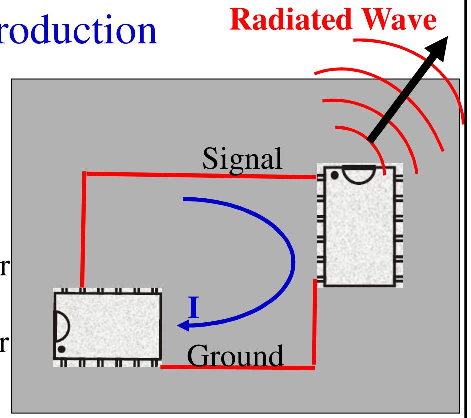
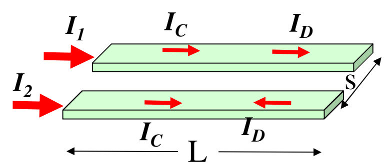
Radiated Emission: Introduction

- In this part, we will discuss the mechanisms by which EM fields are generated in an electronic device.
- Time-varying currents on wires, PCB lands, or any other conductor in a system will radiate (unintentional antennas). Consider a pair of parallel wires or PCB lands of length L and separation s.



- Suppose that the currents at the same cross section are directed to the right and denoted as I_1 and I_2.



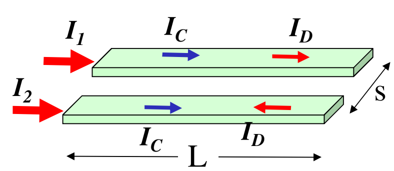
where $\hat{I}_1 = \hat{I}_C + \hat{I}_D$ & $\hat{I}_2 = \hat{I}_C - \hat{I}_D$

with $\hat{I}_C = \frac{\hat{I}_1 + \hat{I}_2}{2}$ & $\hat{I}_D = \frac{\hat{I}_1 - \hat{I}_2}{2}$

More later on I_C and I_D.

1

Differential-mode and common-mode components:

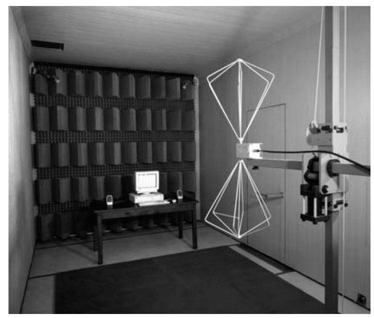


- At a cross section of the line, the differential-mode currents I_D are equal in magnitude but opposite in direction. I_D's are the functional or desired currents on the line.
- The transmission-line model will predict only these differential-mode currents.
- The common-mode currents I_C are undesired currents. At any line cross section the I_C's are equal in magnitude but are directed in the same direction.
- I_C currents are sometimes called "antenna-mode currents."
- The transmission line model will not predict I_C currents.
- Typically, the common-mode currents will be substantially smaller than the differential-mode currents.

2

Radiated emissions: Theoretical model

- To be able to derive simple models for the radiated emissions from the currents on wires and PCB lands, we will briefly present an overview of **antenna fundamentals** (Chapter 7).
- Antennas are devices that convert time-varying currents into a radiated electromagnetic field (transmitting) and vice-versa (receiving).
- An antenna is an essential part of a test system and is used to verify compliance to the governmental regulatory limits.
- In some applications, selecting the best antenna for the job may be very important.

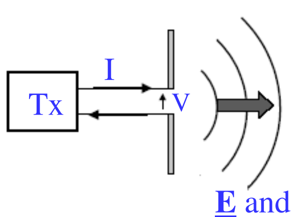


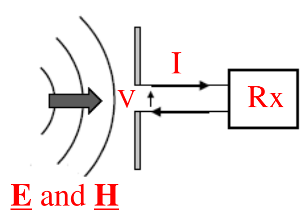
More later on Radiated Emissions (Chapter 8)

3

Antennas: Introduction (Chapter 7)

- An antenna is an electrical conductor or system of conductors
  - **Transmission:** radiates electromagnetic energy into space
  - **Reception:** collects electromagnetic energy from space.





Transmitting antenna transforms power in the form time-dependent electrical current into time-and-space-dependent electromagnetic (EM) wave.

Receiving antenna transforms time-and space-dependent EM wave into time-dependent electrical current (power)

- In two-way communication, the same antenna can be used for transmission and reception

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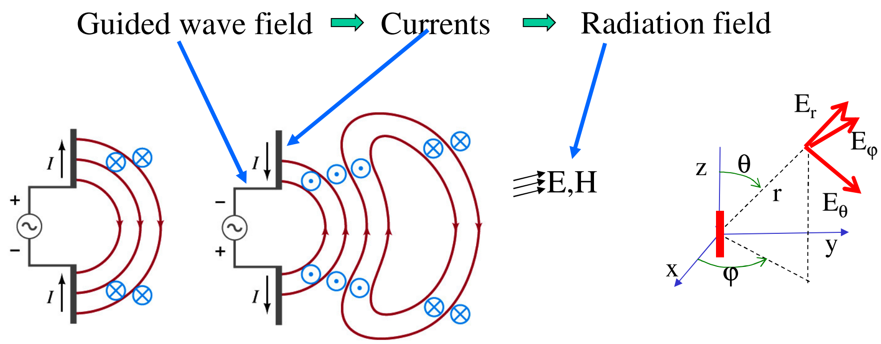
Source of Electromagnetic Waves

- An electric current that periodically changes direction produces time-varying electric and magnetic fields that spread outward from the source.

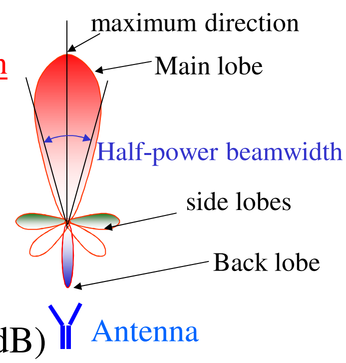


- The magnitudes of *E* and *H* decrease with distance *r* from the source as *1/r*.

5

Antennas: Electrical Parameters

Antenna Radiation Pattern

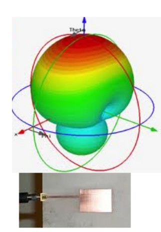


Front end of antenna

- Gain (dBi)
- Beamwidth (degrees)
- Radiation Pattern
- Cross Polarization Discrimination (dB)
- Front to Back Ratio (F/B)

Back end of antenna

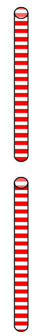
- Voltage Standing Wave Ratio (VSWR)
- Return Loss (RL - dB)

