

- (a) another wire,
- (b) an infinite ground plane, or
- (c) an overall, cylindrical shield.

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons. 5

PCB transmission lines

source Coupled microstrip lines

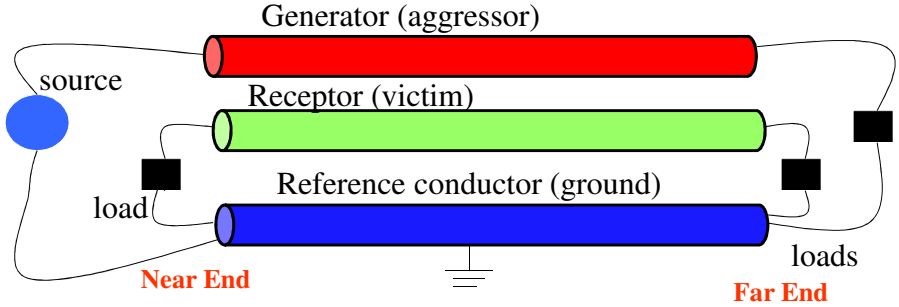
source Coupled coplanar PCBs

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons. 6

Section 9.2: Crosstalk Analysis Techniques

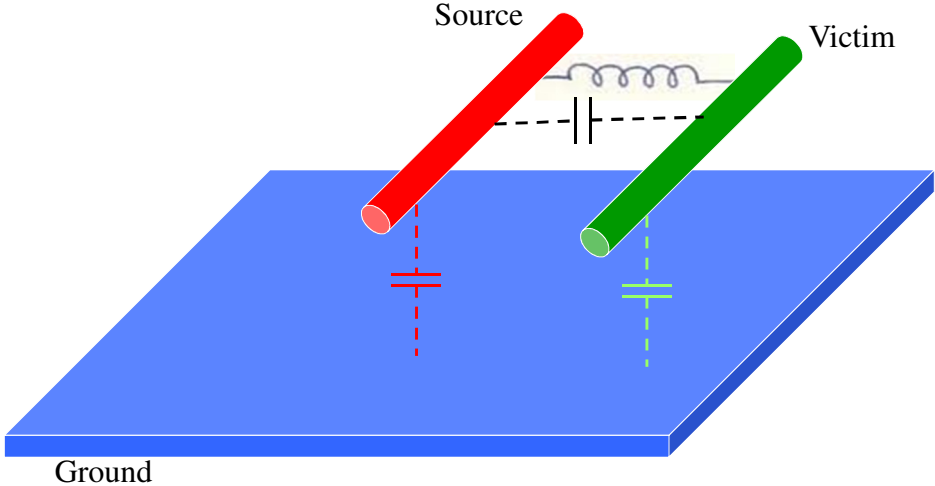
There are two types of analysis:

- 1. Time-domain crosstalk analysis is the determination of the time form of the receptor (victim) terminal voltages $V_{NE}(t)$ and $V_{FE}(t)$ for some general time form of the source voltage $V_S(t)$.
- 2. Frequency-domain crosstalk analysis is the determination of the magnitude and phase of the receptor terminal phasor voltages for a sinusoidal source voltage.



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Field → Circuit

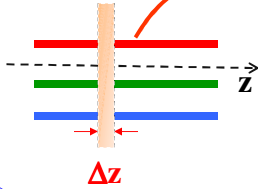


Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

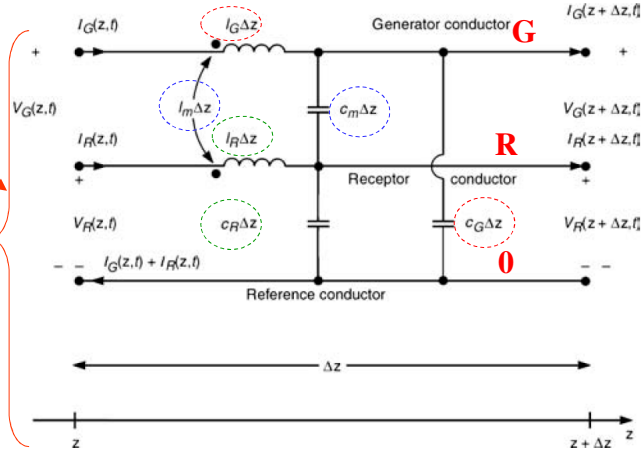
Time-Domain Crosstalk

Assumption: the transverse electromagnetic (TEM) mode of propagation is the only field structure present on the line.

Consider a differential length Δz of a lossless transmission line as shown:



- The total inductance or capacitance is the per-unit-length (puL) value multiplied by Δz



The per-unit-length equivalent circuit of a three-conductor transmission line.

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Time-Domain Crosstalk (Cont.)

By applying KVL & KCL and taken the limit as the differential length Δz approaches zero, we can show that:

$$\frac{\partial V_G(z, t)}{\partial z} = -l_G \frac{\partial I_G(z, t)}{\partial t} - l_m \frac{\partial I_R(z, t)}{\partial t}$$

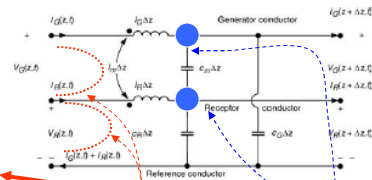
Equations are in terms of L 's

$$\frac{\partial V_R(z, t)}{\partial z} = -l_m \frac{\partial I_G(z, t)}{\partial t} - l_R \frac{\partial I_R(z, t)}{\partial t}$$

$$\frac{\partial I_G(z, t)}{\partial z} = -(c_G + c_m) \frac{\partial V_G(z, t)}{\partial t} + c_m \frac{\partial V_R(z, t)}{\partial t}$$

Equations are in terms of c 's

$$\frac{\partial I_R(z, t)}{\partial z} = c_m \frac{\partial V_G(z, t)}{\partial t} - (c_R + c_m) \frac{\partial V_R(z, t)}{\partial t}$$



Use **two loops** and **two nodes**

These are known as multi-conductor transmission lines (MTL) equations.

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Time-Domain Crosstalk (Cont.)

- The previous MTL equations may be written in a matrix form:



$$\frac{\partial}{\partial z} \mathbf{V}(z, t) = -\mathbf{L} \frac{\partial}{\partial t} \mathbf{I}(z, t)$$

$$\frac{\partial}{\partial z} \mathbf{I}(z, t) = -\mathbf{C} \frac{\partial}{\partial t} \mathbf{V}(z, t)$$

Where:

$$\mathbf{V}(z, t) = \begin{bmatrix} V_G(z, t) \\ V_R(z, t) \end{bmatrix}$$

$$\mathbf{I}(z, t) = \begin{bmatrix} I_G(z, t) \\ I_R(z, t) \end{bmatrix}$$

With:

$$\mathbf{L} = \begin{bmatrix} l_G & l_m \\ l_m & l_R \end{bmatrix}$$

and

$$\mathbf{C} = \begin{bmatrix} (c_G + c_m) & -c_m \\ -c_m & (c_R + c_m) \end{bmatrix}$$

- The matrix form is very powerful and can be extended to lines consisting of more than three conductors.
- These equations may be solved in time domain.



