

Assignment 10: Thermal Physics Ideal Gas Law, Thermal Expansion, Calorimetry

Released: Friday Nov 21

Due Friday Nov 28, 6:00PM

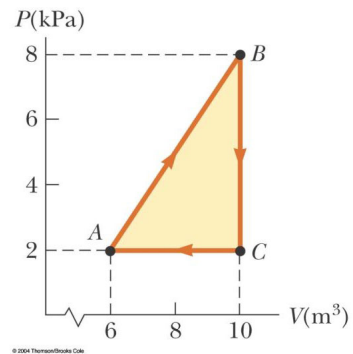
1 Prove that $pV^\gamma = \text{const.}$ for adiabatic process. /Use the First Law of Thermodynamics and Ideal Gas Law in their differential forms./ Use the opposite side of this page for this proof./ FULL DERIVATION WAS PROVIDED IN CLASS -check the your notes./

1. A gas is taken through the cyclic process described in Figure below.
 (a) Find the net energy transferred to the system by heat during one complete cycle. (b) **What If?** If the cycle is reversed—that is, the process follows the path $ACBA$ —what is the net energy input per cycle by heat?

(a) $Q = -W = \text{Area of triangle}$

$$Q = \frac{1}{2}(4.00 \text{ m}^3)(6.00 \text{ kPa}) = \boxed{12.0 \text{ kJ}}$$

(b) $Q = -W = \boxed{-12.0 \text{ kJ}}$



2. One mole of an ideal gas is contained in a cylinder with a movable piston. The initial pressure, volume, and temperature are P_i , V_i , and T_i , respectively. Find the work done on the gas for the following processes and show each process on a PV diagram: (a) An isobaric compression in which the final volume is one-half the initial volume. (b) An isothermal compression in which the final pressure is four times the initial pressure. (c) An isovolumetric process in which the final pressure is three times the initial pressure.

- P20.58** (a) Work done by the gas is the negative of the area under the PV curve

$$W = -P_i \left(\frac{V_i}{2} - V_i \right) = \boxed{+\frac{P_i V_i}{2}}$$

- (b) In this case the area under the curve is $W = -\int P dV$. Since the process is isothermal,

$$PV = P_i V_i = 4P_i \left(\frac{V_i}{4} \right) = nRT_i$$

$$\text{and } W = - \int_{V_i}^{V_i/4} \left(\frac{dV}{V} \right) (P_i V_i) = -P_i V_i \ln \left(\frac{V_i/4}{V_i} \right) = P_i V_i \ln 4$$

$$= \boxed{+1.39 P_i V_i}$$

- (c) The area under the curve is 0 and $\boxed{W = 0}$.

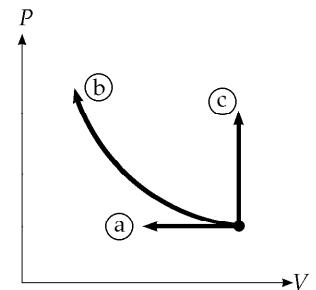
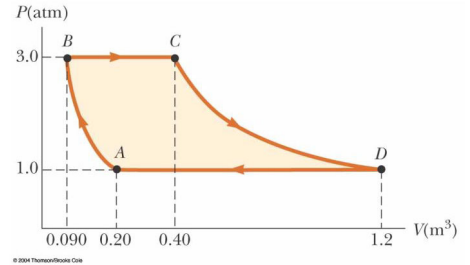


FIG. P20.58

3 A sample of an ideal gas goes through the process shown in Figure P20.32. From A to B, the process is adiabatic; from B to C, it is isobaric with 100 kJ of energy entering the system by heat. From C to D, the process is isothermal; from D to A, it is isobaric with 150 kJ of energy leaving the system by heat. Determine the difference in internal energy $E_{\text{int},B} - E_{\text{int},A}$.



$$W_{BC} = -P_B(V_C - V_B) = -3.00 \text{ atm}(0.400 - 0.090 \text{ m}^3) = -942 \text{ kJ}$$

$$\Delta E_{\text{int}} = Q + W$$

$$E_{\text{int},C} - E_{\text{int},B} = (100 - 94.2) \text{ kJ}$$

$$E_{\text{int},C} - E_{\text{int},B} = 5.79 \text{ kJ}$$

$$E_{\text{int},D} - E_{\text{int},C} = 0$$

$$W_{DA} = -P_D(V_A - V_D) = -1.00 \text{ atm}(0.200 - 1.20) \text{ m}^3 = +101 \text{ kJ}$$

$$E_{\text{int},A} - E_{\text{int},D} = -150 \text{ kJ} + (+101 \text{ kJ}) = -48.7 \text{ kJ}$$

$$\text{Now, } E_{\text{int},B} - E_{\text{int},A} = -[(E_{\text{int},C} - E_{\text{int},B}) + (E_{\text{int},D} - E_{\text{int},C}) + (E_{\text{int},A} - E_{\text{int},D})]$$

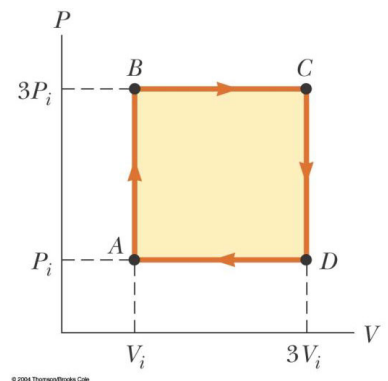
$$E_{\text{int},B} - E_{\text{int},A} = -[5.79 \text{ kJ} + 0 - 48.7 \text{ kJ}] = \boxed{42.9 \text{ kJ}}$$

4 An ideal gas initially at 300 K undergoes an isobaric expansion at 2.50 kPa. If the volume increases from 1.00 m³ to 3.00 m³ and 12.5 kJ is transferred to the gas by heat, what are

(a) $\Delta E_{\text{int}} = Q - P\Delta V = 12.5 \text{ kJ} - 2.50 \text{ kPa}(3.00 - 1.00) \text{ m}^3 = \boxed{7.50 \text{ kJ}}$

(b) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$
 $T_2 = \frac{V_2}{V_1}T_1 = \frac{3.00}{1.00}(300 \text{ K}) = \boxed{900 \text{ K}}$

5. An ideal gas initially at P_i , V_i , and T_i is taken through a cycle as in Figure P20.38. (a) Find the net work done on the gas per cycle. (b) What is the net energy added by heat to the system per cycle? (c) Obtain a numerical value for the net work done per cycle for 1.00 mol of gas initially at 0°C.



(a) The work done during each step of the cycle equals the negative of the area under that segment of the PV curve.

$$W = W_{DA} + W_{AB} + W_{BC} + W_{CD}$$

$$W = -P_i(V_i - 3V_i) + 0 - 3P_i(3V_i - V_i) + 0 = \boxed{-4P_iV_i}$$

(b) The initial and final values of T for the system are equal.

$$\text{Therefore, } \Delta E_{\text{int}} = 0 \text{ and } Q = -W = \boxed{4P_iV_i}.$$

(c) $W = -4P_iV_i = -4nRT_i = -4(1.00)(8.314)(273) = \boxed{-9.08 \text{ kJ}}$