École de gestion TELFER School of Management

LIÉE au LINKED with LEADERSHIP

ADM 2304 – Final Examination

APPLIED STATISTICAL METHODS IN BUSINESS

15 April 2013

PRINT YOUR NAME: ____

STUDENT NUMBER: ________________________________

SECTION (please circle one): M N P Q R S

Exam: **10 pages (including cover page)**

Appendices: **10 pages of Minitab output and 4 pages of statistical tables**

Complete all tests with hypotheses, test statistic, critical value, decision, and conclusion. Use the .05 significance level unless otherwise indicated. Explain your answers if asked.

Calculators and one sheet of notes (8.5 x 14 in.) are allowed.

Please hand back all exam materials (exam booklet, appendices and tables), but keep your personal sheet of notes for future reference.

Question	Value	Mark
1	8	
2	14	
3	7	
4	18	
5	23	
Total	**70**	

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ____

Problem 1. [8 marks]

A manufacturing plant for recreational vehicles receives shipments from three different parts vendors. There has been a quality issue with some of the electrical wiring in the recreational vehicles manufactured at the plant. The plant manager claims that the quality varies with the parts vendor. He asks for a review of a sample of quality assurance inspections from the last six months.

	Vendor		
Inspection Result	*Perfect Parts Co.*	*Made-4-U Co.*	*25 Hour Parts Co.*
Unacceptable	54	48	70
Perfect	94	72	88
Not perfect but acceptable	22	30	22

a) What is the probability that a randomly selected shipment is Unacceptable?

[1]

b) What is the probability that a randomly selected shipment is Unacceptable, given that it came from the Perfect Parts Co.?

[1]

c) State the appropriate null and alternative hypothesis to test whether the plant manager is correct in his belief that the quality varies with the parts vendor.

[1]

d) Compute the expected frequency and the contribution to the chi-square statistic that comes from the top-left cell (Unacceptable and Perfect Parts Co.), assuming the null hypothesis is true.

[2] Expected Frequency = Chi-square contribution =

e) What are the degrees of freedom for the chi-squared statistic in this situation?

[1]

f) The chi-squared statistic for this test is 6.63. Find the most precise bound for the p-value.

[1]

g) At the 5% significance level, what do you conclude?

[1]

Problem 2. [14 marks]

a) Automobile insurance appraisers examine cars that have been involved in collisions to assess the cost of repairs. A company wanted to examine whether different appraisers produce significantly different assessments of minor collisions. An experiment was designed in such a way that a total of 10 cars were used and each car was assessed by each appraiser. Here are the data:

Car	1	2	3	4	5	6	7	8	9	10
Appraiser 1	1650	360	660	1050	890	750	470	1270	550	730
Appraiser 2	1440	380	600	920	930	650	410	1080	480	770

An analyst was not sure which procedure was more appropriate to analyze the data – the matched pairs t-test or the two-sample t-test, but she completed the analyses in Appendix A.

i) Explain briefly which of the two tests is the better choice.

[1]

ii) Test whether there is enough evidence at the 5% significance level to show that the appraisers differ in their assessments.

[4]

iii) Explain what the Type I Error is in this situation.

[1]

iv) Explain by reference to the appropriate boxplot(s), if the t-test is appropriate or if the Wilcoxon signed ranks test would be better.

[1]

(b) Suppose that the same data came from a differently designed experiment where 20 cars were used; 10 were shown to Appraiser 1 and the other 10 to Appraiser 2.

Car	1	2	3	4	5	6	7	8	9	10
Appraiser 1	1650	360	660	1050	890	750	470	1270	550	730
Car	11	12	13	14	15	16	17	18	19	20
Appraiser 2	1440	380	600	920	930	650	410	1080	480	770

i) Which test is more appropriate in this case? (Circle your choice)

[1] 2-sample t-test Matched pairs t-test

ii) What is your conclusion from the test you chose?

[1]

iii) Do your conclusions in parts (a) and (b) agree? Explain why or why not.

[1]

iv) Construct a 95% confidence interval for the difference in mean appraisals between the two appraisers. Use the formula based on the pooled standard deviation; to simplify your calculations, use the s_p value of 361. Report your answer to the nearest whole numbers.

[3]

v) Explain, by reference to the appropriate boxplot(s), if the 2-sample t-test is appropriate or if the Mann-Whitney test would be better.

[1]

Problem 3. [7 marks]

A common metric by which emergency physicians are assessed is the rate at which patients they have seen in the emergency department return within 72 hours for the same complaint. The thinking is that such patients were sent home inappropriately. A certain hospital wanted to compare two of their emergency physicians based on this statistic. They analysed all patients seen by each physician over the last year. Physician A saw a total of 1443 patients and 97 patients returned within 72 hours with the same complaint. Physician B saw a total of 1558 patients and 122 of them returned within 72 hours with the same complaint.

(a) Test whether there is a difference between the two physicians in terms of the rate of return visits within 72 hours at the 0.10 significance level. Assume that the patients seen by each physician represent random samples.

[5]

(b) If you wanted to be within 0.01 of the true proportion of returns within 72 hours for physician A with 99% confidence, what sample size would be required?

[2]

Problem 4. [18 marks]

A researcher randomly assigns 60 rats to one of 6 diets – 10 rats to each feeding treatment. The treatments are comprised of each of three food sources (beef, cereal or pork) at either a high or low protein level. At the end of the experiment, the weight gain of each rat is recorded. Treatment means are shown in the table below:

Protein level	Food source	Weight Gain
High	Beef	100.0
High	Cereal	85.9
High	Pork	99.5
Low	Beef	79.2
Low	Cereal	83.9
Low	Pork	78.7

(a) Is this an observational study or a designed experiment? Explain briefly.

[1]

(b) How many treatment combinations are there?

[1]

(c) Identify the blocks, if applicable, and the number of replications of each treatment.

[1]

(d) The researcher wants to perform an ANOVA on the results. Consider the boxplots of the data and the residual plots in Appendix B. Explain with reference to the appropriate plot(s) whether the key assumptions of the ANOVA model are justified.

[2]

(e) Complete the ANOVA table for the data set.
[3]

Source	DF	Sum of Squares	Mean Square	F
Protein		3168.3		
Food			133.3	
Interaction		1178.1		
Error				NA
Total		16198.93	NA	NA

(f) Test whether the protein level and food source interact. Explain in plain language what the lack of interaction means in this case.
[4]

(g) Examine the interaction plots. Do these correspond to the result of the test in (f)? If not, what explanation can you offer?
[2]

(h) Construct a 95% Bonferroni confidence interval for the mean difference between a low protein beef diet and a high protein cereal diet. Hint: the relevant critical value of the t distribution is 3.07, but you must explain where this number comes from (i.e. specify the appropriate degrees of freedom and tail probability).