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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Frequency-Domain Crosstalk

- For a single-frequency, sinusoidal steady-state excitation (phasor form) we simply replace time derivatives in the MTL equations with $j\omega$,

$$\frac{\partial}{\partial z} \mathbf{V}(z, t) = -\mathbf{L} \frac{\partial}{\partial t} \mathbf{I}(z, t)$$

$$\frac{\partial}{\partial z} \mathbf{I}(z, t) = -\mathbf{C} \frac{\partial}{\partial t} \mathbf{V}(z, t)$$



$$\frac{d}{dz} \hat{\mathbf{V}}(z) = -j\omega \mathbf{L} \hat{\mathbf{I}}(z)$$

$$\frac{d}{dz} \hat{\mathbf{I}}(z) = -j\omega \mathbf{C} \hat{\mathbf{V}}(z)$$

- We'll give later the solution for these coupled differential equations.
- Then the time-domain voltages and currents can be found from these phasor voltages and currents as:

$$\mathbf{V}(z, t) = \Re\{\hat{\mathbf{V}}(z)e^{j\omega t}\}$$

$$\mathbf{I}(z, t) = \Re\{\hat{\mathbf{I}}(z)e^{j\omega t}\}$$

- But in order to obtain the final solution, one needs to calculate the pul parameters which will be presented in the next slides.***

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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Section 9.3: The Per-Unit-Length (pul) Parameters

For homogeneous medium, the pul parameters matrices are related

$$LC = CL = \mu\epsilon I$$

Where, for 3-conductor

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \mu\epsilon L^{-1} = \frac{1}{v^2} L^{-1}$$

Once we know expressions for one of them (L or C), we can compute the other:

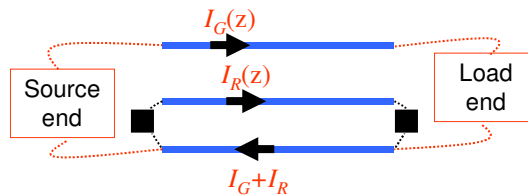
$$\begin{bmatrix} c_G + c_m & -c_m \\ -c_m & c_R + c_m \end{bmatrix} = \frac{1}{v^2(l_G l_R - l_m^2)} \begin{bmatrix} l_R & -l_m \\ -l_m & l_G \end{bmatrix}$$

More later on how to calculate the PUL (end of this set).

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Sinusoidal Steady-State Crosstalk Inductive-Capacitive Coupling Model (9.4)

- The transmission-line equations in the frequency domain (two 1st order coupled ODEs for sinusoidal steady-state excitation, see slide #11) :



$$\begin{aligned} \frac{\partial}{\partial z} \mathbf{V}(z, t) &= -\mathbf{L} \frac{\partial}{\partial t} \mathbf{I}(z, t) \\ \frac{\partial}{\partial z} \mathbf{I}(z, t) &= -\mathbf{C} \frac{\partial}{\partial t} \mathbf{V}(z, t) \end{aligned}$$



$$\begin{aligned} \frac{d}{dz} \hat{\mathbf{V}}(z) &= -j\omega \mathbf{L} \hat{\mathbf{I}}(z) \\ \frac{d}{dz} \hat{\mathbf{I}}(z) &= -j\omega \mathbf{C} \hat{\mathbf{V}}(z) \end{aligned}$$

L and C: inductance and capacitance matrices (self and mutual terms)

- We can get uncoupled second-order ODE as

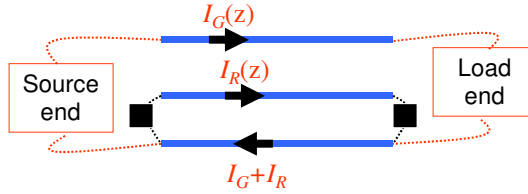
Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Sinusoidal Steady-State Crosstalk

- in the frequency domain:

$$\frac{d}{dz} \hat{V}(z) = -j\omega L \hat{I}(z)$$

$$\frac{d}{dz} \hat{I}(z) = -j\omega C \hat{V}(z)$$



$$\frac{d^2 \hat{V}(z)}{dz^2} = -\omega^2 LC \hat{V}(z) \quad \& \quad \frac{d^2 \hat{I}(z)}{dz^2} = -\omega^2 LC \hat{I}(z) \quad \text{where} \quad V = \begin{bmatrix} V_G \\ V_R \end{bmatrix}$$

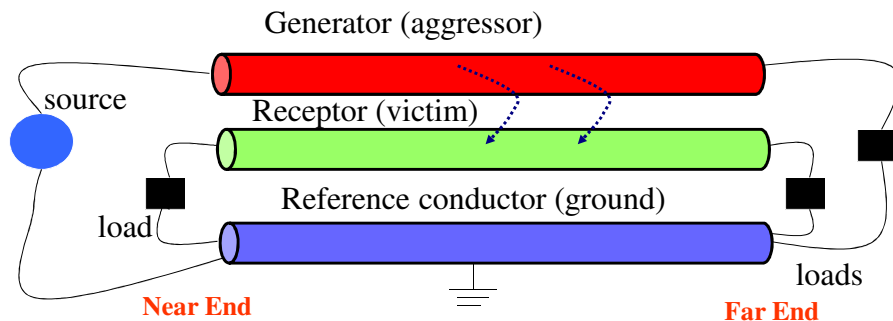
- The above ODEs can be solved for V or I .
- Instead, we'll derive approximate expressions for the near and far ends coupling

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Approximate Solution for Lossless Lines in Homogeneous Media: Inductive and Capacitive Coupling

Assumptions:

- the line is electrically short at the frequency of interest: $L \ll \lambda$.
- the generator and receptor circuits are weakly coupled
- the frequency of excitation is sufficiently small

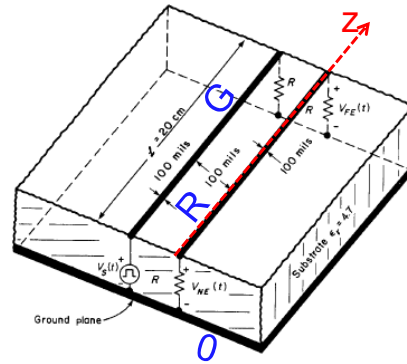


Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Inductive and Capacitive Coupling (Cont.)

- We are interested in the voltages and currents at the end points of the line: near and far ends

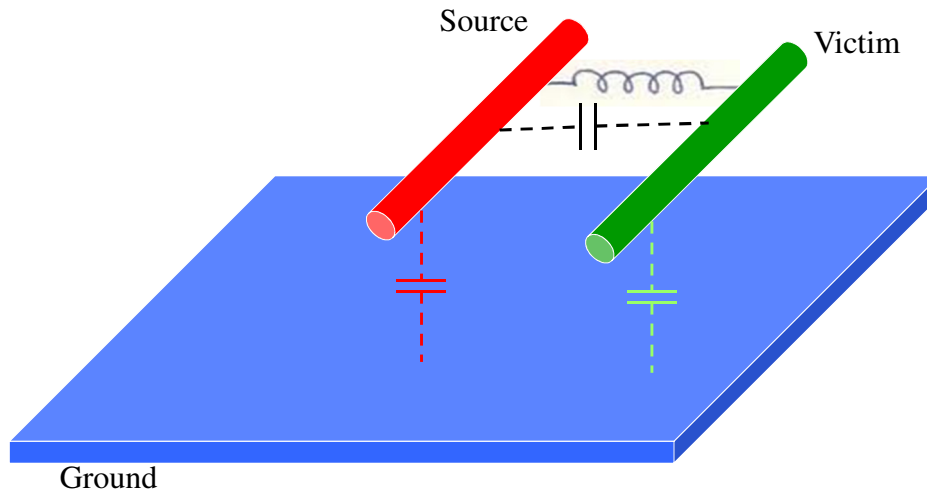
$$V_R(0) = V_{NE} \quad \& \quad V_R(L) = V_{FE}$$



- Under the previous assumptions, we can deduce a simple and approximate solution for the voltages.
- We represent the induced voltages in the receptor circuit via mutual inductance and capacitance together with the voltage and current of (isolated) generator circuit.