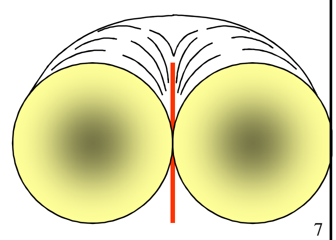


6

Types of Antennas: Dipole Antenna

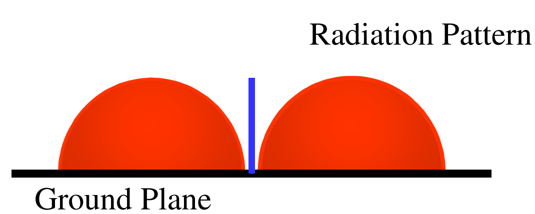
- Dimensions - about 1/2 lambda. Fairly rugged.
- Gain - about (1.64) 2.15 dB
- Bandwidth - 5 percent of center frequency
- Polarization - linear
- Power - up to 10's of Watts
- Weight - light
- Radiation resistance ≅ 73 Ω





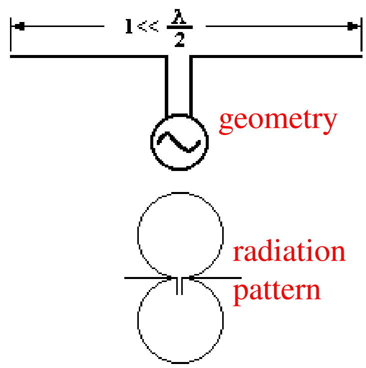
Types of Antennas: Monopole Antenna

- In many engineering applications, antennas are mounted on ground planes
- In practice, ground planes are metallic finite size and may not be planar
- A monopole antenna is ¼ wavelength fed at one end
- For a car antenna, the car is the ground plane
- Gain - about (3.28) 5.15 dB
- Radiation resistance ≅ 36.5 Ω



Types of Antennas:
The Hertzian (infinitesimal) Dipole

- The length is much less than λ/2.
- The self impedance is generally capacitive.
- The radiation resistance is quite small.
- SWR bandwidth is quite small, ~ 2% of design frequency.
- Directivity is ~ (1.5) 1.8 dB.



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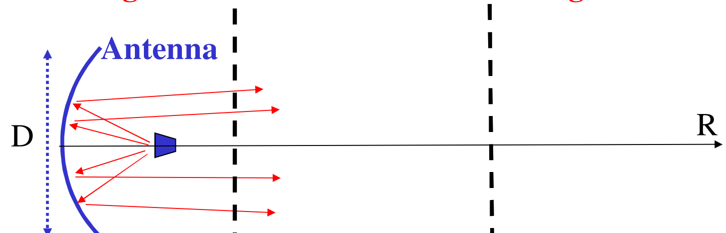
Distance to Antenna Far Field

Region 1 **Region 2** **Region 3**



- Non Radiating Near Field region: R < λ

  R < 0.62√(D³/λ)

  Induction (reactive) field

- Radiating Near Field region:

  λ < R < 2D²/λ

- Far Field region (Radiated field):

  R > 2D²/λ

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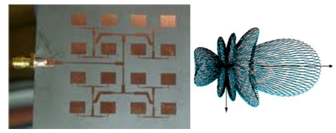
Mathematical Tools for Antenna Analysis

Source ⇒ Vector potential ⇒ Fields



For electric sources J,

the magnetic vector potential A is

$$\vec{A} = \frac{\mu}{4\pi}\iiint_{volume} \vec{J}\,\frac{e^{-jkR}}{R}\,dv$$

For magnetic and

electric fields are given by

$$\vec{H}_A = \nabla \times \vec{A}/\mu$$

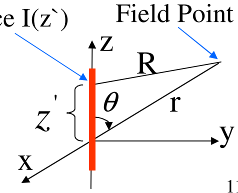
$$\vec{E}_A = -j\omega\vec{A} - j\nabla\left(\nabla\cdot\vec{A}\right)/(\omega\mu\varepsilon)$$

Special case:

For z directed line sources (Far field)

$$\vec{A} = A_z\hat{z} = \hat{z}\,\mu\frac{e^{-jkr}}{4\pi r}\int_{length} I(z')e^{jkz'\cos\theta}dz'$$

$$k = \beta = 2\pi/\lambda$$



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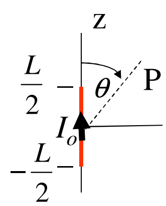
Hertzian dipole L << λ : General expressions

Assume the current is given by:

$$I(z') = \begin{cases} I_0 & x = 0, y = 0, |z| \le L/2, \quad L << \lambda \\ 0 & \text{elsewhere} \end{cases}$$



One can show that, at every point P in space (including near field region), except at the source,

$$\vec{A} = \hat{z}\frac{\mu I_0 \ell}{4\pi r}e^{-jkr}, \quad \Rightarrow \quad \vec{H} = \frac{1}{\mu}\nabla x\vec{A}, \quad \vec{E} = -j\omega\vec{A} - j\frac{\nabla(\nabla\bullet\vec{A})}{\omega\mu\varepsilon}$$

$$\mathbf{H}_r = 0, \quad \mathbf{H}_\theta = 0, \quad \& \quad \mathbf{H}_\phi = j\frac{kI_0\ell\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{I_0\ell\sin\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}, \quad E_\theta = j\eta\frac{kI_0\ell\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(jkr)^2}\right]e^{-jkr},$$

$$E_\phi = 0$$

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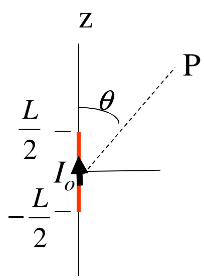
Far Field



$$I(z') = \begin{cases} I_0 & x' = 0, y' = 0, |z'| \le L/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$E_\theta = j\frac{60\pi \sin\theta \, e^{-jkr}}{r\lambda} \int_{-l/2}^{l/2} I(z') \, e^{jkz'\cos\theta} dz'$$

$\cong 1$

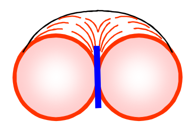
$$\therefore E_\theta = j\frac{60\pi I_0 L}{\lambda} \frac{e^{-jkr}}{r} \sin\theta$$

Radiation Pattern:
Max = 1 @ θ=90°

$$H_\phi = \frac{E_\theta}{\eta} = \frac{jI_0 L}{2\lambda} \frac{e^{-jkr}}{r} \sin\theta,$$

$$\eta = 120\pi \cong 377\Omega$$

All Other E & H fileds components = 0



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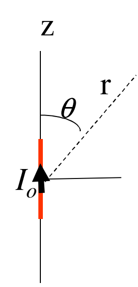
Review Exercise 7.1

The magnitude of the far electric field of a Hertzian dipole is measured at a distance of 100 m as 1 mV/m. Determine the magnitude of the electric field at 1000 m.

$$\therefore E_\theta = j\frac{60\pi I_0 L}{\lambda} \frac{e^{-jkr}}{r} \sin\theta$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{R_2}{R_1}$$



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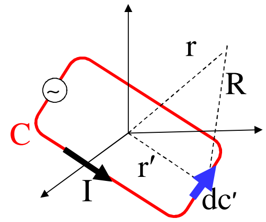
Loop Antennas