[4]

Problem 5. [23 marks]

The Canadian venture capital (VC) sector suffers from a shortage of large Canadian VC funds.[1] This induces a reliance on US funds. As startups grow, they require larger amounts of capital for further growth. However, few Canadian VC funds can supply the necessary sums of money. Consequently, startups typically seek sources of capital in the US. As a result, there is a debate: does such reliance on US sources affect entrepreneurial activity in Canada negatively? To address this question, Finance Canada decided to conduct research into this issue, and you are asked to help. Appendix C defines the variables and shows output from four different models.

(a) Looking at the output from Model 1 and Model 2, explain which assumptions of the regression model are not justified. State these assumptions briefly and explain why they may not be warranted.

[2]

(b) Based on Model 1, write down the estimated regression equations for the "AmountFinanced", one for companies receiving funds from a Canadian fund and the other for companies receiving funds from a U.S. fund.

[2]

(c) What does Model 1 assume regarding the relationship between "AmountFinanced" and "Age" for startups receiving Canadian funds and for those receiving U.S. funds?

[1]

[1] A large fund is referred to as one with a large amount capital under management – one that manages a large amount of money thus has a large amount of money to invest in Canadian startups. The average size (the amount of capital under management) of Canadian venture capital funds is $118 million CDN; whereas that of US funds is $335 million US.

(d) Based on Model 2, write down the estimated regression equations for the "AmountFinanced", one for companies receiving funds from a Canadian fund and one for companies receiving funds from a U.S. fund.

[2]

(e) What does Model 2 assume regarding the relationship between "AmountFinanced" and "Age" for startups receiving Canadian funds and for those receiving U.S. funds?

[1]

(f) Ignoring possible problems with the assumptions of the regression model as discussed in (a), use the summary statistics to explain briefly whether Model 1 or Model 2 fit the data better.

[2]

To resolve some of the problems with the model assumptions, the basic variables were transformed. These are described in the appendix. Using these new variables, two more models (Model 3 and Model 4) were estimated.

(g) Looking at the residual plots for Models 3 and 4, explain whether the problems with the assumptions of the regression model as identified in part (a) have been resolved. State these assumptions briefly and explain if they are now justified.

[2]

(h) Explain briefly whether Model 3 or Model 4 fits the data better, looking only at the summary statistics.

[2]

(i) Looking now at **Model 3** only, test the overall usefulness of the model. Use the critical value approach and the 0.05 level of significance.

[3]

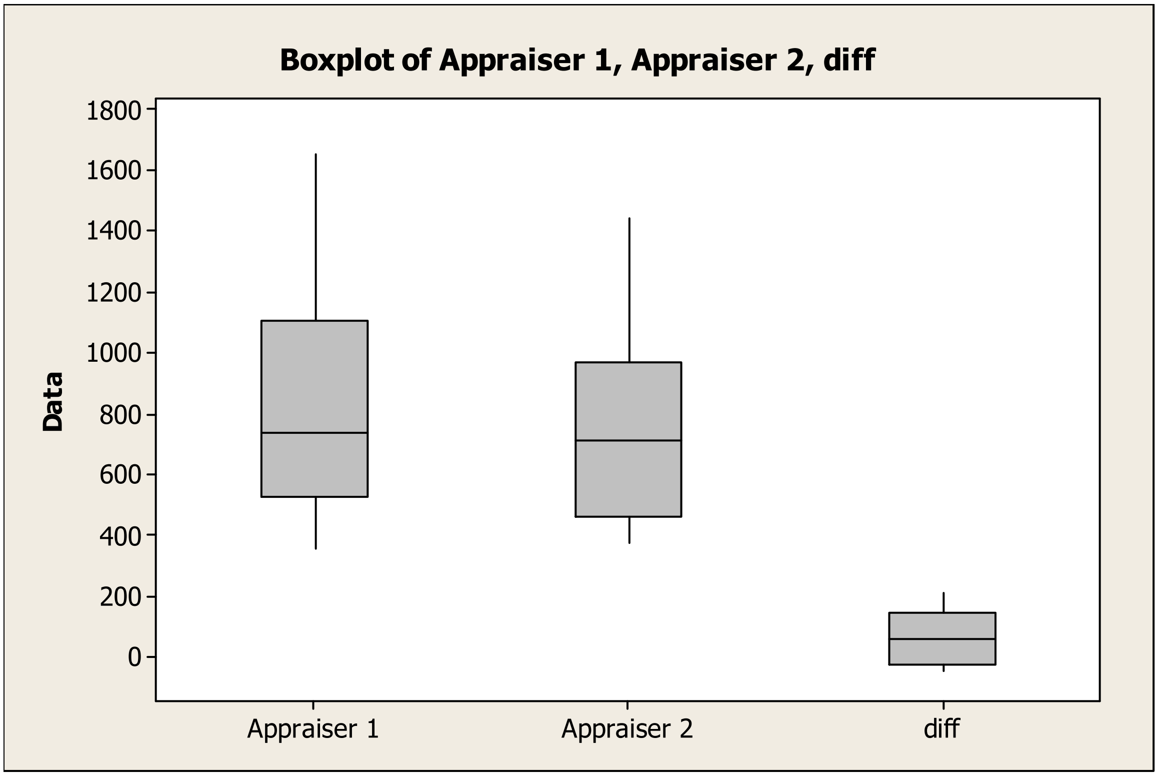
(j) Looking now at **Model 3** only, test whether there is an overall difference between the amounts financed by US and Canadian funds. Use the 0.05 level of significance.

[3]

(k) Using **Model 3**, calculate a 95% interval for the actual value of "AmountFinanced" if the logarithm of the firm's age is 2 and it obtained financing from a US fund. Note that the "Fit" value given is the logarithm of the predicted amount financed.

[3]

Appendix A.



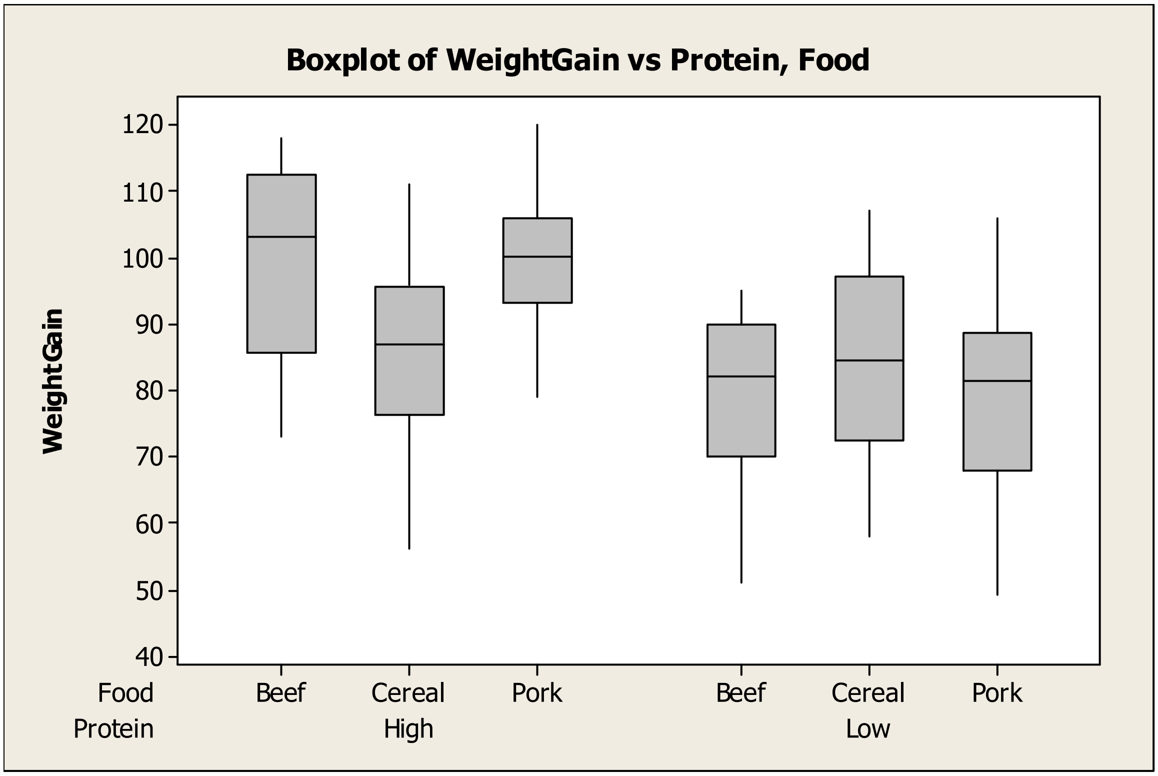
Two-Sample T-Test and CI: Appraiser 1, Appraiser 2