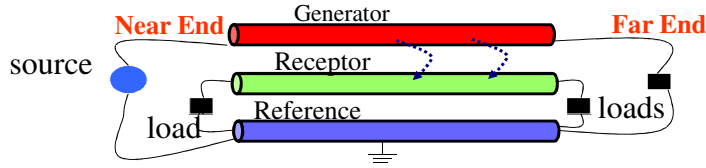
Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Coupling



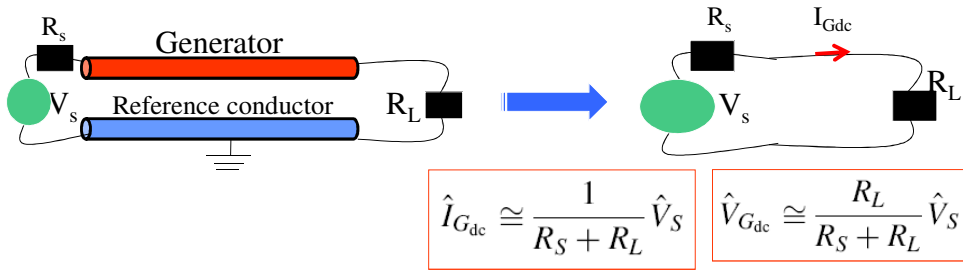
Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Inductive and Capacitive Coupling (Cont.)



- For a short line, and @ low frequencies, the signal on the generator may be assume to be constant (dc).
- The voltage and current of (isolated) generator circuit are:

generator circuit

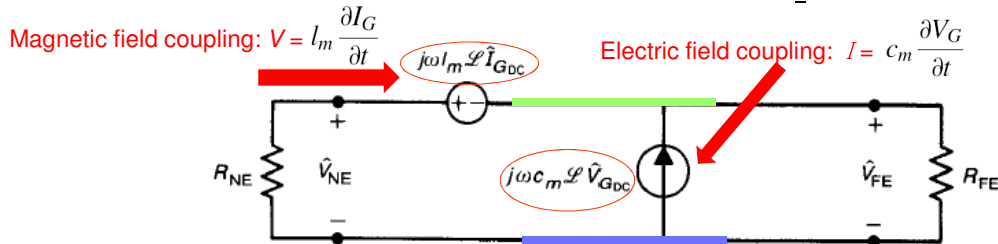
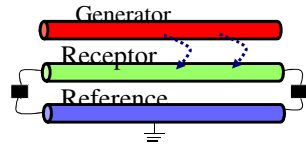


Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Inductive and Capacitive Coupling (Cont.)

Receptor circuit

- We represent the induced voltages in the receptor circuit via mutual inductance and capacitance together with the voltage and current of (isolated) generator circuit.
- A simple equivalent circuit for the inductive – capacitive coupling in the receptor circuit (line) is shown.

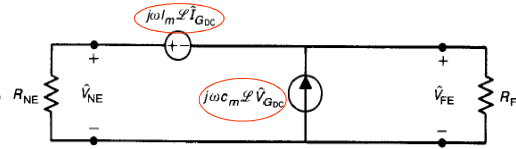


- In this model, we ignore the induced signals due to self inductance and capacitance terms.
- Only *mutual coupling* terms are included.

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Inductive and Capacitive Coupling (Cont.)

- From the equivalent circuit for the receptor, the solutions for the near and far ends voltages are expressed as:



$$\hat{V}_{NE} = \underbrace{\frac{R_{NE}}{R_{NE} + R_{FE}} j\omega L_m \hat{I}_{Gdc}}_{\text{inductive coupling}} + \underbrace{\frac{R_{NE} R_{FE}}{R_{NE} + R_{FE}} j\omega C_m \hat{V}_{Gdc}}_{\text{capacitive coupling}}$$

$$\hat{V}_{FE} = -\underbrace{\frac{R_{FE}}{R_{NE} + R_{FE}} j\omega L_m \hat{I}_{Gdc}}_{\text{inductive coupling}} + \underbrace{\frac{R_{NE} R_{FE}}{R_{NE} + R_{FE}} j\omega C_m \hat{V}_{Gdc}}_{\text{capacitive coupling}}$$

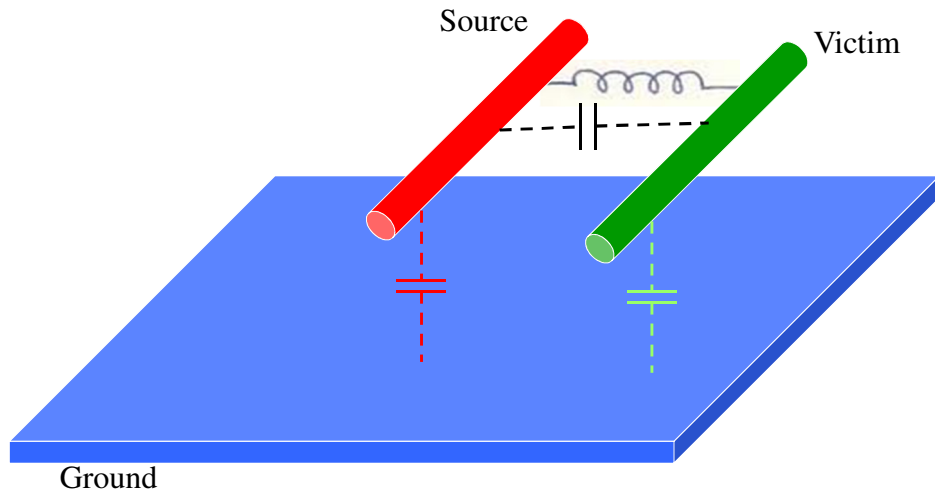
with $\hat{I}_{Gdc} \cong \frac{1}{R_S + R_L} \hat{V}_S$ $\hat{V}_{Gdc} \cong \frac{R_L}{R_S + R_L} \hat{V}_S$

- Where the total mutual capacitance and inductance of the line are:

$$C_m = c_m \mathcal{L} \quad L_m = l_m \mathcal{L}$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

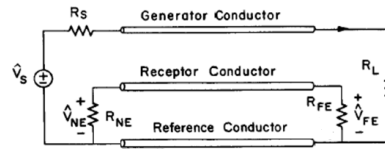
Coupling



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Inductive and Capacitive Coupling (Cont.)