- Usually small in comparison with wavelength
- Used in AM receivers and direction finders
- May be air-wound or wound on a ferrite rod
- Bidirectional pattern



$$\mathbf{A} = \mu \frac{e^{-jkr}}{4\pi r}\int_{V'} \mathbf{J} e^{jk\bar{r}'\cdot\hat{r}} dv' = \mu \frac{e^{-jkr}}{4\pi r}\int_{C} \mathbf{I} e^{jk\bar{r}'\cdot\hat{r}} dc'$$

$$\text{E \& H fields: E} = -j\omega \text{A} \Downarrow$$

$$\mathbf{E} = \eta\beta^2 S \frac{\mu I e^{-j\beta r}}{4\pi r}\sin\theta\ \hat{\phi}$$

$$\mathbf{H} = \frac{1}{\eta}\hat{r}\times\mathbf{E} = -\beta^2 S \frac{\mu I e^{-j\beta r}}{4\pi r}\sin\theta\ \hat{\theta}$$

$S = Loop's\ area$

$$k = \beta = 2\pi/\lambda$$

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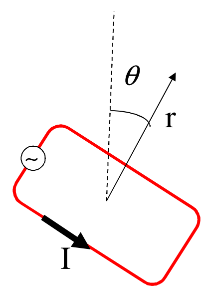
Example 7.2

consider a 1 x 1 cm current loop on a PCB Suppose that the loop carries a 100 mA current at a frequency of 50 MHz.

At a distance of 3 m we calculate E_{max}, 40.8 dBμV/m,.



$$\mathbf{E} = \eta\beta^2 S \frac{\mu I e^{-j\beta r}}{4\pi r}\sin\theta\hat{\phi}$$

$S = Loop's\ area$

$$k = \beta = 2\pi/\lambda$$

The FCC Class B limit from 30 to 88 MHz is 40 dBμV/m. Therefore this loop will cause a radiated emission that will fail to comply with the FCC Class B limit!

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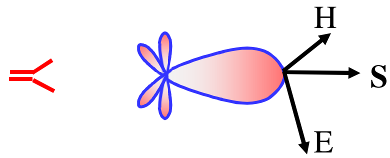
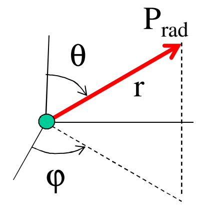
Antenna Characteristics: Radiated Power

- Power Strength at different angular points in 3D space is defined by ***Poynting Vector S*** (Radiated average power density *S*):





$$\vec{S}(r,\theta,\varphi) = \frac{1}{2}\mathrm{Re}\left[\vec{E} \times \vec{H}^*\right] = S\,\hat{a}_r \qquad [\mathrm{W/m^2}]$$

$$S = \frac{1}{2\eta}|E|^2 = \frac{\eta}{2}|H|^2$$

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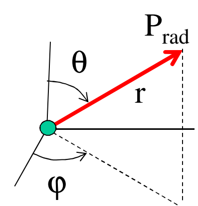
Antenna Characteristics: Radiated Power

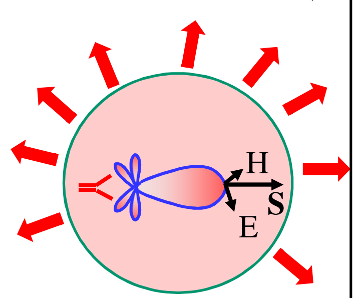
- Radiated power through a sphere (**total radiated power** P_{rad}):

$$P_{rad} = \iint \vec{S} \bullet d\vec{s} = \int_0^{2\pi}\int_0^{\pi} S(r,\theta,\phi) r^2 \sin\theta d\theta d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi) \sin\theta d\theta d\phi,$$



where $U = \mathrm{r}^2\left|\vec{\mathrm{S}}\right| = \frac{1}{2\eta}|E|^2 = \frac{\eta}{2}|H|^2$



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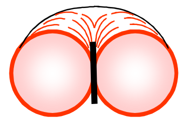
Antenna Characteristics: Radiated Power

- For **Hertzian** Dipole Source:

$$U(\theta) = \frac{\eta I_o^2 L^2}{8\lambda^2}\sin^2\theta \quad \text{[W/solid angle]}$$
$$= Radiation\ Intensity$$



$$P_{rad} = \iint_{4\pi} U(\theta,\phi)\sin\theta d\theta d\phi = \frac{\eta\pi I_o^2 L^2}{3\lambda^2} \quad \text{[W]}$$

Review Exercise 7.2
A Hertzian dipole is of length 1 cm
and carries a peak current of 100 mA at a frequency of 10 MHz.
Determine the total radiated power.

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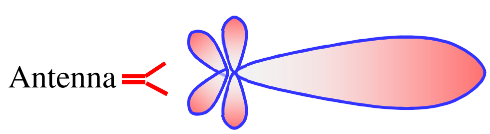
Antenna Characteristics: Directivity

The ability of an antenna to concentrate the radiated power, or conversely to absorb effectively the incident power from that direction, is specified in terms of its Gain, or Directivity.



Radiation Intensity
U (θ, φ)

- Directivity is defined as:

$$D_o = \frac{U_{max}}{U_o} = \frac{U_{max}}{P_{rad}/4\pi} = \frac{4\pi U_{max}}{P_{rad}}$$

- **Note**: D_o has no units

- For **Hertzian** dipole

$$D_o = \frac{4\pi \bullet 1}{\iint_{4\pi}\sin\theta^3 d\theta d\phi} = 1.5$$

- For isotropic case [U=1], $U_{max} = 1$ & $P_{rad} = 4\pi \Rightarrow D_o = 1$

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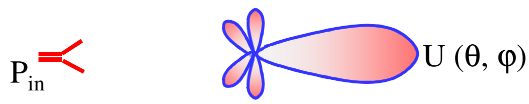
Antenna Characteristics: <u>Gain</u>

- <u>Antenna gain</u>: Power output compared to that produced by a perfect omnidirectional antenna (isotropic antenna).
- ***The gain of an antenna takes into account the losses of the antenna.***
- For <u>lossless antenna</u>, the gain and directivity are identical.



- Antenna Gain is defined as $G = \frac{4\pi U_{\max}}{P_{in}}$ ⇒ $G_{dB} = 10\log G$

- For **<u>Hertzian</u>** l dipole, $D_{dB} = 10\log 1.5 = 1.75 dBi$

<u>Note</u>: dBi, unit using isotropic reference
dBd, unit using half-wave dipole reference

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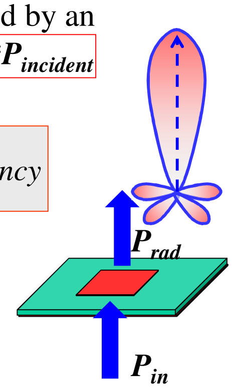
Antenna Characteristics: <u>Effective Area</u>

- <u>Effective area</u>: Related to physical size and shape of antenna
- It measure of the effective absorption area presented by an antenna to an incident plane wave: $\boldsymbol{P_{received} = A_e * P_{incident}}$
- It depends on the antenna gain and wavelength

$$A_e = e_{cd}\frac{\lambda^2}{4\pi}D = \frac{\lambda^2}{4\pi}G \quad [\text{m}^2], \quad e_{cd} = radiation \quad efficiency$$