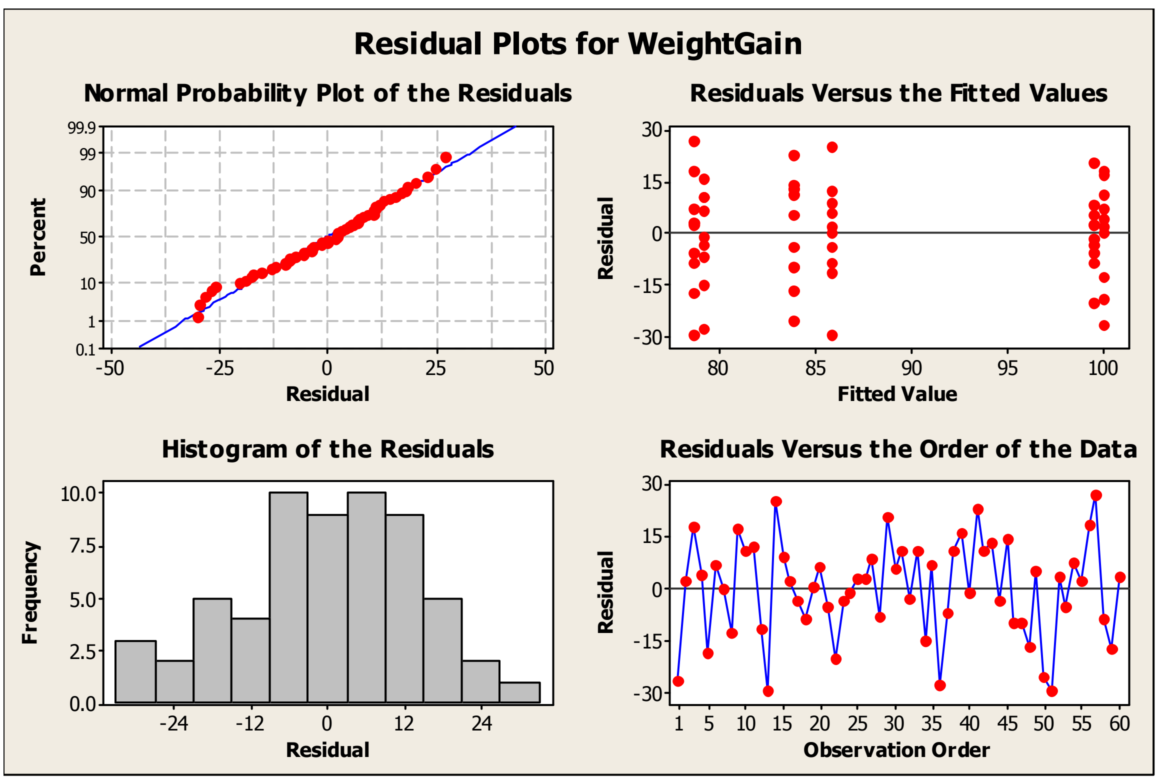
```
Two-sample T for Appraiser 1 vs Appraiser 2

              N  Mean  StDev  SE Mean
Appraiser 1  10   838    393      124
Appraiser 2  10   766    334      106


Difference = mu (Appraiser 1) - mu (Appraiser 2)
Estimate for difference:  72.0000
95% CI for difference:  (-272.0718, 416.0718)
T-Test of difference = 0 (vs not =): T-Value = 0.44  P-Value = 0.664  DF = 17
```

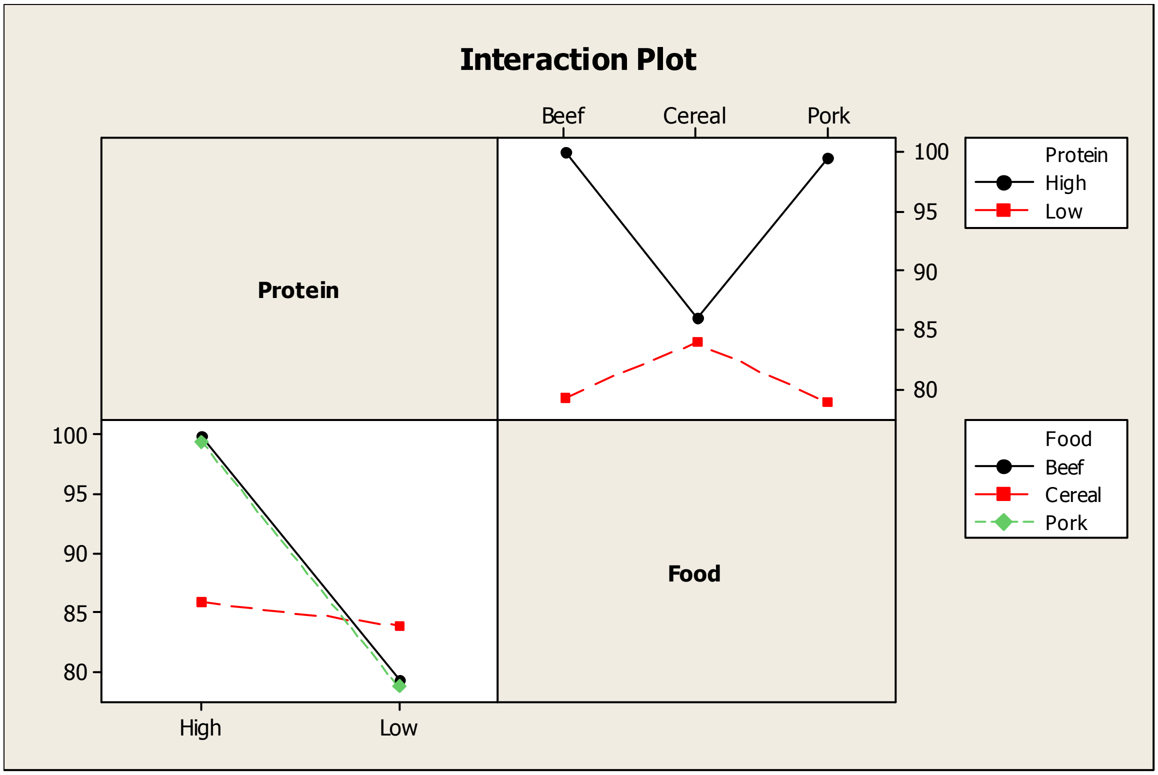
Paired T-Test and CI: Appraiser 1, Appraiser 2

```
Paired T for Appraiser 1 - Appraiser 2

              N     Mean    StDev  SE Mean
Appraiser 1  10  838.000  393.215  124.345
Appraiser 2  10  766.000  333.673  105.517
Difference   10  72.0000  88.7944  28.0793


95% CI for mean difference: (_____ , ______)
T-Test of mean difference = 0 (vs not = 0): T-Value = ______  P-Value = _____
```

