

CARLETON UNIVERSITY

FINAL
EXAMINATION
December 2011

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004 A, B, C, D.
Course Instructor(s): Dr. A. B. Mingarelli (Sect. A), Dr. P. K. Chan (Sect. B), Dr. Z. Montazeri (Sect. C), Mr. L. Bourbonnais (Sect. D), Dr. M. Sadeghi (Sect. E).

AUTHORIZED MEMORANDA
NON-PROGRAMMABLE CALCULATOR PERMITTED.
BLANK SHEETS OF PAPER FOR ROUGH WORK

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1. Please verify that you are in possession of a Scantron FORM
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination is out of a total of 100** and consists of 20 multiple choice questions each worth 5 marks. **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question).

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1. [5 marks] Let $f(x) = \ln(\text{Arcsin } x)$. Evaluate $f'(1/2)$. In other words, find the derivative of f at $x = 1/2$.
(a) $f'(\frac{1}{2}) = \frac{4\sqrt{3}}{\pi}$ (b) $f'(\frac{1}{2}) = \frac{1}{\pi}$ (c) $f'(\frac{1}{2}) = \frac{\sqrt{3}}{4}$ (d) $f'(\frac{1}{2}) = 12$
2. [5 marks] Let $f(x) = |x - 1| + 1$. Calculate $L = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$. (a) $L = 0$ (b) $L = 1$ (c) $L = -1$ (d) This limit does not exist
3. [5 marks] Let $f(x) = e^{x^3} \ln(x^2)$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.
(a) $f'(1) = 0$ (b) $f'(1) = 2e$ (c) $f'(1) = e^2$ (d) $f'(1) = -1$
4. [5 marks] Let $f(x) = |x - 1| + 1$. Calculate $L = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$.
(a) $L = 0$ (b) $L = 12$ (c) $L = 1$ (d) The limit does not exist.
5. [5 marks] Two differentiable functions f, g are defined by $f(x) = \sin x$ and $g(x) = \cos x$. What is the value of the derivative of their composition $g(f(x))$ at $x = \frac{\pi}{2}$?
(a) -3 (b) -1 (c) 3.2 (d) 0
6. [5 marks] Let $f(x) = 3^{5 \cos x}$. Evaluate $f'(x)$. In other words, find the derivative of f at x .
(a) $f'(x) = 3^{-5 \sin x}$ (b) $f'(x) = 5 \cdot 3^{5 \cos x}$ (c) $f'(x) = 5 \cos x \cdot 3^{4 \cos x - 1}$ (d) $f'(x) = -(5 \sin x \ln 3) \cdot 3^{5 \cos x}$
7. [5 marks] Evaluate the limit: $L = \lim_{x \rightarrow \infty} \{-2 + x^2 4^{-2x}\}$.
(a) $L = 1$ (b) $L = -2$ (c) $L = 3.61$ (d) $L = 1.22$
8. [5 marks] Evaluate $\lim_{x \rightarrow 0^+} \frac{d}{dx} \int_1^{\sqrt{x}} \frac{\sin t^2}{t} dt$.
(a) $I = 0$ (b) $I = \frac{5}{2}$ (c) This limit does not exist (d) $I = \frac{1}{2}$

9. [5 marks] Let u be given implicitly as a differentiable function of v by $e^{uv} + u^2v = 1$. Calculate the value of the derivative $\frac{du}{dv}$ at the point (u, v) where $u = 0, v = 1$:
 (a) 4, (b) -1 , (c) 5, (d) 0
10. [5 marks] Find the critical points of the function f defined by $f(x) = \frac{\sqrt{x}}{x-2}$.
 (a) $x = 0, x = 1, x = 2$ (b) $x = -1$ (c) $x = 0$ only (d) $x = 0$ and $x = 2$ only.
11. [5 marks] Determine where the function f whose values are given by $f(x) = x^4 - 2x^3 + 6$ is concave up.
 (a) $\{x : x < 0\}$ (b) $\{x : 1 < x\}$ (c) $\{x : 0 < x < 1\}$ (d) $\{x : 2 < x < \infty\}$
12. [5 marks] Find all the asymptotes of the function defined by $p(x) = \frac{x+1}{(x-1)(x-2)}$.
 (a) Vertical asymptotes at $x = -1, 1, 2$ only. No horizontal asymptotes. (b) Vertical asymptotes at $x = 1$ and $x = 2$. Horizontal asymptote given by $y = 0$. (c) Vertical asymptotes at $x = -1$ and $x = 1$ only. Horizontal asymptote given by $y = 2$. (d) Vertical asymptotes at $x = 1$ and $x = 2$. Horizontal asymptote at $y = -1$.
13. [5 marks] An antiderivative of $f(x) = \tan(x+1)$ is given by
 (a) $\ln(\sec^2(x+1)) - 1$ (b) $\ln|\cos(x+1)| + 2$ (c) $\ln(\sin x)$ (d) $-\ln|\cos(x+1)|$.
14. [5 marks] Which of the following functions has a point of inflection at $x = 0$?
 (a) $f(x) = x^3 + x - 2$ (b) $f(x) = -x^2 - 4x + 8$ (c) $f(x) = x^4 + x + 4$ (d) $f(x) = x^2 + x + 1$
15. [5 marks] Evaluate $\int_0^3 x \sqrt{x+1} dx$
 (a) $\frac{116}{15}$ (b) $\frac{124}{15}$ (c) $\frac{2428}{25}$ (d) $\frac{298}{15}$
16. [5 marks] The improper integral $\int_0^\infty x 3^{-x^2} dx$ is given by
 (a) $\frac{1}{2 \ln 3}$ (b) 1 (c) $\frac{1}{9}$ (d) $\frac{1}{3} \ln 3$
17. [5 marks] The value of $\int_{1/3}^1 2t^3 \ln 3t dt$ is given by
 (a) $2 \ln 3$ (b) $\frac{61}{162}$ (c) $\frac{1}{3} \ln 3 - \frac{8}{27}$ (d) $\frac{1}{2} \ln 3 - \frac{10}{81}$
18. [5 marks] Evaluate and simplify the indefinite integral: $\int (x^2 + 1) e^{-3x} dx$.
 (a) $\frac{e^{-3x}}{9} (6x^2 + 9x + 2) + C$ (b) $-\frac{e^{-3x}}{27} (x^2 + 6x + 1) + C$ (c) $-\frac{e^{-3x}}{27} (9x^2 + 6x + 11) + C$ (d) $\frac{e^{-3x}}{27} (9x^2 - 6x + 2) + C$
19. [5 marks] Find an expression for the area between the curves defined by $y = 2$ and $y = 2x^2$ in the first quadrant.
 (a) $2 \int_0^1 (1 - x^2) dx$ (b) $\int_0^1 (x - \sqrt{x}) dx$ (c) $\int_0^1 (x - 2x^2) dx$ (d) $\int_0^1 (x - x^2) dx$
20. [5 marks] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \cos x$ between $x = 0$ and $x = \pi/2$ about the y -axis.
 (a) $\pi \int_0^{\pi/2} x^2 \cos x dx$ (b) $\int_0^{\pi/2} \cos x dx$ (c) $\int_0^{\pi/2} x \sin x dx$ (d) $\pi \int_0^{\pi/2} x \cos x dx$