- <u>Radiation efficiency</u>:

$$e_{cd} = P_{rad} / P_{in}$$

$$\left.\begin{array}{l} G = 4\pi U_{\max} / P_{in} \\ D = 4\pi U_{\max} / P_{rad} \end{array}\right\} \Rightarrow G = e_{cd} \cdot D$$

Typically: e_{cd} = 50-80%

- <u>Aperture efficiency</u>: $e_{aperture} = A_e / A$
  where A: physical area of antenna's aperture,

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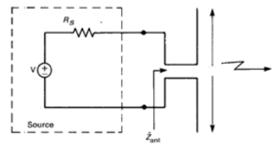
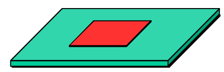
Example For a **Hertzian dipole**, show that $A_e = 3\lambda^2/(8\pi)$

Circuit theory :
assume matched circuit for max power transfer
$P_{rec} = V_a^2/(8R_{rad})$



with (antenna theory)

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \qquad and \qquad V_a = E_o l,$$



$P_{received} = A_e * P_{incident}$

also (EM theory) :
$P_{rec} = E_o^2 A_e/(240\pi)$

$S = |E|^2/2\eta$

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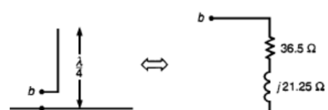
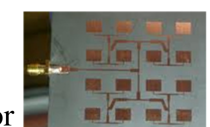
Antenna Characteristics: Input Impedance

- Transmitting Mode: the antenna has a specific input impedance: $Z_A = R_A + jX_A$
- Receiving Mode: the antenna acts as a voltage source with a series source impedance $Z_A = R_A + jX_A$.

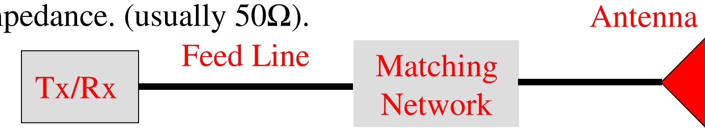


Monopole antenna



Array antenna

- The impedance Z_A is the same whether transmitting or receiving.
- Antenna impedance is frequency dependent so that most antennas only operate over a limited frequency range.
- To get maximum signal, the impedance of the antenna must be "matched" to the feeder line characteristic impedance. (usually 50Ω).



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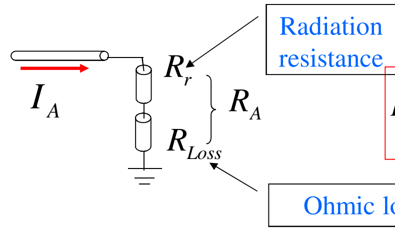
Antenna Characteristics: Circuit Model

$$P_{in} = \frac{1}{2} R_A |I_A|^2 = P_r + P_{loss}$$
$$= \frac{1}{2} R_r |I_A|^2 + \frac{1}{2} R_{loss} |I_A|^2$$



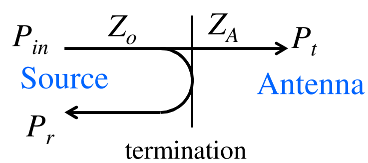
$$R_r = \frac{2P_r}{|I_A|^2}$$

$$R_{loss} = \frac{l_{wire}}{4\pi r_{wire} \sigma \delta}$$

- Antenna radiation efficiency:

$$e_{cd} = \frac{P_r}{P_{in}} = \frac{P_r}{P_r + P_{loss}} = \frac{R_r}{R_r + R_{loss}} = \frac{R_r}{R_A}$$

- Antenna impedance matching:



Reflection Coefficient Γ:

$$\Gamma = \frac{Z_A - Z_o}{Z_A + Z_o} = \frac{VSWR - 1}{VSWR + 1}$$

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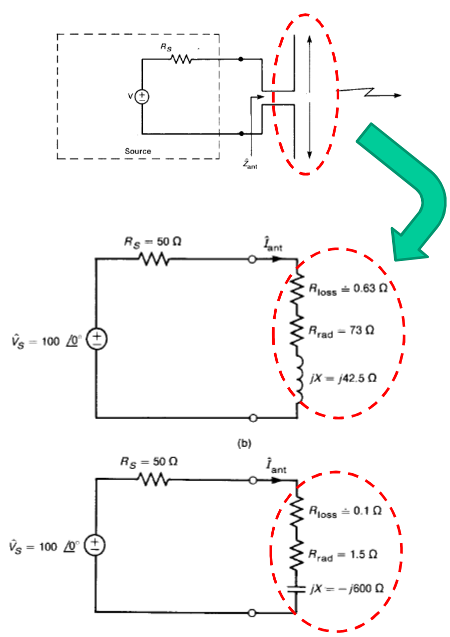
Circuit representation

Examples

Half-wave dipole

Hertzian dipole



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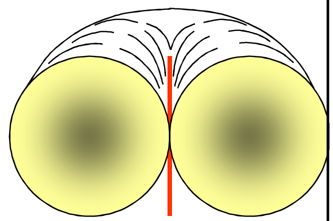
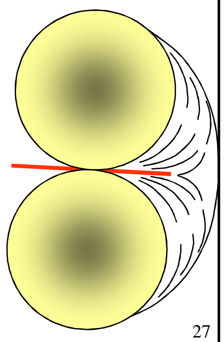
Antenna Examples: Infinitesimal (Hertzian) Dipole Source: L<<λ (L<λ/50)



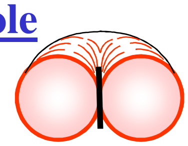
3D Radiation Pattern (intensity)

$$U(\theta) = \frac{\eta I_o^2 L^2}{8\lambda^2} \sin^2\theta$$



- Independent of ϕ
- Minimum @ $\theta = 0^o$ & Maximum @ $\theta = 90^o$
- Half Power Beamwidth (HPBW) = 90^o
- Directivity = 1.5
- $R_{rad} = 2P_{rad}/I^2 = 80\pi\,(L/\lambda)^2$

Antenna Examples: Half-Wave Dipole



Normalized field pattern: $$f(\theta) = \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$$

Power Density $$S_r(\theta) = \frac{\eta I_o^2}{8\pi^2 r^2}\left(\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right)^2 \text{ W/m}^2$$

Example: For 1 W/m^2 average radiated power density from a half-wave dipole, calculate the corresponding E.

Radiation Intensity

$$U(\theta) = \frac{\eta I_o^2}{8\pi^2}\left(\frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}\right)^2 \text{ W/solid angle}$$

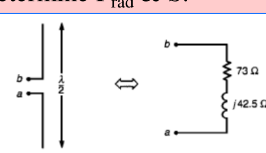
Review Exercise 7.3 A half-wave dipole carries a 100 MHz current whose magnitude (RMS) at the center of the dipole (the excitation point) is 100 mA. Determine P_{rad} & S.