Appendix B.


Boxplot of WeightGain vs Protein, Food
WeightGain
120
110
100
90
80
70
60
50
40
Food
Protein
Beef
Cereal
Pork
High
Beef
Cereal
Pork
Low

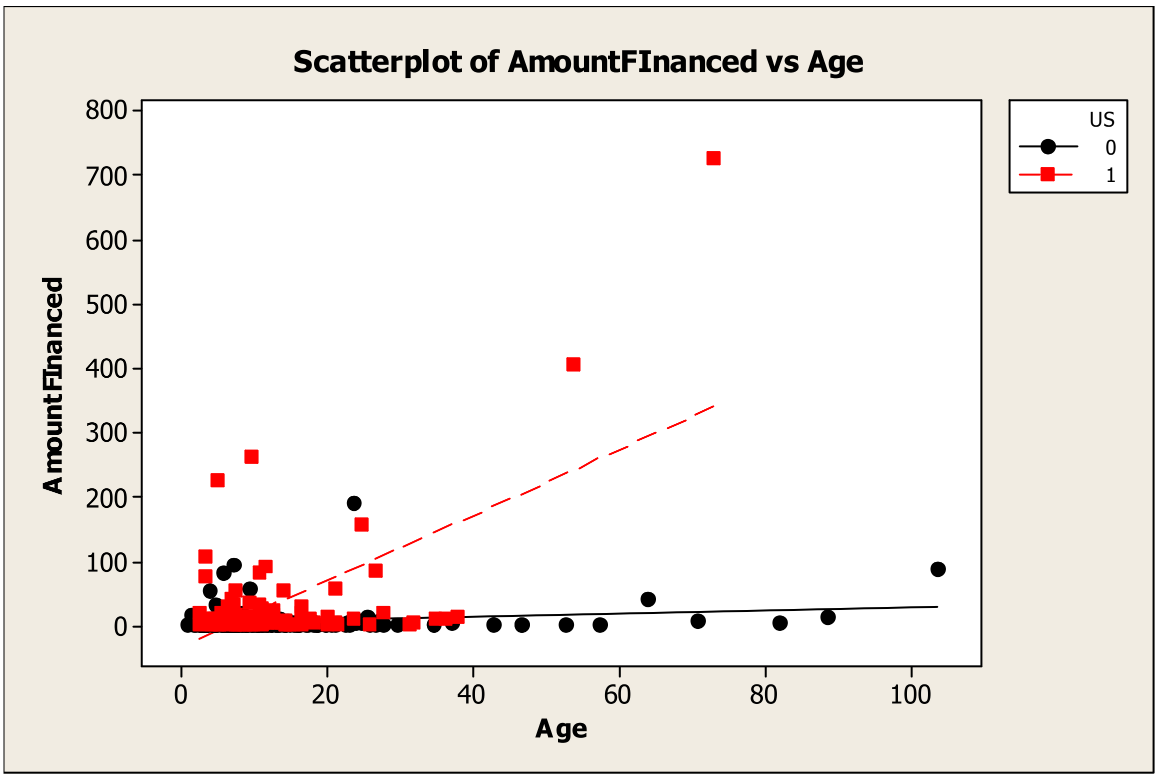

Residual Plots for WeightGain
Normal Probability Plot of the Residuals
Percent
99.9
99
90
50
10
1
0.1
-50
-25
0
25
50
Residual
Residuals Versus the Fitted Values
Residual
30
15
0
-15
-30
80
85
90
95
100
Fitted Value
Histogram of the Residuals
Frequency
10.0
7.5
5.0
2.5
0.0
-24
-12
0
12
24
Residual
Residuals Versus the Order of the Data
Residual
30
15
0
-15
-30
1 5 10 15 20 25 30 35 40 45 50 55 60
Observation Order


Interaction Plot
Beef
Cereal
Pork
100
95
90
85
80
Protein
High
Low
Protein
100
95
90
85
80
High
Low
Food
Beef
Cereal
Pork
Food

Appendix C

The basic variables are defined as follows:

- **"AmountFinanced": amounts of VC invested in startups(in $millions CDN);**
- **"US": A dummy variable coded 1 if the VC fund investing is a US fund; 0 if the fund is Canadian;**
- **"Age": Age of the startup company (number of years since inception). As a startup typically requires a larger amount of capital infusion as it grows, a positive coefficient is expected for this variable.**
- **"Age_US": defined as the product of "US" and "Age".**



Model 1.

Regression Analysis: AmountFinanced versus US, Age

```
The regression equation is
AmountFinanced = - 11.3 + 25.8 US + 1.31 Age


Predictor      Coef  SE Coef      T      P
Constant    -11.340    4.858  -2.33  0.020
US           25.847    6.377   4.05  0.000
Age          1.3055   0.2252   5.80  0.000

S = 51.8565   R-Sq = 14.3%   R-Sq(adj) = 13.7%
```