- The cross talk terms may be expressed in terms of transfer functions between the input (source) and outputs (near & far end voltages):



$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega \left(\frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_m}{R_S + R_L} + \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}} \frac{R_L C_m}{R_S + R_L} \right)$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega \left(-\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{L_m}{R_S + R_L} + \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}} \frac{R_L C_m}{R_S + R_L} \right)$$

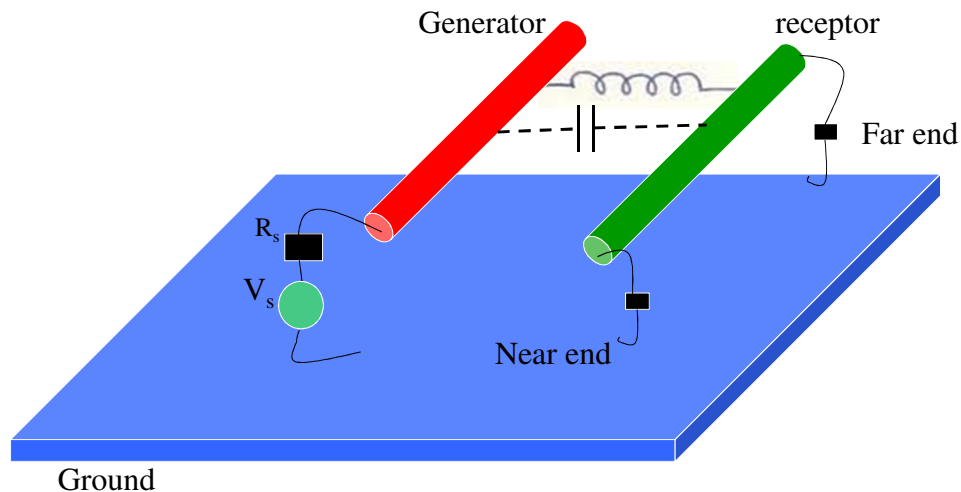
- Each can be written in a form to represent the two types of coupling (inductive (IND) and capacitive (CAP)):

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega (M_{NE}^{IND} + M_{NE}^{CAP})$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega (M_{FE}^{IND} + M_{FE}^{CAP})$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Coupling



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Inductive and Capacitive Coupling (Cont.)

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega(M_{NE}^{IND} + M_{NE}^{CAP})$$

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega(M_{FE}^{IND} + M_{FE}^{CAP})$$

Where the inductive and capacitive coupling terms are given by

$$M_{NE}^{IND} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{L_m}{R_S + R_L}$$

$$M_{FE}^{IND} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{L_m}{R_S + R_L}$$

$$M_{NE}^{CAP} = \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}} \frac{R_L C_m}{R_S + R_L}$$

$$M_{FE}^{CAP} = M_{NE}^{CAP} = \frac{R_{NE}R_{FE}}{R_{NE} + R_{FE}} \frac{R_L C_m}{R_S + R_L}$$

$$C_m = c_m \mathcal{L} \quad \& \quad L_m = l_m \mathcal{L}$$

- Note:**
- If $L_m/C_m > R_{FE} R_L$, the near end inductive coupling dominates the NE capacitive coupling.
 - If $L_m/C_m > R_{NE} R_L$, the far end inductive coupling dominates the FE capacitive coupling.
 - Capacitive coupling dominates if the above inequalities are reversed.

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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Inductive and Capacitive Coupling (Cont.)

- For a homogenous system, the previous conclusions can be represented in different forms for **dominating inductive coupling**:

- **NE coupling condition:**

$$\frac{R_{FE}R_L}{(L_m/C_m)} = \frac{R_{FE}R_L}{Z_{CG}Z_{CR}} < 1$$

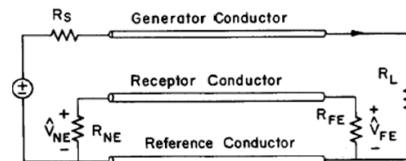
- **FE coupling condition:**

$$\frac{R_{NE}R_L}{(L_m/C_m)} = \frac{R_{NE}R_L}{Z_{CG}Z_{CR}} < 1$$

where: $Z_{CG} = \sqrt{\frac{l_G}{c_G + c_m}} \quad \& \quad Z_{CR} = \sqrt{\frac{l_R}{c_R + c_m}}$

are the characteristic impedances of each circuit (generator (G) and receptor (R)) in the presence of the other circuit

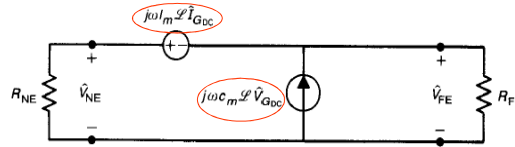
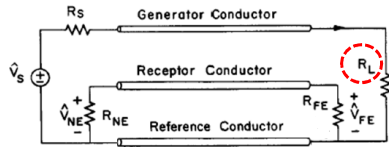
- **Reverse the inequality for a dominating capacitive coupling**



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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Inductive and Capacitive Coupling (Cont.)



Let us consider two cases:

low and high impedance terminations:

Case I:

For low-impedance loads with respect to the lines characteristic impedance (high currents, low voltages), the inductive coupling component will dominate the capacitive coupling component:

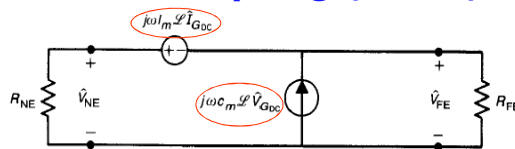
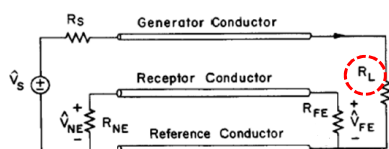
$$\text{in } \hat{V}_{NE} \text{ if } \frac{R_L}{Z_{CG}} \frac{R_{FE}}{Z_{CR}} < 1 \quad \text{then} \quad \hat{V}_{NE}^{\text{IND}} = \frac{R_{NE}}{R_{NE} + R_{FE}} j\omega l_m \mathcal{L} \hat{I}_{GDC}$$

$$\text{in } \hat{V}_{FE} \text{ if } \frac{R_L}{Z_{CG}} \frac{R_{NE}}{Z_{CR}} < 1 \quad \text{then} \quad \hat{V}_{FE}^{\text{IND}} = -\frac{R_{FE}}{R_{NE} + R_{FE}} j\omega l_m \mathcal{L} \hat{I}_{GDC}$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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Inductive and Capacitive Coupling (Cont.)



Case II: For high-impedance loads (low currents, high voltages), the capacitive coupling component will dominate the inductive coupling component:

$$\text{in } \hat{V}_{NE} \text{ if } \frac{R_L}{Z_{CG}} \frac{R_{FE}}{Z_{CR}} > 1 \quad \text{then} \quad \hat{V}_{NE}^{\text{CAP}} = \frac{R_{NE} R_{FE}}{R_{NE} + R_{FE}} j\omega c_m \mathcal{L} \hat{V}_{GDC}$$

$$\text{in } \hat{V}_{FE} \text{ if } \frac{R_L}{Z_{CG}} \frac{R_{NE}}{Z_{CR}} > 1 \quad \text{then} \quad \hat{V}_{FE}^{\text{CAP}} = \frac{R_{NE} R_{FE}}{R_{NE} + R_{FE}} j\omega c_m \mathcal{L} \hat{V}_{GDC}$$

Note that for both cases, the induced signals are directly proportional to the source frequency

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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Frequency Response

$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega(M_{FE}^{IND} + M_{FE}^{CAP})$$

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega(M_{NE}^{IND} + M_{NE}^{CAP})$$

20 dB/decade
~f

(a) Low-impedance loads

$$\hat{V}_{NE}^{IND} = \frac{R_{NE}}{R_{NE} + R_{FE}} j\omega l_m \mathcal{L} \hat{I}_{Gdc}$$

$$\hat{V}_{FE}^{IND} = -\frac{R_{FE}}{R_{NE} + R_{FE}} j\omega l_m \mathcal{L} \hat{I}_{Gdc}$$