Directivity of half-wave dipole:

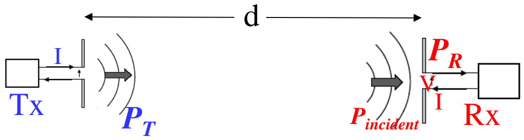
$$D \approx 1.64$$

For half-wave dipole, $R_{rad} = 2P_{rad}/I_o^2 \cong 73\ \Omega$

and $X_{in} \cong +42.5\ \Omega$



Friis Transmission Equation in Free Space



$$P_R = P_{incident} \cdot A_e$$
$$= \left(e_T \frac{D_T P_T}{4\pi d^2} \right) \left(e_R \frac{\lambda^2 D_R}{4\pi} \right)$$
$$= e_T e_R P_T D_T D_R \left(\frac{\lambda}{4\pi d} \right)^2$$
$$P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi d} \right)^2$$
$$= P_T A_{eT} A_{eR} / (\lambda d)^2$$
$$P_R(dB) = P_T + G_T + G_R - 20\log(4\pi d / \lambda)$$

- λ: wavelength [m]
- P_R: power available at the receiving antenna
- P_T: power delivered to the transmitting antenna
- D_T (G_T): directivity (gain) of the transmitting antenna
- D_R (G_R): directivity (gain) of the receiving antenna
- e_T: radiation efficiency of the transmitting antenna
- Matched polarizations
- Note field at distance d:

$$|E_T|^2 = \frac{60 P_T G_T}{d^2}$$

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Examples

1. What is the power received from GEO satellite (λ=0.1m, P_T =440 W, D_T=1000) at Trieste (distance ~38'000 km, D_R=1)? Assume 100% efficiencies.

$$P_R = P_T D_T D_R \left(\frac{\lambda}{4\pi r} \right)^2 = 4.4 \bullet 10^2 \bullet 10^3 \bullet \left(\frac{0.1}{4 \bullet \pi \bullet 38 \bullet 10^6} \right)^2$$
$$\approx 1 \bullet 10^{-15}\ \text{W} = -150\ \text{dB(W)}$$

2. What is the power from a transmitter (λ=0.1m, P_T=440 mW, D_T=1) received at distance of 3.8 cm (D_R=1)?

$$P_R = P_T D_T D_R \left(\frac{\lambda}{4\pi r} \right)^2 = 4.4 \bullet 10^{-1} \bullet 1 \bullet 1 \bullet \left(\frac{0.1}{4 \bullet \pi \bullet 3.8 \bullet 10^{-2}} \right)^2$$
$$\approx 10^{-5}\text{W} = -50\ \text{dB(W)}$$

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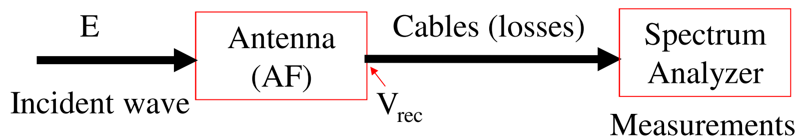
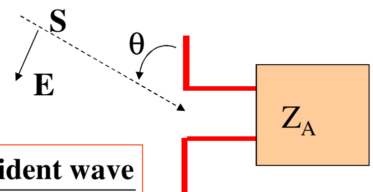
Antenna Factor



For EMC applications (measurements), the Antenna Factor (AR) is defined as



$$AF = \frac{|E_{inc}|}{|V_{rec}|} = \frac{E - Field\ Strength\,(V/m)\ \textbf{in incident wave}}{\textbf{V received}}$$

$$\mathrm{AF_{dB}} = \mathrm{dB\mu V/m\ (incident\ field) - dB\mu V\ (received\ voltage)}$$

Note for maximum power transfer: $Z_A = Z_S^*$ **&** $P_R = \dfrac{|V_{O.C.}|^2}{8R_{rad}}$

For an infinitesimal dipole: $|V_{O.C.}| = |E| \bullet dl \bullet \sin\theta$

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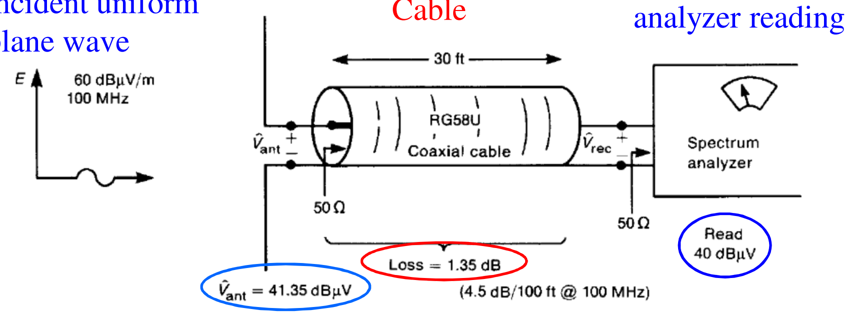
Example 7.10:

linearly polarized, incident uniform plane wave

Cable

Spectrum analyzer reading



Add Cable loss to the reading

What is the antenna factor?

$$\mathrm{AF_{dB}} = \mathrm{dB\mu V/m\ (incident\ field) - dB\mu V\ (received\ voltage)}$$

$$\mathrm{AF_{dB}} = 60\,\mathrm{dB\mu V/m} - 41.35\,\mathrm{dB\mu V}$$
$$= 18.65\,\mathrm{dB}$$

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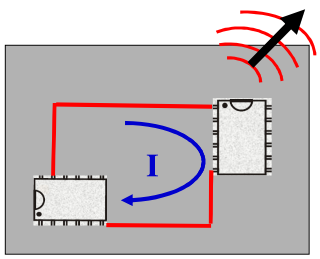
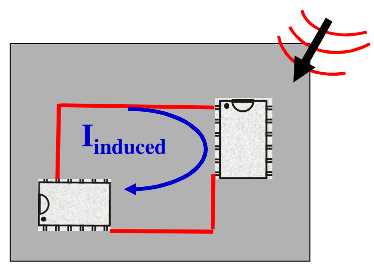
Radiated Emission/Susceptibility: Wires, PCB lands, or any other conductor

- The antenna theory presented so far will be used next to evaluate radiation emission by wires, PCB lands, or any other conductor: i.e., radiation by differential and common mode currents
- Also, we will investigate the ability of a product to be susceptible to radiated emissions from other electronic devices by deriving simple models that give the voltages and currents induced in parallel-conductor lines by an incident uniform plane.