Analysis of Variance

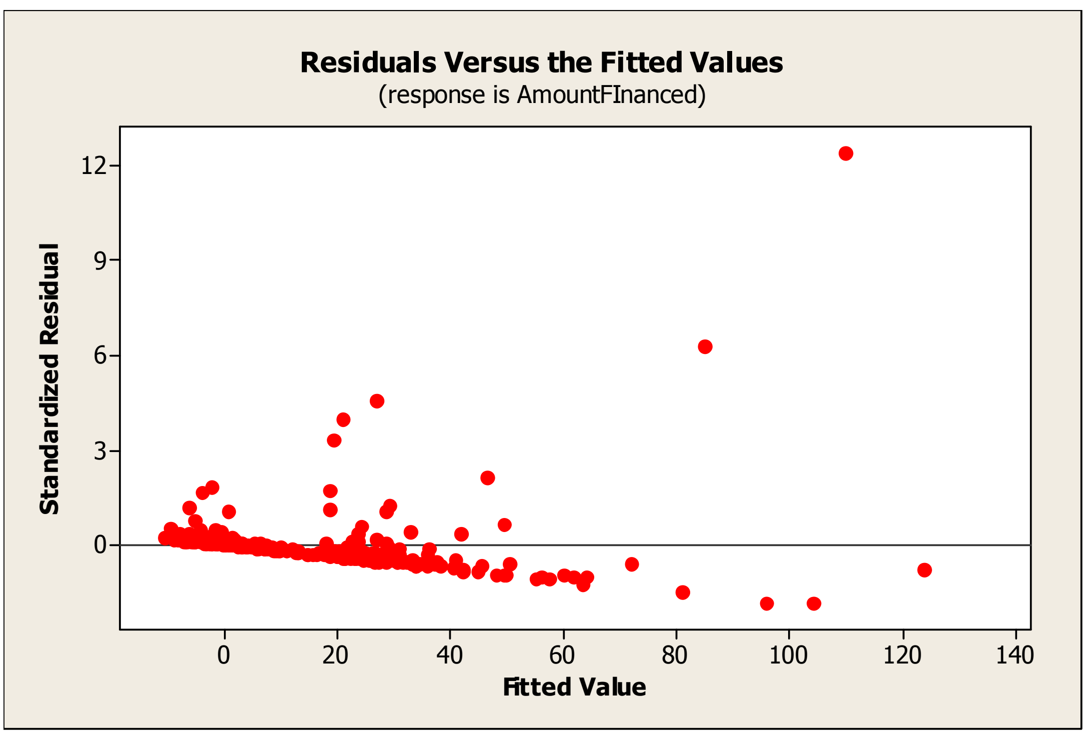
Source	DF	SS	MS	F	P
Regression	2	129368	64684	24.05	0.000
Residual Error	288	774460	2689		
Total	290	903828			

Source	DF	Seq SS
US	1	38965
Age	1	90402

Unusual Observations

Obs	US	AmountFinanced	Fit	SE Fit	Residual	St Resid
30	1.00	260.87	27.00	5.17	233.86	4.53R
32	0.00	6.94	81.12	13.43	-74.18	-1.48 X
35	1.00	404.67	84.91	10.67	319.77	6.30RX
40	0.00	15.00	104.36	17.31	-89.36	-1.83 X
43	0.00	41.67	72.07	11.94	-30.40	-0.60 X
46	0.00	2.00	57.59	9.60	-55.59	-1.09 X
47	1.00	224.50	21.14	5.39	203.36	3.94R
60	0.00	5.63	95.91	15.89	-90.27	-1.83 X
63	0.00	0.00	63.56	10.55	-63.56	-1.25 X
137	0.00	190.00	19.43	4.39	170.57	3.30R
176	0.00	87.00	123.91	20.61	-36.91	-0.78 X
209	1.00	157.16	46.68	5.83	110.48	2.14R
274	1.00	725.63	109.97	14.61	615.66	12.37RX

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large influence.



Model 2.

Regression Analysis: AmountFinanced versus Age, Age_US

The regression equation is
AmountFinanced = - 7.40 + 0.596 Age + 3.34 Age_US

Predictor	Coef	SE Coef	T	P
Constant	-7.404	3.775	-1.96	0.051
Age	0.5958	0.2085	2.86	0.005
Age_US	3.3362	0.3253	10.26	0.000

S = 45.6305 R-Sq = 33.7% R-Sq(adj) = 33.2%

Analysis of Variance

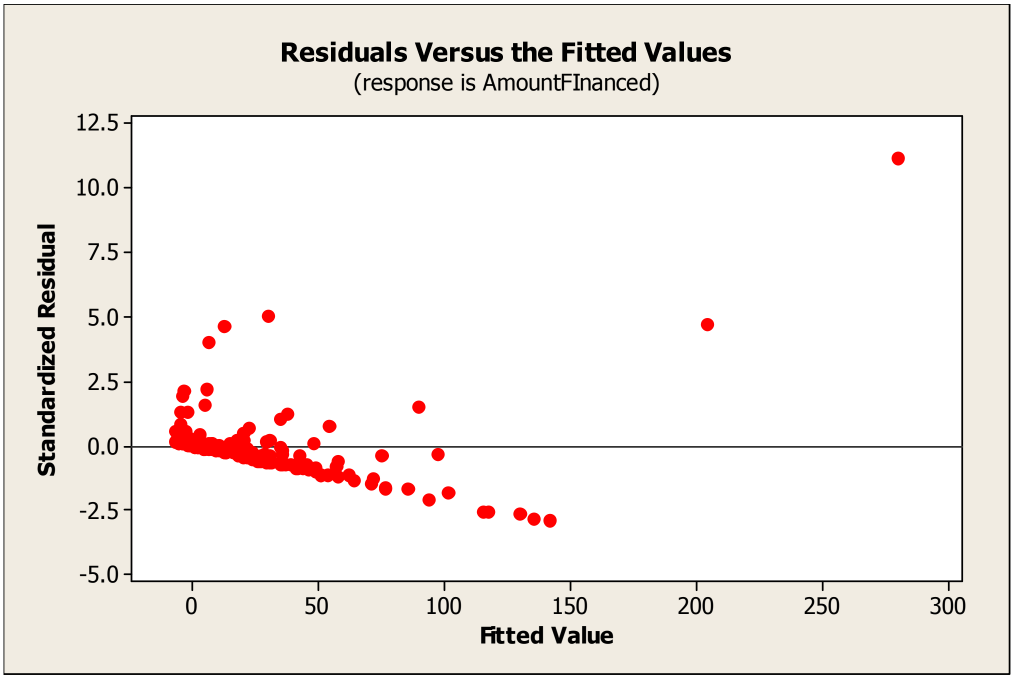
Source	DF	SS	MS	F	P
Regression	2	304171	152085	73.04	0.000
Residual Error	288	599657	2082		
Total	290	903828			