(b) High-impedance loads

$$\hat{V}_{NE}^{CAP} = \frac{R_{NE} R_{FE}}{R_{NE} + R_{FE}} j\omega c_m \mathcal{L} \hat{V}_{Gdc}$$

$$\hat{V}_{FE}^{CAP} = \frac{R_{NE} R_{FE}}{R_{NE} + R_{FE}} j\omega c_m \mathcal{L} \hat{V}_{Gdc}$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons. 29

Inclusion of losses: Common-Impedance Coupling

- The assumption of a lossless medium is usually reasonable for frequencies below the GHz range.
- However, **imperfect conductors** can produce significant crosstalk at the lower frequencies known as *the common-impedance coupling*.
- For an electrically short line we may lump the per-unit-length resistance of the reference conductor, r_0 , as a single resistance $R_0 = r_0 \mathcal{L}$. The voltage drop across the reference conductor is given by

$$\hat{V}_0 = R_0 \hat{I}_G = \frac{R_0}{R_S + R_L} \hat{V}_S$$

Common-Impedance Coupling

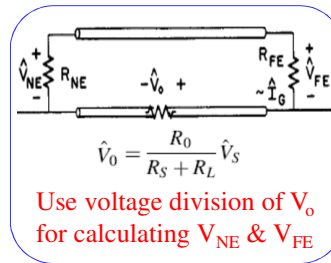
Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons. 30

Inclusion of losses: Common-Impedance Coupling (Cont.)

- The voltage V_0 appears directly in the receptor circuit, producing contributions to the crosstalk transfer functions **at low frequencies**:

$$\frac{\hat{V}_{NE}^{CI}}{\hat{V}_S} = M_{NE}^{CI} \quad \text{where} \quad M_{NE}^{CI} = \frac{R_{NE}}{R_{NE} + R_{FE}} \frac{R_0}{R_S + R_L}$$

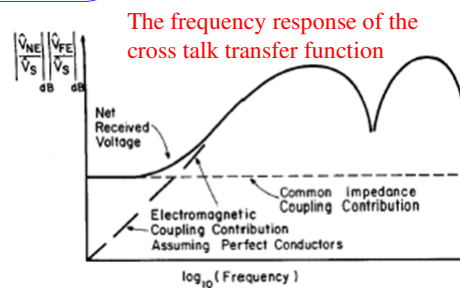
$$\frac{\hat{V}_{FE}^{CI}}{\hat{V}_S} = M_{FE}^{CI} \quad M_{FE}^{CI} = -\frac{R_{FE}}{R_{NE} + R_{FE}} \frac{R_0}{R_S + R_L}$$



Then the total couplings are:

$$\frac{\hat{V}_{NE}}{\hat{V}_S} = j\omega(M_{NE}^{IND} + M_{NE}^{CAP}) + M_{NE}^{CI}$$

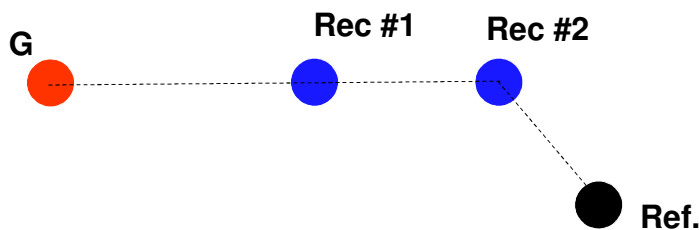
$$\frac{\hat{V}_{FE}}{\hat{V}_S} = j\omega(M_{FE}^{IND} + M_{FE}^{CAP}) + M_{FE}^{CI}$$



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

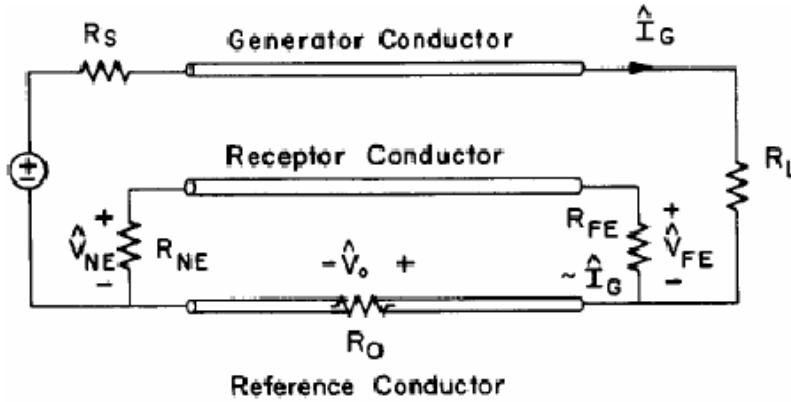
Crosstalk Calculations: Summary

- For a given configuration, calculate the per unit length parameters
- For a given source and loads, calculate the inductive and capacitive coupling transfer terms for near and far ends at the required frequency.
- For a lossy medium, calculate the common-impedance coupling
- Add up all terms



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

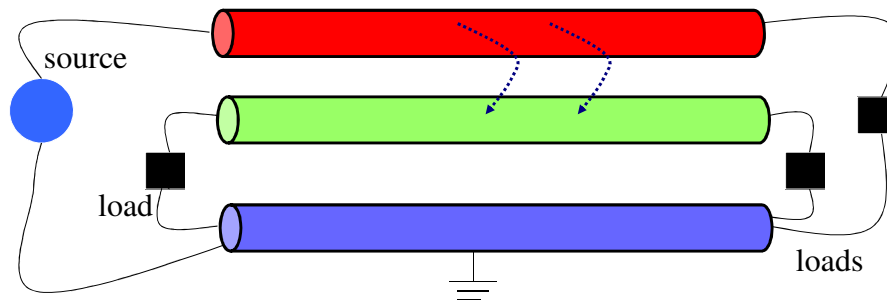
Section 9.5: A Lumped Circuit Model For Computing Cross Talk in a 3-Conductor Line