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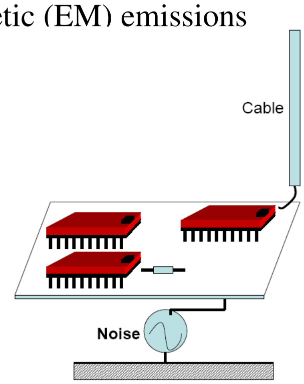
Radiated Emissions and Susceptibility (Chapter 8)

All electronic devices produce electromagnetic (EM) emissions (radiation), but we classify them as



Intentional Radiation

designed to produce EM radiation (TV trans., cell phones, radar etc)

Unintentional Radiation

not designed to produce EM radiation (computer, VCR, auto ignition etc)

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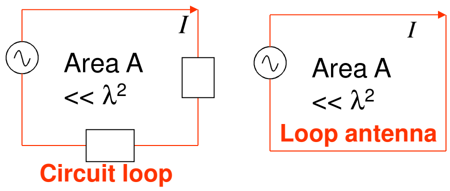
Radiation from Circuits

- Just like antennas – electronic circuits radiate (and by reciprocity receive) EM energy
- Currents radiate whether they are on antennas or in electric circuits



- The total power radiated is

$$P_{rad} == \int_0^{2\pi}\int_0^{\pi} |E|^2 /(2\eta) \bullet r^2 \sin\theta d\theta d\phi$$

In both cases,

$$P_{rad} = \frac{4\pi^3}{3}\eta I^2 \left[\frac{A}{\lambda^2}\right]^2 \quad \begin{cases} \eta \approx 377\Omega \\ I : \text{phasor current (peak)} \end{cases}$$

- Note that the power radiated depends upon the electric size of the loop, i.e. its size in wavelengths $\lambda = c/f = 3\times10^8 / f$

f	λ(m)	Loop P_{rad}(W)
60Hz	5×10^6	2.5×10^{-31}
1kHz	3×10^5	1.9×10^{-26}
1MHz	300	1.9×10^{-14}
1GHz	0.3	1.9×10^{-2}

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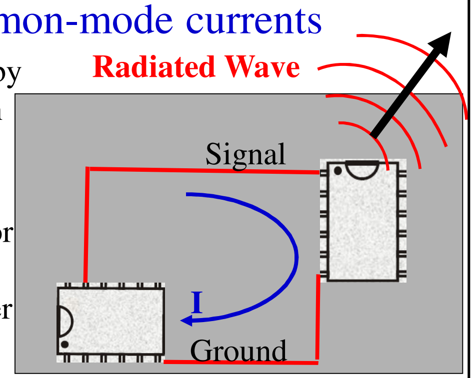
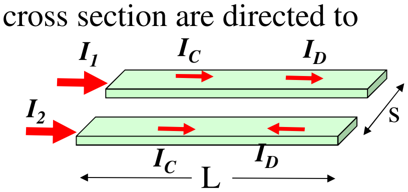
Radiated Emission: Differential & common-mode currents

- We will discuss the mechanisms by which EM fields are generated in an electronic device.
- Time-varying currents on wires, PCB lands, or any other conductor in a system will radiate (unintentional antennas). Consider a pair of parallel wires or PCB lands of length L and separation s.



- Suppose that the currents at the same cross section are directed to the right and denoted as I_1 and I_2.



where $\hat{I}_1 = \hat{I}_C + \hat{I}_D$ & $\hat{I}_2 = \hat{I}_C - \hat{I}_D$

with $\hat{I}_C = \frac{\hat{I}_1 + \hat{I}_2}{2}$ & $\hat{I}_D = \frac{\hat{I}_1 - \hat{I}_2}{2}$

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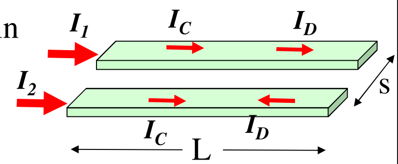
Differential-mode and common-mode currents

- At a cross section of transmission line, the differential-mode currents I_D are equal in magnitude but opposite in direction.
  - I_D's are the functional or desired currents on the line.
  - The transmission-line model will predict only these differential-mode currents.



- The common-mode currents I_C are undesired currents.
  - At any line cross section the I_C's are equal in magnitude but are directed in the same direction.
  - I_C currents are sometimes called "antenna-mode currents."
  - The transmission line model will not predict I_C currents.
  - Typically, the common-mode currents I_C are substantially smaller than the differential-mode currents I_D.

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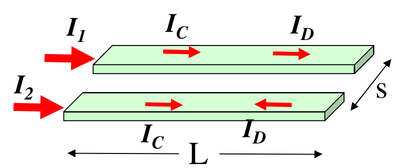
Simple Emission Models for Wires and PCB Lands

We may decompose the current in a conductor in terms of auxiliary currents: common and differential mode currents,



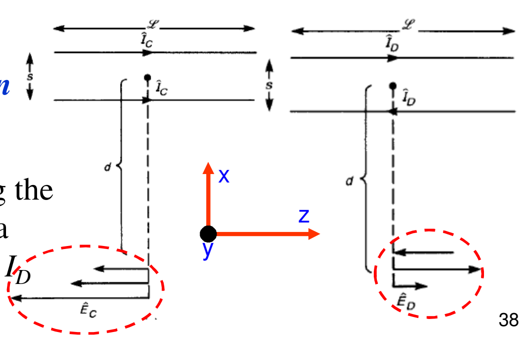
$$\hat{I}_1 = \hat{I}_C + \hat{I}_D \quad \& \quad \hat{I}_2 = \hat{I}_C - \hat{I}_D$$

Differential-Mode & Common-Mode Currents are given by:

$$\hat{I}_D = \frac{\hat{I}_1 - \hat{I}_2}{2} \quad \& \quad \hat{I}_C = \frac{\hat{I}_1 + \hat{I}_2}{2}$$

Both I_C & I_D will be present in practical systems.

We are interested in calculating the far-field radiated emissions of a two-conductor line due to I_C & I_D



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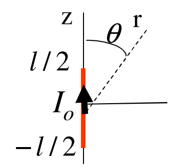
Computation of the far-field radiated emissions of a two-conductor line:

Use Antenna Background Theory **(Chapter 7)**

1. Hertzian Dipole Source:

$$E_\theta = j\frac{60\pi I_0 L}{\lambda}\frac{e^{-j\beta r}}{r}\sin\theta \Big|_{\theta=90^o} = j\frac{60\pi I_0 L}{\lambda}\frac{e^{-j\beta r}}{r}$$



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Computation of the far-field radiated emissions of a two-conductor line:

2. Array of two elements

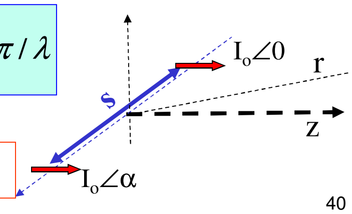
Radiated emission in the plane of the wires and broadside to the wires (worst case)

Element pattern **Array factor**

$$E_{tot} = E_1 + E_2 = j\frac{60\pi I_0 l}{\lambda}\frac{e^{-j\beta r}}{r} * \left(2\cos\left(\frac{\beta s}{2}\cos\phi + \frac{\alpha}{2}\right)\right)$$

$$E_{total-\max} = j\frac{120\pi I_0 l}{\lambda}\frac{e^{-j\beta r}}{r}\cos\left(\frac{\beta s}{2} + \frac{\alpha}{2}\right), \quad \beta = 2\pi/\lambda$$