Source	DF	Seq SS
Age	1	85191
Age_US	1	218980

Unusual Observations

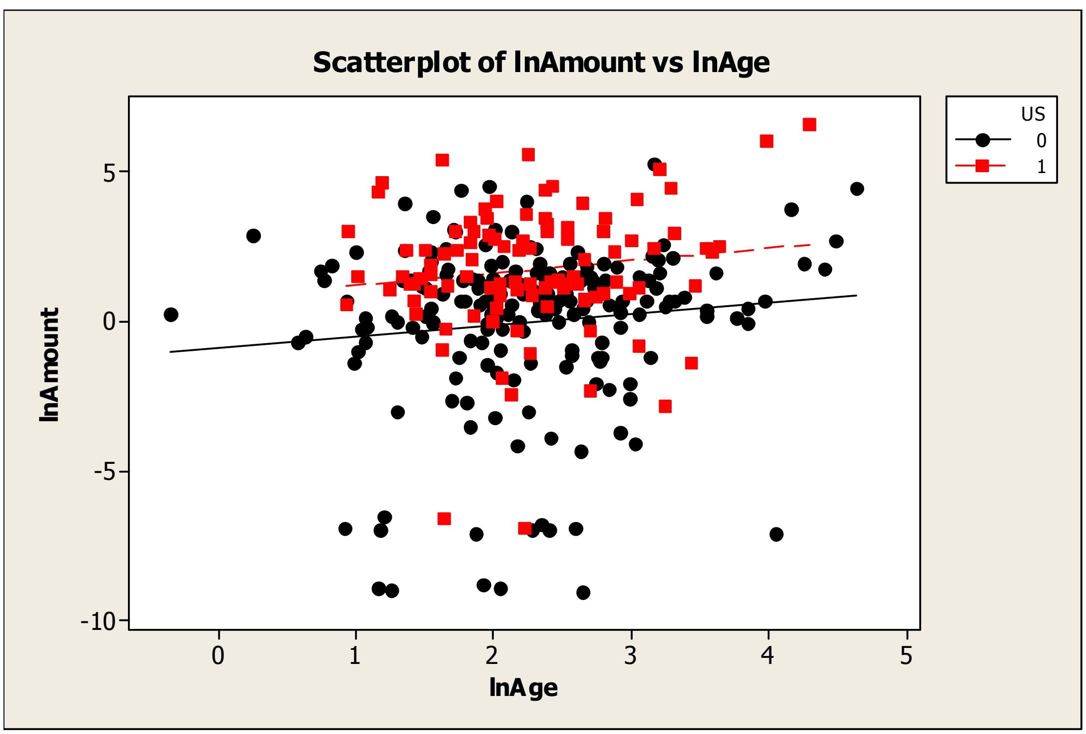
Obs	Age	AmountFinanced	Fit	SE Fit	Residual	St Resid
30	10	260.87	30.22	3.38	230.64	5.07R
32	71	6.94	34.79	12.82	-27.85	-0.64 X
35	54	404.67	204.62	15.92	200.05	4.68RX
40	89	15.00	45.40	16.45	-30.40	-0.71 X
43	64	41.67	30.66	11.41	11.01	0.25 X
46	53	2.00	24.05	9.20	-22.05	-0.49 X
47	5	224.50	12.57	3.21	211.93	4.66R
59	31	0.25	115.22	8.77	-114.97	-2.57RX
60	82	5.63	41.54	15.12	-35.90	-0.83 X
63	57	0.00	26.78	10.11	-26.78	-0.60 X
137	24	190.00	6.64	3.97	183.36	4.03R
171	7	93.63	-3.18	3.10	96.81	2.13R
176	104	87.00	54.32	19.52	32.68	0.79 X
187	26	0.06	93.95	7.16	-93.90	-2.08R
194	38	12.50	141.88	10.87	-129.38	-2.92RX
240	35	11.18	129.78	9.91	-118.61	-2.66RX
244	36	9.95	135.36	10.35	-125.42	-2.82RX
245	32	3.29	117.51	8.95	-114.22	-2.55RX
274	73	725.63	280.13	22.11	445.51	11.16RX
279	3	105.58	5.44	3.33	100.15	2.20R
285	47	1.50	20.47	8.03	-18.97	-0.42 X

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large influence.



Three additional variables were defined and used to estimate Model 3 and Model 4:

- **LnAmount: the natural logarithm of the "AmountFinanced";**
- **LnAge: the natural logarithm of the "Age" variable;**
- **LnAge_US: the product of the LnAge and US variables.**

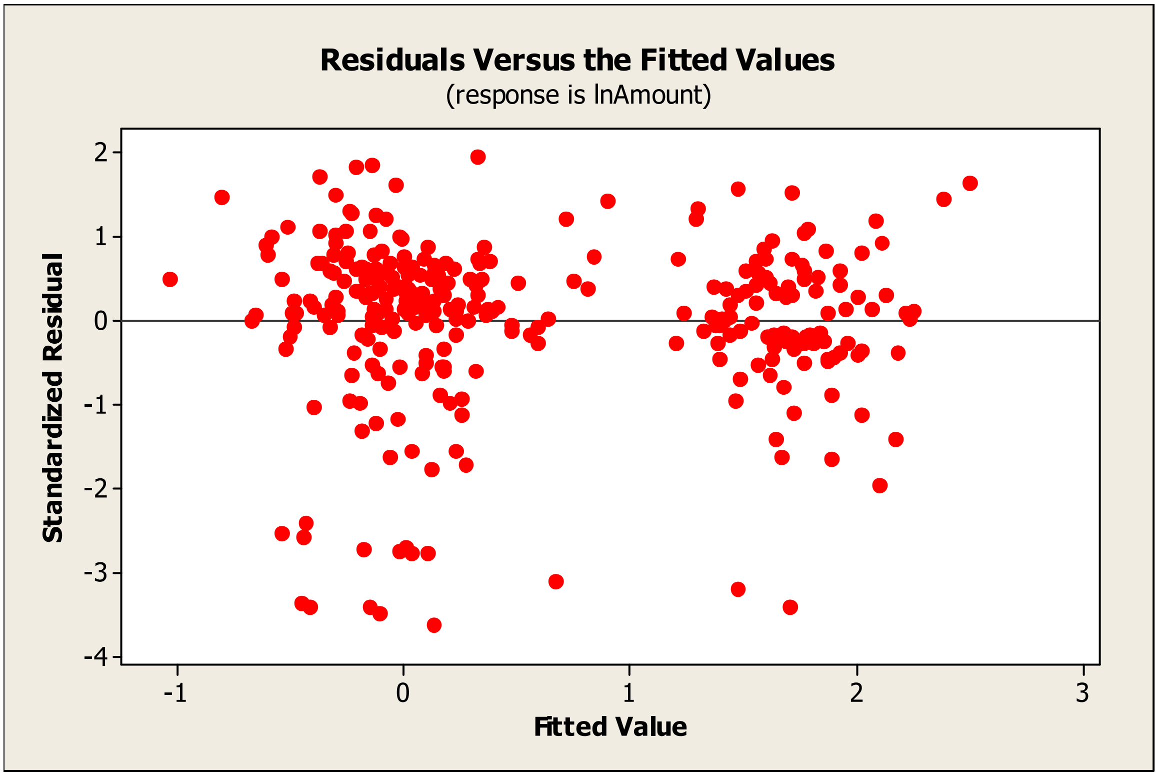


Model 3.