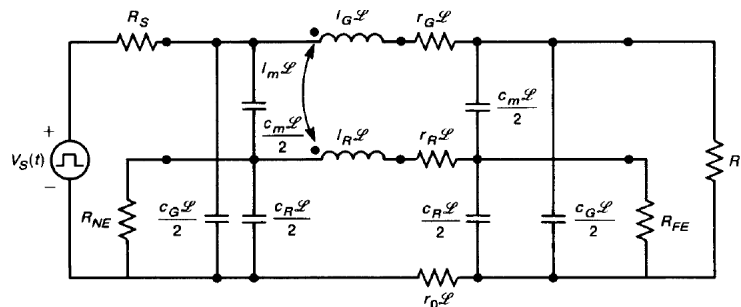
Example: Fig. 9.42

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

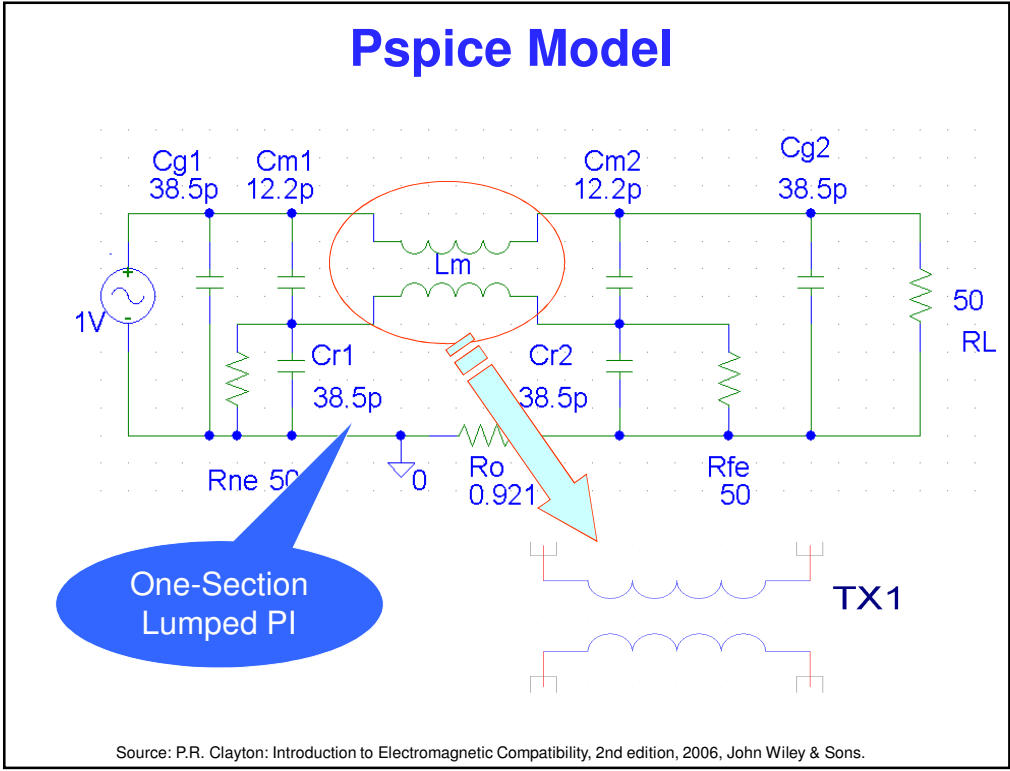
A Lumped Circuit Model For a 3-Conductor Line



Lumped pi



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.



XFRM_LINEAR

Property Editor

Color	COUPLING	Designator	Graphic	ID
Default	0.31578		XFRM_LINEAR.Normal	

COUPLING	L1_VALUE	L2_VALUE
0.31578	3.6uH	3.6uH

Experimental / simulations Results: Frequency Domain

(a)

(b)

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

Section 9.3: PUL parameters: Wire-Type Structures

Wide-separation approximation: assume that the wires are separated sufficiently that the charge and current distributions around the peripheries of the wires are essentially uniform.

The per-unit-length external inductance is defined in terms of the flux penetrating a unit length surface between the wires as:

Flux due to a current-carrying wire:

$$\psi_m = \int_S \vec{B}_T \cdot d\vec{s} \quad \text{where} \quad \vec{B}_T = \mu_0 \vec{H}_T$$

$$\oint_C \vec{H}_T \cdot d\vec{l} = I_{\text{enclosed}} \implies H_T = \frac{I}{2\pi r}$$

∴ Flux per unit wire length

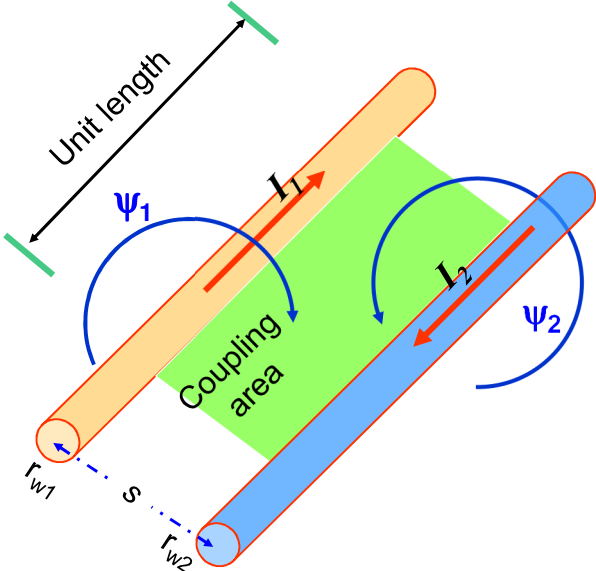
$$\psi_m = \int_{r=R_1}^{R_2} \frac{\mu_0 I}{2\pi r} dr = \frac{\mu_0 I}{2\pi} \ln\left(\frac{R_2}{R_1}\right)$$

$$l = \frac{\psi_m}{I}$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

PUL: Note on flux directions

Two-wire transmission line

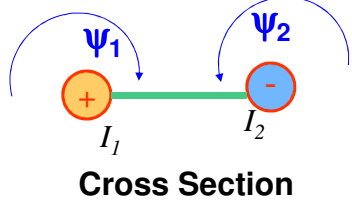


Flux per unit length:

$$\psi_1 = \frac{\mu I_1}{2\pi} \cdot \ln\left(\frac{s - r_{w2}}{r_{w1}}\right)$$

$$\psi_2 = \frac{\mu I_2}{2\pi} \cdot \ln\left(\frac{s - r_{w1}}{r_{w2}}\right)$$

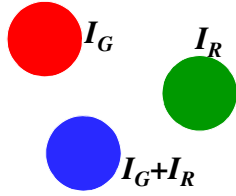
$$\psi_{\text{tot}} = \psi_1 + \psi_2$$



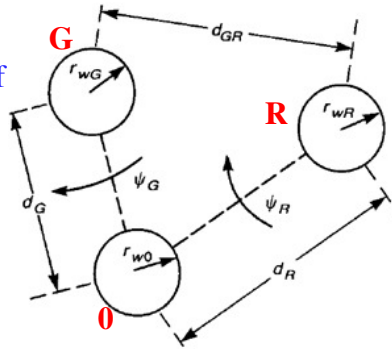
Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

General Formulation for three wires:



Cross section of three parallel wires



Assume that the wires are separated sufficiently large enough that the current distributions around the peripheries of the wires are uniform.

The magnetic fluxes penetrating generator and receptor circuits are:

$$\psi_G = l_G I_G + l_m I_R$$

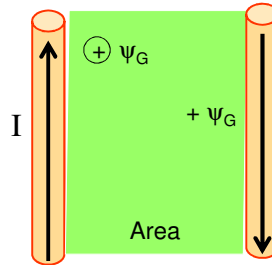
$$\psi_R = l_m I_G + l_R I_R$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

General Formulation for three wires:

The pul inductances can be found by applying a current on one conductor (and returning on the reference conductor), setting the other current equal to zero:



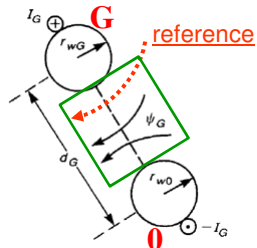
a) self-inductances calculation

$$\psi_G = l_G I_G + l_m I_R$$



$$l_G = \left. \frac{\psi_G}{I_G} \right|_{I_R=0}$$

self-inductance of the Generator



Apply the current at the generator, And find flux penetrating the G circuit: Area between G & O

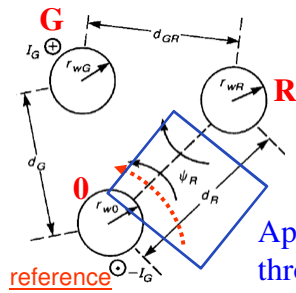
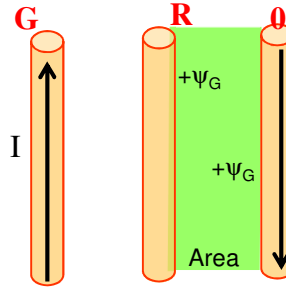
Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