Note: for I_C: $\alpha = 0$ & For I_D: $\alpha = 180^o$



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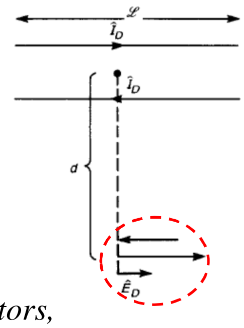
A) Differential-Mode Current Emission Model



For I_D: $\alpha = 180^o$, and $I_o = I_D$
At a distance $r=d$, the maximum radiated emission in the plane of the wires and broadside to the wires (worst case)

$$\therefore \left|\mathrm{E}_{D-\max}\right| = \frac{120\pi}{\lambda}\frac{|I_D| l}{d}\sin\left(\frac{\beta s}{2}\right)$$

For *small electrical spacing between the two conductors,*

$$\text{For } s \ll \lambda,\ \sin(\beta s/2) \cong \beta s/2$$

Using $\beta=2\pi/\lambda$ and $c=\lambda f$, the maximum radiated emission due to a differential-mode current reduces to:

$$\left|\mathrm{E}_{D-\max}\right| = 1.316 \times 10^{-14}\frac{|I_D| f^2 l s}{d}$$

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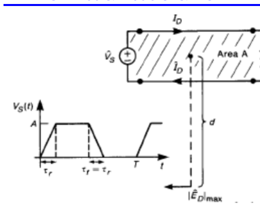
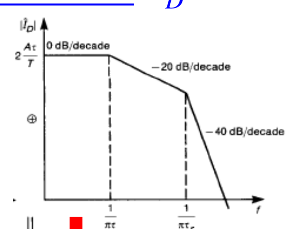
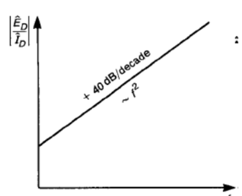
The radiated emissions for a trapezoidal pulse train: I_D

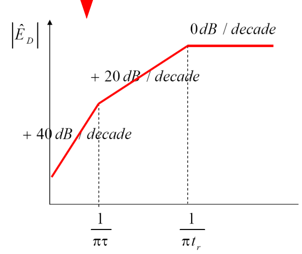


Bounds of a trapezoidal pulse train: Section 3.2.2



$$\left|\mathrm{E}_{D-\max}\right| = \mathrm{Constant} \bullet |I_D| f^2$$





Radiated emission problems due to differential mode currents tend to be confined to the upper frequencies of the radiated emission regulatory limit, typically above 100MHz.

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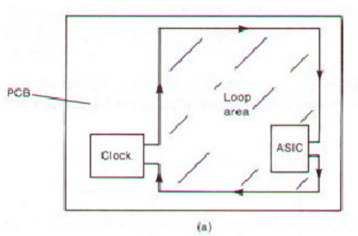
Common mistakes that lead to unnecessarily large differential-mode emissions

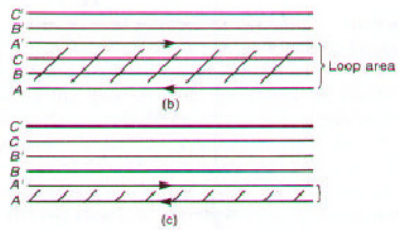
$$\left|E_{D-\max}\right| = 1.316\times10^{-14}\frac{\left|I_D\right| f^2 l s}{d}$$

Options to reduce differential-mode EM:

1. Reduce the current level
2. Reduce the loop area



Place the ASIC close to the clock;



choices of connector pin assignments in ribbon cables to minimize loop areas.

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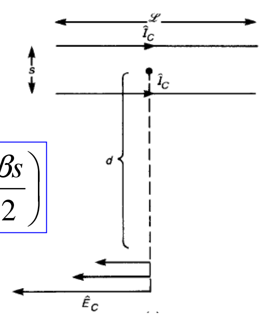
B) Common-Mode Current Emission Model

For I_C: $\alpha = 0°$, and $I_o = I_C$

At a distance $r=d$, the maximum radiated emission in the plane of the wires and broadside to the wires (worst case)

$$E_{C-\max} = \frac{120\pi}{\lambda}\frac{\left|I_C\right| l}{d}\cos\left(\frac{\beta s}{2}\right)$$



For *small electrical spacing between the two conductors:* with $s \ll \lambda$, $\cos(\beta s/2) \cong 1$

Using c=λf, the maximum radiated emission due to a common-mode current reduces to:

$$\left|E_{C-\max}\right| = 1.257\times10^{-6}\left|I_C\right| f\, l/d$$

Options to reduce common-mode radiated emissions:

1. Reduce the current level
2. Reduce the line length
3. Using ferrite bead / common choke.

Note: Properties of ferrite bead and common choke are discussed in sec. 5.8-5.9.

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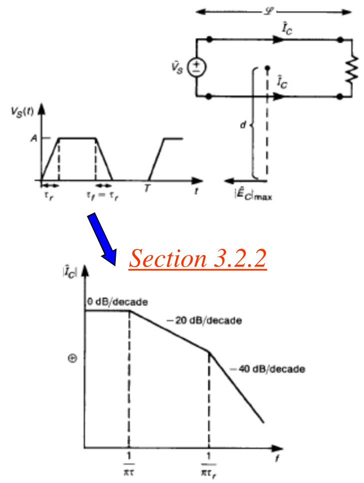
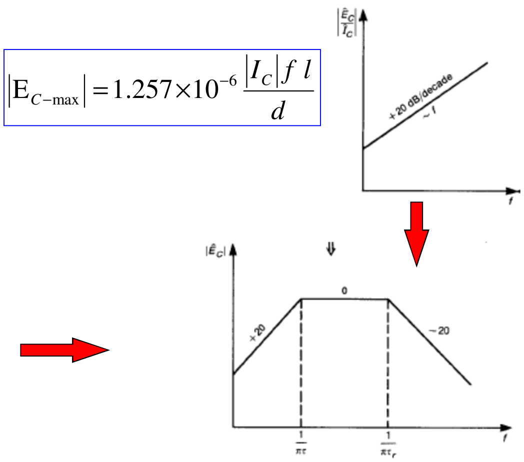
The radiated emissions for a trapezoidal pulse train: I_C



$$|E_{C-\max}| = 1.257\times10^{-6}\frac{|I_C|\,f\,l}{d}$$



Radiated emission problems due to common mode currents tend to be confined to the lower frequencies of the radiated emission regulatory limit, typically below 200MHz

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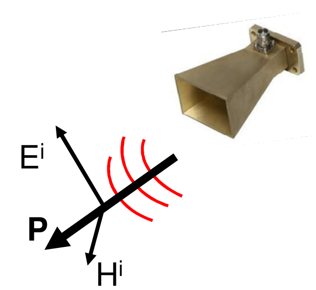
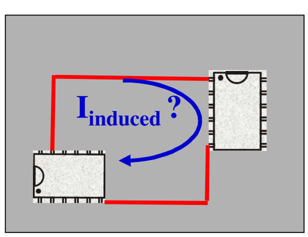
Simple Susceptibility Models for Wires





Given an incident EM wave, what is the induced signal in a PCB?

