University of Ottawa
ECO 3145 Mathematical Economics I
Fall 2014, Professor Shiell

Exam I (midterm)

Total time: 80 minutes. Total marks: 50. Marks are noted in parentheses beside each question.

1. (8) Mr. Greenthumb wishes to marks out a rectangular flower bed along the side wall of his house. The other three sides are to be marked by wire netting, of which he has only 32 ft. available. What are the length L and width W of the rectangle that would give him the largest possible planting area? Prove that your answer gives the largest, not the smallest area.

2. Consider the function $f(x_1, x_2) = 100 - 5x_1 + 4x_1^2 - 9x_2 + 5x_2^2 + 8x_1x_2$.

   a.) (9) Find the stationary point of the function.

   b.) (7) Use the second-order conditions to determine whether the point is a local maximum, local minimum, or neither.

3. (5) Sketch the graph of the set $\{(x, y)\ |xy \geq 1, x > 0, y > 0\}$. Is this set convex? Why or why not?

4. Consider the function $y = 2x_1^2 - x_1x_2 + x_2^2$.

   a.) (9) Using the definition of concave and convex functions, determine whether the function is concave, convex, strictly concave, strictly convex, or neither.

   b.) (4) What is the implication of your answer in (a.) for a stationary point of the function?

5. Consider the equation $x^2 + 3xy + 2yz + y^2 + z^2 - 11 = 0$.

   a.) (5) Show that the equation implicitly defines z as a function of x and y in the neighbourhood of $(x = 1,\ y = 2, z = 0)$.

   b.) (3) Find the derivative dz/dx at this point.

Menu of useful formulas and definitions

$$f(y) - f(x) \geq \sum_{i=1}^{n} f_i'(x)(y_i - x_i)$$

$$z = (1-\lambda)x + \lambda y$$

$$\lambda f(x) + (1-\lambda) f(y) \leq f(\lambda x + (1-\lambda) y)$$

$$\frac{\partial y}{\partial x_i} = -\frac{F_i}{F_y}$$

$$f(y) - f(x) \leq \sum_{i=1}^{n} f_i'(x)(y_i - x_i)$$

$$\lambda f(x) + (1-\lambda) f(y) \geq f(\lambda x + (1-\lambda) y)$$

$$H = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ f_{n1} & f_{n2} & \cdots & f_{nn} \end{bmatrix}$$

$$\left|J_y\right| = \begin{vmatrix} \frac{\partial F^1}{\partial y_1} & \frac{\partial F^1}{\partial y_2} & \cdots & \frac{\partial F^1}{\partial y_n} \\ \frac{\partial F^2}{\partial y_1} & \frac{\partial F^2}{\partial y_2} & \cdots & \frac{\partial F^2}{\partial y_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F^n}{\partial y_1} & \frac{\partial F^n}{\partial y_2} & \cdots & \frac{\partial F^n}{\partial y_n} \end{vmatrix}$$