General Formulation for three wires:

b) mutual inductance calculation

The pul inductances can be found by applying a current on one conductor (and returning on the reference conductor), setting the other current equal to zero:



$$\psi_R = l_m I_G + l_R I_R$$

$$l_m = \frac{\psi_R}{I_G} \Big|_{I_R=0} \quad \text{Mutual inductance}$$

Apply the current at the generator (return through the ground 0),
And find flux penetrating the Area between R & O

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

(a) Computation of the self-inductances:

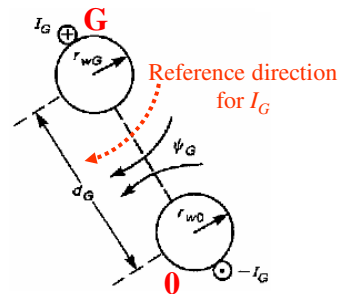
For l_G : assume the generator conductor carries I_G which returns on the reference conductor with $I_R=0$. Coupling area is between the generator and reference conductors.

l_G calculation Start with: $l_G = \frac{\psi_G}{I_G} \Big|_{I_R=0}$

Note: There are two sources for ψ_G

$$l_G = \frac{\mu_0}{2\pi} \ln\left(\frac{d_G}{r_{wG}}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{d_G}{r_{wO}}\right)$$

$$= \frac{\mu_0}{2\pi} \ln\left(\frac{d_G^2}{r_{wG} r_{wO}}\right)$$



Similarly for l_R :

$$l_R = \frac{\mu_0}{2\pi} \ln\left(\frac{d_R^2}{r_{wR} r_{wO}}\right)$$

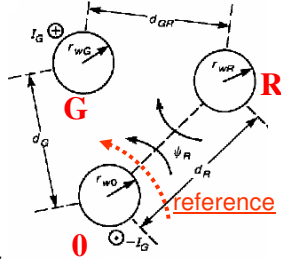
It is very important to determine the correct direction of the resulting flux.

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

(b) Computation of the mutual inductance:

Assume the generator conductor carries I_G that returns on the reference conductor. Coupling area is between the receptor and reference conductors with $I_R = 0$.



Start with:
$$l_m = \frac{\psi_R}{I_G} \Big|_{I_R=0}$$

Note: There are two sources for ψ_R

$$l_m = \frac{\mu_0}{2\pi} \ln\left(\frac{d_G}{d_{GR}}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{d_R}{r_{w0}}\right)$$

$$= \frac{\mu_0}{2\pi} \ln\left(\frac{d_G d_R}{d_{GR} r_{w0}}\right)$$

NOTE:

- 1) the per-unit-length capacitances can be obtained by matrix inversion of the L matrix.
- 2) we can also compute: $v = 1/\sqrt{lc}$ & $Z_c = \sqrt{l/c}$
- 3) the above procedure will be applied to different configurations.

$$C = \mu \epsilon L^{-1}$$

$$= \frac{1}{v^2} L^{-1}$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

An example:

The three-wire ribbon cable.



By applying the previous procedure, we can show that:

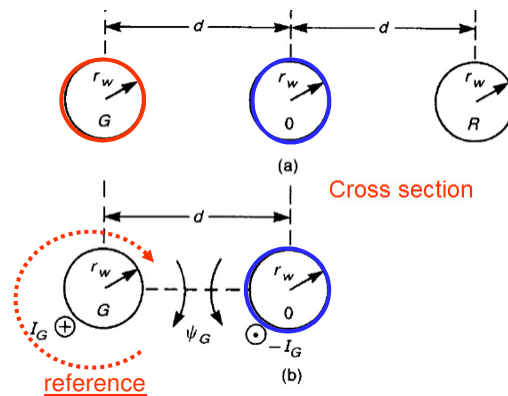


a) self-inductances calculation

Apply I_G , set $I_R=0$, with coupling through generator circuit

$$l_G = \frac{\mu_0}{2\pi} \ln\left(\frac{d}{r_w}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{d}{r_w}\right)$$

$$= \frac{\mu_0}{\pi} \ln\left(\frac{d}{r_w}\right)$$



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

An example: The three-wire ribbon cable.

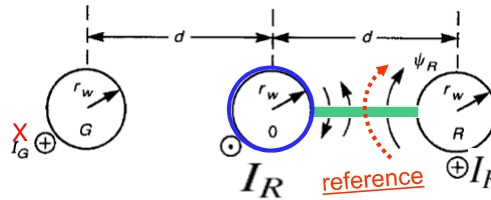
a) self-inductances calculation



Similarly

Receptor self inductance :

$$l_R = \frac{\mu_0}{\pi} \ln\left(\frac{d}{r_w}\right)$$



$$\psi_R = l_m I_G + l_R I_R$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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The Per-Unit-Length Parameters (Cont.)

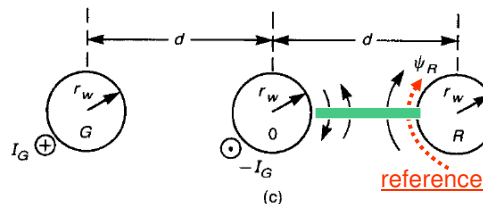
An example: The three-wire ribbon cable.

b) mutual-inductance calculation

Apply I_G , set $I_R=0$, with coupling through receptor circuit

$$l_m = -\frac{\mu_0}{2\pi} \ln\left(\frac{2d}{d}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{d}{r_w}\right) = \frac{\mu_0}{2\pi} \ln\left(\frac{d}{2r_w}\right)$$

$$\psi_R = l_m I_G + l_R I_R$$

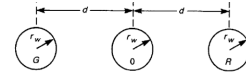


Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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Example (Numerical values)

For the ribbon cable with $r_w = 7.5$ mils and $d = 50$ mils



- We obtain $l_G = l_R = 0.759$ mH/m and $l_m = 0.24$ mH/m.
- The per-unit-length capacitances can be computed from these results, using matrix inversion: as $c_G = c_R = 11.1$ pF/m. and $c_m = 5.17$ pF/m

$$\mathbf{C} = \mu\epsilon\mathbf{L}^{-1}$$

- We can compute the characteristic impedance of each isolated circuit from (assume air, $v_o =$ speed of light)

$$Z_C = v_o l_G = v_o l_R = 227.7 \Omega$$

- We can also compute the characteristic impedance of one circuit in the presence of the other circuit,

$$Z_C = \sqrt{l_G / (c_G + c_m)} = 216 \Omega$$

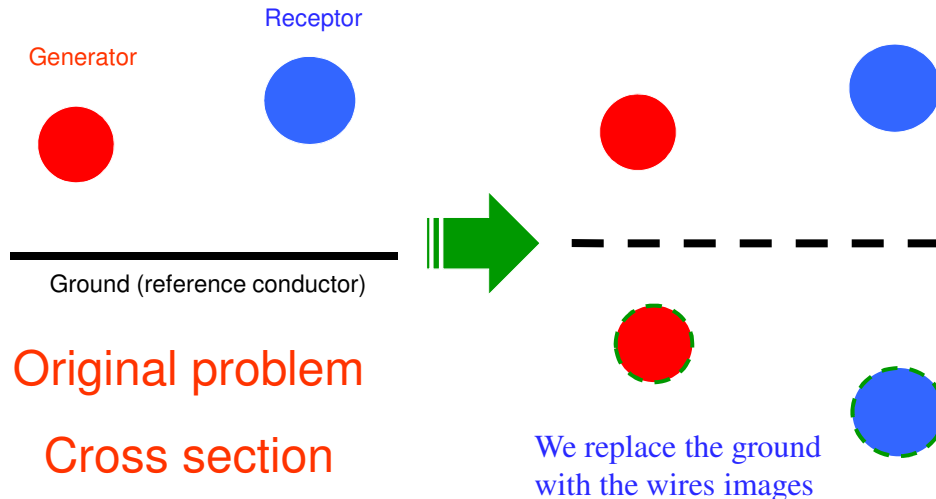
- The characteristic impedance of one circuit (line) is affected by the presence of the other circuits (lines).