Regression Analysis: LnAmount versus LnAge, US

```
The regression equation is
lnAmount = - 0.892 + 0.386 lnAge + 1.74 US

Predictor      Coef   SE Coef       T       P
Constant    -0.8919    0.4691   -1.90   0.058
lnAge        0.3862    0.1917    2.01   0.045
US           1.7358    0.3115   _____   ______


S = 2.53568   R-Sq = 10.9%   R-Sq(adj) = 10.3%

Analysis of Variance

Source           DF       SS      MS      F       P
Regression        2   226.60  113.30  ______  ______
Residual Error  288  1851.75    6.43
Total           290  2078.35


Source  DF   Seq SS
lnAge    1    27.01
US       1   199.59


Unusual Observations

Obs  lnAge  lnAmount     Fit  SE Fit  Residual  St Resid
  3   2.28    -6.984  -0.012   0.185    -6.972    -2.76R
 29   1.87    -7.080  -0.170   0.198    -6.910    -2.73R
 40   4.48     2.708   0.840   0.466     1.868     0.75 X
 56   1.63    -6.627   1.475   0.278    -8.103    -3.21R
 60   4.41     1.729   0.811   0.453     0.918     0.37 X
 63   4.05    -7.099   0.672   0.391    -7.771    -3.10R
 84   2.23    -6.914   1.706   0.251    -8.619    -3.42R
 86   1.20    -6.538  -0.427   0.273    -6.111    -2.42R
 88   2.05    -8.938  -0.102   0.189    -8.836    -3.49R
108  -0.35     0.223  -1.027   0.532     1.251     0.50 X
122   2.40    -6.960   0.035   0.187    -6.996    -2.77R
133   1.16    -8.935  -0.444   0.279    -8.491    -3.37R
159   0.92    -6.907  -0.535   0.314    -6.371    -2.53R
164   1.92    -8.809  -0.151   0.195    -8.658    -3.42R
176   4.64     4.466   0.900   0.494     3.566     1.43 X
181   1.18    -6.970  -0.437   0.276    -6.534    -2.59R
206   2.34    -6.825   0.013   0.185    -6.838    -2.70R
220   2.65    -9.062   0.132   0.200    -9.194    -3.64R
232   1.26    -9.023  -0.407   0.265    -8.617    -3.42R
274   4.29     6.587   2.502   0.464     4.085     1.64 X
275   2.60    -6.918   0.111   0.196    -7.029    -2.78R

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large influence.

Predicted Values for New Observations

New
Obs    Fit  SE Fit      95% CI             95% PI
  1  1.616   0.256  (______. ______)   (______, ______)
```

```
Values of Predictors for New Observations

New
Obs  lnAge    US
  1   2.00  1.00
```



Model 4.

Regression Analysis: LnAmount versus LnAge, LnAge_US

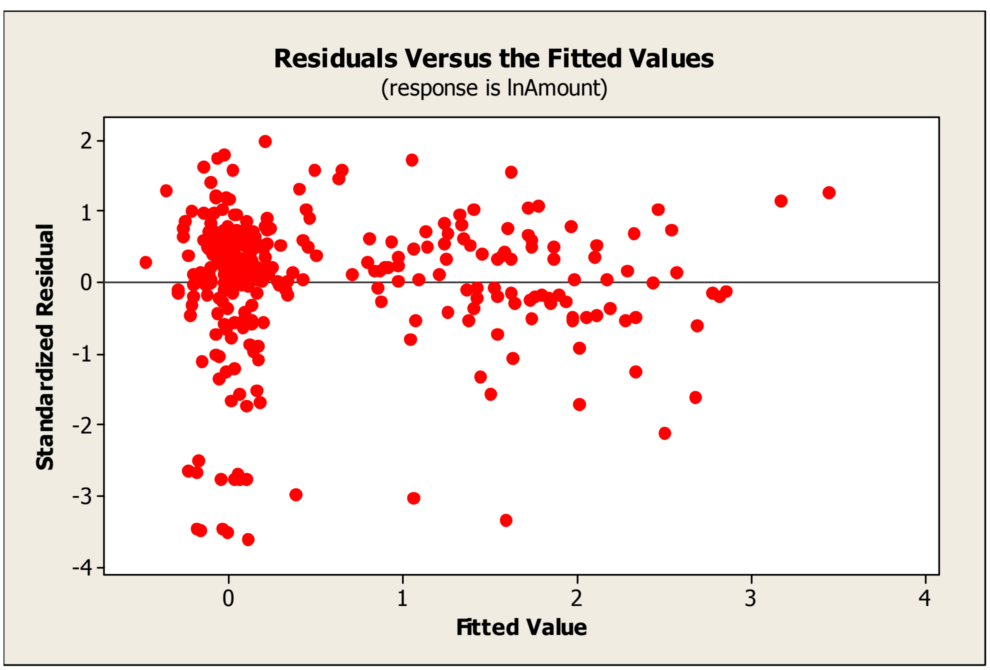
```
The regression equation is
lnAmount = - 0.408 + 0.195 lnAge + 0.701 lnAge_US


Predictor      Coef  SE Coef      T      P
Constant    -0.4079   0.4594  -0.89  0.375
lnAge        0.1949   0.1961   0.99  0.321
lnAge_US     0.7013   0.1320   5.31  0.000


S = 2.54694   R-Sq = 10.1%   R-Sq(adj) = 9.5%

Analysis of Variance

Source           DF       SS      MS      F      P
Regression        2   210.13  105.06  16.20  0.000
Residual Error  288  1868.23    6.49
Total           290  2078.35
```

Source	DF	Seq SS
lnAge	1	27.01
lnAge_US	1	183.11

Unusual Observations

Obs	lnAge	lnAmount	Fit	SE Fit	Residual	St Resid
3	2.28	-6.984	0.036	0.183	-7.020	-2.76R
29	1.87	-7.080	-0.044	0.189	-7.037	-2.77R
32	4.26	1.937	0.422	0.451	1.515	0.60 X
35	3.99	6.003	3.166	0.511	2.837	1.14 X
40	4.48	2.708	0.466	0.492	2.242	0.90 X
56	1.63	-6.627	1.057	0.233	-7.685	-3.03R
60	4.41	1.729	0.451	0.478	1.278	0.51 X
63	4.05	-7.099	0.381	0.414	-7.481	-2.98R
84	2.23	-6.914	1.592	0.242	-8.505	-3.35R
86	1.20	-6.538	-0.173	0.260	-6.365	-2.51R
88	2.05	-8.938	-0.009	0.182	-8.929	-3.51R
108	-0.35	0.223	-0.476	0.523	0.699	0.28 X
122	2.40	-6.960	0.060	0.188	-7.020	-2.76R
133	1.16	-8.935	-0.182	0.266	-8.753	-3.46R
159	0.92	-6.907	-0.228	0.302	-6.679	-2.64R
164	1.92	-8.809	-0.034	0.187	-8.775	-3.45R
176	4.64	4.466	0.496	0.521	3.969	1.59 X
181	1.18	-6.970	-0.178	0.263	-6.792	-2.68R
187	3.25	-2.830	2.504	0.376	-5.335	-2.12R
206	2.34	-6.825	0.049	0.185	-6.874	-2.71R
220	2.65	-9.062	0.109	0.206	-9.171	-3.61R
232	1.26	-9.023	-0.163	0.252	-8.860	-3.50R
274	4.29	6.587	3.439	0.570	3.148	1.27 X
275	2.60	-6.918	0.098	0.201	-7.016	-2.76R

R denotes an observation with a large standardized residual.
X denotes an observation whose X value gives it large influence.