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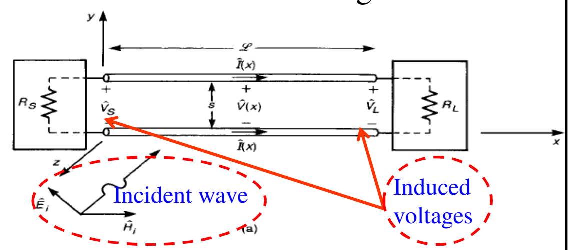
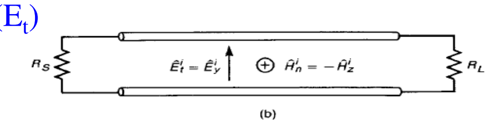
Simple Susceptibility Models for Wires

Consider a two-conductor line with an incident EM wave.
The objective is to determine the induced terminal voltages.

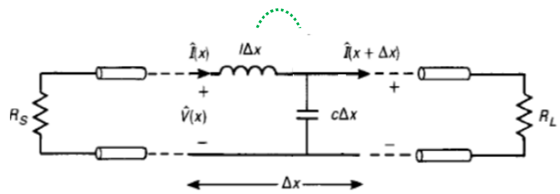
a. Problem definition;



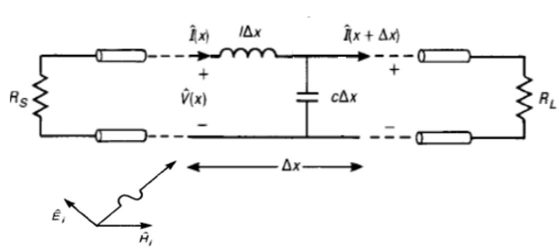
b. The transverse electric field (E_t) and the normal magnetic field (H_n) components



c. a per-unit-length equivalent circuit.

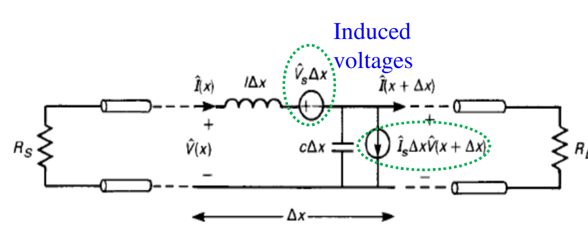


Simple Susceptibility Models for Wires



Incident wave will induce signal

d. a per-unit-length equivalent circuit.



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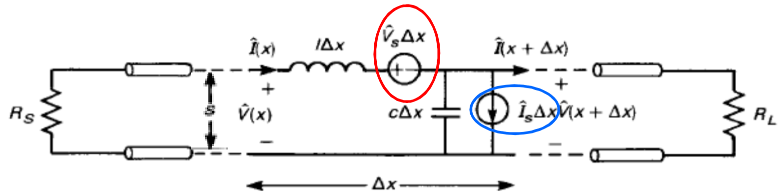
Simple Susceptibility Models for Wires (Cont.)



Due to the incident EM wave, we can show that the equivalent per-unit-length induced voltage and current sources:

$$\hat{V}_s(x) = j\omega\mu_0 \int_{y=0}^{s} \hat{H}_n^i dy$$

Faraday's law; induced emf due to H^i

$$\hat{I}_s(x) = j\omega c \int_{y=0}^{s} \hat{E}_t^i dy$$

Induced emf due to E^i → Norton equivalent

c is the per-unit-length capacitance of the line

The incident fields (E^i, H^i) at the position of the line may be produced by some distant antenna and can be determined using antenna theory.

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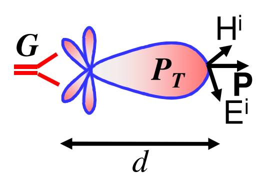
Simple Susceptibility Models (Cont.)

- The antenna producing these incident fields is assumed to be:
  - transmitting a total radiated power P_T ,
  - is located a distance d away, and
  - has a gain G in the direction of the line.



- The incident fields are:

$$|E^i| = \sqrt{\frac{60 P_T G}{d}}, \qquad |H^i| = \frac{|E^i|}{\eta}$$

where $\eta = 120\pi = 377\Omega$

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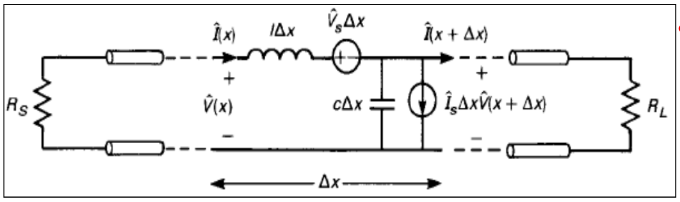
Simple Susceptibility Models (Cont.)

- From the per-unit-length model in Fig. (c) we may derive the transmission line equations that relate the voltage and current along the line and solve them to obtain V and I at any point on the line.



- Note: hat symbol means phasor variable.

$$\hat{V}(x+\Delta x)-\hat{V}(x)=-j\omega l\Delta x\hat{I}(x)-\hat{V}_s(x)\Delta x$$

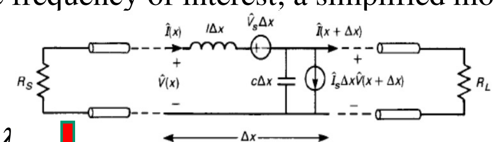
$$\frac{d\hat{V}(x)}{dx}+j\omega l\hat{I}(x)=-\hat{V}_s(x)=-j\omega\mu_0\int_{y=0}^{s}\hat{H}_n^i dy = H_n^i * s$$

$$\hat{I}(x+\Delta x)-\hat{I}(x)=-j\omega c\Delta x\hat{V}(x+\Delta x)-\hat{I}_s(x)\Delta x$$

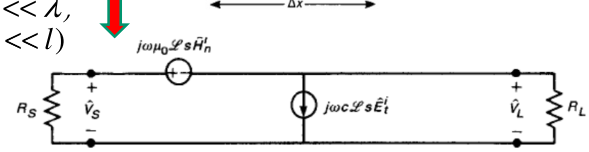
$$\frac{d\hat{I}(x)}{dx}+j\omega c\hat{V}(x)=-\hat{I}_s(x)=-j\omega c\int_{y=0}^{s}\hat{E}_t^i dy = E_n^i * s$$

Simple Susceptibility Models (Cont.)

- For many cases of practical interest the line length is electrically short at the frequency of interest; a simplified model is shown



($l << \lambda$,
($s << l$)



- From this model we can compute the induced terminal voltages as:

(Apply circuit theory: superposition and voltage division rules)

$$\hat{V}_S=\frac{R_S}{R_S+R_L}j\omega\mu_0\mathscr{L}s\hat{H}_n^i-\frac{R_SR_L}{R_S+R_L}j\omega c\mathscr{L}s\hat{E}_t^i$$

$$\hat{V}_L=-\frac{R_L}{R_S+R_L}j\omega\mu_0\mathscr{L}s\hat{H}_n^i-\frac{R_SR_L}{R_S+R_L}j\omega c\mathscr{L}s\hat{E}_t^i$$