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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The Per-Unit-Length Parameters (Cont.)

Example: Two wires above a ground plane.



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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The Per-Unit-Length Parameters (Cont.)

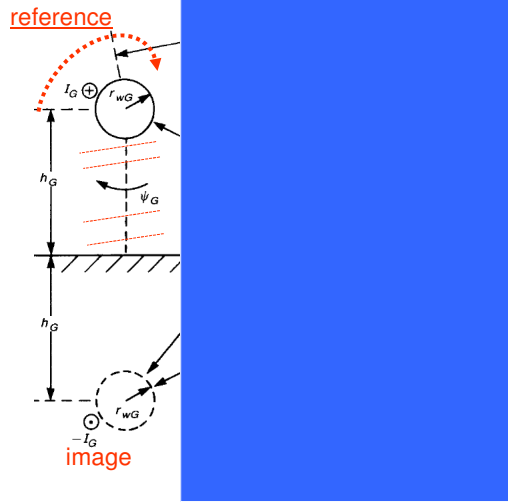
Example: Two wires above a ground plane (Cont.)

We replace the ground with the wires images and apply the same procedure:

a) self-inductances calculation

Apply I_G , set $I_R=0$, with coupling through generator circuit

$$\begin{aligned}
 l_G &= \left. \frac{\psi_G}{I_G} \right|_{I_R=0} \\
 &= \frac{\mu_0}{2\pi} \ln\left(\frac{h_G}{r_{wG}}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{2h_G}{h_G}\right) \\
 &= \frac{\mu_0}{2\pi} \ln\left(\frac{2h_G}{r_{wG}}\right)
 \end{aligned}$$



Similarly:

$$l_R = \frac{\mu_0}{2\pi} \ln\left(\frac{2h_R}{r_{wR}}\right)$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

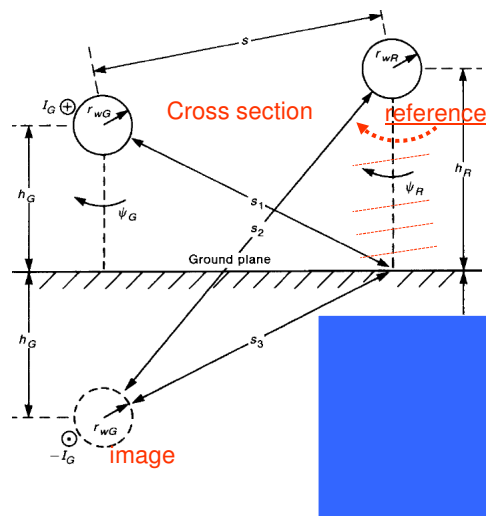
Example: two wires above a ground plane(Cont.)

We replace the ground with the wires images and apply the same procedure:

b) mutual-inductance calculation

Apply I_G , set $I_R=0$, with coupling through receptor circuit

$$\begin{aligned}
 l_m &= \left. \frac{\psi_R}{I_G} \right|_{I_R=0} \\
 &= \frac{\mu_0}{2\pi} \ln\left(\frac{s_1}{s}\right) + \frac{\mu_0}{2\pi} \ln\left(\frac{s_2}{s_3}\right) \\
 &= \frac{\mu_0}{2\pi} \ln\left(\frac{s_2}{s}\right)
 \end{aligned}$$

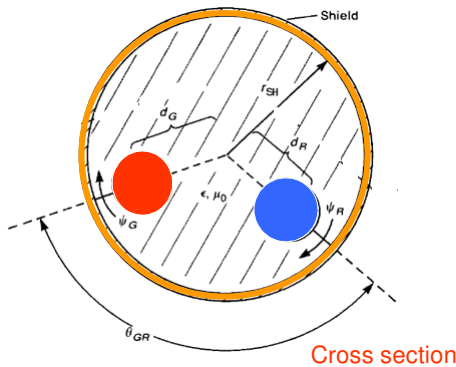


Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

The Per-Unit-Length Parameters (Cont.)

Example: two wires within an overall, cylindrical shield.

Just use the formula, no need for the background theory



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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The Per-Unit-Length Parameters (Cont.)

Example: two wires within an overall, cylindrical shield.

Just use the formula, no need for the background theory

a) self-inductances

We replace the shield with the wires images.

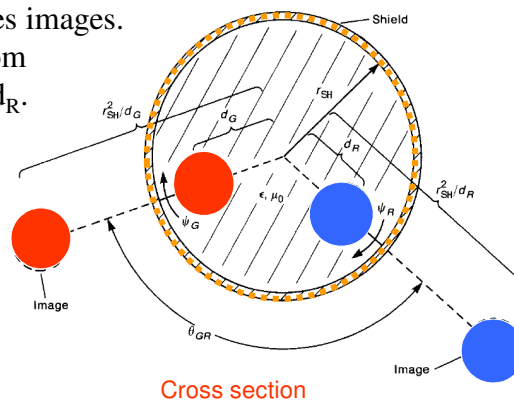
Each image lies on a radial line from

the shield center: r_{SH}^2/d_G and r_{SH}^2/d_R .

It can be shown that:

$$l_G = \frac{\mu_0}{2\pi} \ln \left(\frac{r_{SH}^2 - d_G^2}{r_{SH} r_{wG}} \right)$$

$$l_R = \frac{\mu_0}{2\pi} \ln \left(\frac{r_{SH}^2 - d_R^2}{r_{SH} r_{wR}} \right)$$



b) mutual-inductance

$$l_m = \frac{\mu_0}{2\pi} \ln \left[\frac{d_R}{r_{SH}} \sqrt{\frac{(d_G d_R)^2 + r_{SH}^4 - 2d_G d_R r_{SH}^2 \cos \theta_{GR}}{(d_G d_R)^2 + d_R^4 - 2d_G d_R^3 \cos \theta_{GR}}} \right]$$

Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

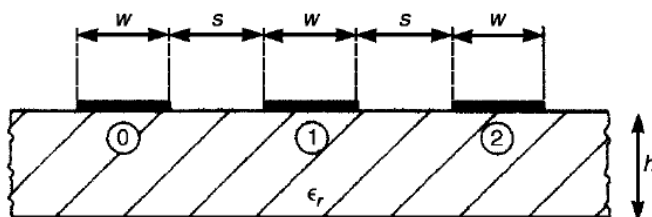
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The Per-Unit-Length Parameters (Cont.)

Numerical techniques are usually used to calculate the pul for general cases involving:

- General cross sections
- Dielectric materials
- PCBs

Commercial and other CAD tools are available to determine the pul for these general practical structures.



Source: P.R. Clayton: Introduction to Electromagnetic Compatibility, 2nd edition, 2006, John Wiley & Sons.

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