Standard Normal Distribution

P(Z < z) **(z negative)**

Second decimal place in z

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	0.0

Standard Normal Distribution

P(Z < z) (z positive)

Second decimal place in z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Student's t distribution

t_α

$\alpha = P(t > t_\alpha)$ = one-tail probability

df	0.100	0.050	0.025	0.010	0.009	0.008	0.005	0.001	0.0005	0.0004	0.0001
1	3.08	6.31	12.71	31.82	35.36	39.78	63.66	318.31	636.62	837.66	3183.10
2	1.89	2.92	4.30	6.96	7.35	7.81	9.92	22.33	31.60	36.25	70.70
3	1.64	2.35	3.18	4.54	4.72	4.93	5.84	10.21	12.92	14.18	22.20
4	1.53	2.13	2.78	3.75	3.87	4.01	4.60	7.17	8.61	9.25	13.03
5	1.48	2.02	2.57	3.36	3.46	3.57	4.03	5.89	6.87	7.29	9.68
6	1.44	1.94	2.45	3.14	3.23	3.32	3.71	5.21	5.96	6.28	8.02
7	1.41	1.89	2.36	3.00	3.07	3.16	3.50	4.79	5.41	5.67	7.06
8	1.40	1.86	2.31	2.90	2.97	3.04	3.36	4.50	5.04	5.26	6.44
9	1.38	1.83	2.26	2.82	2.89	2.96	3.25	4.30	4.78	4.98	6.01
10	1.37	1.81	2.23	2.76	2.83	2.89	3.17	4.14	4.59	4.77	5.69
11	1.36	1.80	2.20	2.72	2.78	2.84	3.11	4.02	4.44	4.60	5.45
12	1.36	1.78	2.18	2.68	2.74	2.80	3.05	3.93	4.32	4.47	5.26
13	1.35	1.77	2.16	2.65	2.71	2.77	3.01	3.85	4.22	4.37	5.11
14	1.35	1.76	2.14	2.62	2.68	2.74	2.98	3.79	4.14	4.28	4.99
15	1.34	1.75	2.13	2.60	2.66	2.71	2.95	3.73	4.07	4.21	4.88
16	1.34	1.75	2.12	2.58	2.64	2.69	2.92	3.69	4.01	4.15	4.79
17	1.33	1.74	2.11	2.57	2.62	2.67	2.90	3.65	3.97	4.09	4.71
18	1.33	1.73	2.10	2.55	2.60	2.66	2.88	3.61	3.92	4.04	4.65
19	1.33	1.73	2.09	2.54	2.59	2.64	2.86	3.58	3.88	4.00	4.59
20	1.33	1.72	2.09	2.53	2.58	2.63	2.85	3.55	3.85	3.97	4.54
21	1.32	1.72	2.08	2.52	2.57	2.62	2.83	3.53	3.82	3.93	4.49
22	1.32	1.72	2.07	2.51	2.56	2.61	2.82	3.50	3.79	3.91	4.45
23	1.32	1.71	2.07	2.50	2.55	2.60	2.81	3.48	3.77	3.88	4.42
24	1.32	1.71	2.06	2.49	2.54	2.59	2.80	3.47	3.75	3.85	4.38
25	1.32	1.71	2.06	2.49	2.53	2.58	2.79	3.45	3.73	3.83	4.35
26	1.31	1.71	2.06	2.48	2.53	2.58	2.78	3.43	3.71	3.81	4.32
27	1.31	1.70	2.05	2.47	2.52	2.57	2.77	3.42	3.69	3.79	4.30
28	1.31	1.70	2.05	2.47	2.51	2.56	2.76	3.41	3.67	3.78	4.28
29	1.31	1.70	2.05	2.46	2.51	2.56	2.76	3.40	3.66	3.76	4.25
30	1.31	1.70	2.04	2.46	2.50	2.55	2.75	3.39	3.65	3.75	4.23
31	1.31	1.70	2.04	2.45	2.50	2.55	2.74	3.37	3.63	3.73	4.22
32	1.31	1.69	2.04	2.45	2.49	2.54	2.74	3.37	3.62	3.72	4.20
33	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.36	3.61	3.71	4.18
34	1.31	1.69	2.03	2.44	2.49	2.54	2.73	3.35	3.60	3.70	4.17
35	1.31	1.69	2.03	2.44	2.48	2.53	2.72	3.34	3.59	3.69	4.15
36	1.31	1.69	2.03	2.43	2.48	2.53	2.72	3.33	3.58	3.68	4.14
37	1.30	1.69	2.03	2.43	2.48	2.52	2.72	3.33	3.57	3.67	4.13
38	1.30	1.69	2.02	2.43	2.47	2.52	2.71	3.32	3.57	3.66	4.12
39	1.30	1.68	2.02	2.43	2.47	2.52	2.71	3.31	3.56	3.65	4.10
40	1.30	1.68	2.02	2.42	2.47	2.52	2.70	3.31	3.55	3.65	4.09
50	1.30	1.68	2.01	2.40	2.45	2.49	2.68	3.26	3.50	3.59	4.01
60	1.30	1.67	2.00	2.39	2.43	2.48	2.66	3.23	3.46	3.55	3.96
100	1.29	1.66	1.98	2.36	2.41	2.45	2.63	3.17	3.39	3.47	3.86
1000	1.28	1.65	1.96	2.33	2.37	2.41	2.58	3.10	3.30	3.38	3.73

Chi-square distribution

$\chi2_{\alpha;\nu}$

$\alpha = P(\chi^2 \geq \chi^2_{\alpha;\nu})$

ν	0.990	0.980	0.975	0.950	0.900	0.100	0.050	0.025	0.020	0.010
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	5.41	6.63
2	0.02	0.04	0.05	0.10	0.21	4.61	5.99	7.38	7.82	9.21
3	0.11	0.18	0.22	0.35	0.58	6.25	7.81	9.35	9.84	11.34
4	0.30	0.43	0.48	0.71	1.06	7.78	9.49	11.14	11.67	13.28
5	0.55	0.75	0.83	1.15	1.61	9.24	11.07	12.83	13.39	15.09
6	0.87	1.13	1.24	1.64	2.20	10.64	12.59	14.45	15.03	16.81
7	1.24	1.56	1.69	2.17	2.83	12.02	14.07	16.01	16.62	18.48
8	1.65	2.03	2.18	2.73	3.49	13.36	15.51	17.53	18.17	20.09
9	2.09	2.53	2.70	3.33	4.17	14.68	16.92	19.02	19.68	21.67
10	2.56	3.06	3.25	3.94	4.87	15.99	18.31	20.48	21.16	23.21

Fisher's F distribution

$F_{\alpha;\nu1;\nu2}$

$\alpha = P(F \geq f_{\alpha;\nu1;\nu2}) =$ **0.050**

ν_1

ν_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95	1.92	1.89
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.93	1.89	1.86	1.84
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.91	1.88	1.84	1.82
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.90	1.86	1.83	1.80
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.89	1.85	1.82	1.79
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.84	1.80	1.77	1.74
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.82	1.78	1.75	1.72