CARLETON UNIVERSITY

FINAL EXAMINATION
MATH 1004 A, B, C, D, E, G
December 2011

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1004 A, B, C, D, E.
Course Instructor(s): Dr. A.B. Mingarelli (Sect. A), Dr. P.K. Chan (Sect. B), Dr. Z. Montazeri (Sect. C), Mr. L. Bourbonnais (Sect. D), Dr. M. Sadeghi (Sect. E)/G

AUTHORIZED MEMORANDA
NON-PROGRAMMABLE CALCULATOR PERMITTED.
BLANK SHEETS OF PAPER FOR ROUGH WORK

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1. Please verify that you are in possession of a Scantron FORM
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination is out of a total of 100** and consists of 20 multiple choice questions each worth 5 marks **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.*, you get 0 marks for that question).

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1. **[5 marks] Let $f(x) = \ln(\text{Arctan}\, x)$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.**
   (a) $f'(1) = \frac{2}{\pi}$ (b) $f'(1) = \frac{1}{\pi}$ (c) $f'(1) = \pi$ (d) $f'(1) = 2\pi$

2. **[5 marks] Let $f(x) = |x - 2| + 2$. Calculate $L = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$.**
   (a) $L = 0$ (b) $L = 1$ (c) $L = -1$ (d) This limit does not exist

3. **[5 marks] Let $f(x) = 3^{x^2+1} \log_3(x^3)$. Evaluate $f'(1)$. In other words, find the derivative of f at $x = 1$.**
   (a) $f'(1) = 0$ (b) $f'(1) = 81$ (c) $f'(1) = 27/\ln 3$ (d) $f'(1) = 3 \ln 3$

4. **[5 marks] Evaluate $L = \lim_{x \to \infty} x^2 \sin \frac{1}{x^2}$.**
   (a) 1 (b) 0 (c) 2 (d) This limit does not exist

5. **[5 marks] A differentiable function f has the property that $f(2) = 3$, $f'(3) = 2$, $f'(2) = 1$, and $f(3) = 5$, which is the value of the derivative of $f(f(x))$ at $x = 2$?**
   (a) 3 (b) 10 (c) 5 (d) 2

6. **[5 marks] Let $f(x) = 4^{3 \sin x}$. Evaluate $f'(x)$. In other words, find the derivative of f at x.**
   (a) $f'(x) = 4^{-3 \sin x}$ (b) $f'(x) = (3 \cos x)\, 4^{3 \sin x - 1}$ (c) $f'(x) = (3 \cos x)(\ln 4)\, 4^{3 \sin x}$
   (d) $f'(x) = (3 \sin x)(\ln 4)\, 4^{3 \sin x}$

7. **[5 marks] Evaluate the limit: $L = \lim_{x \to \infty} \{3 + x^3 e^{-x}\}$.**
   (a) $L = 1$ (b) $L = 3$ (c) $L = 0$ (d) $L = \infty$

8. [5 marks] Evaluate $L = \dfrac{d}{dx}\displaystyle\int_0^{\frac{x^4}{4}} \cos(16\,t^2)\ dt.$

(a) $L = x^3\cos(x^8)$ (b) $L = \cos(x^8)$ (c) $L = \cos(x^8) - 1$ (d) $L = 32x\sin(x^8)$

9. [5 marks] Let y be given implicitly as a differentiable function of x by $x^3 + y^2 + e^y = 2$. Calculate the value of the derivative $\dfrac{dy}{dx}$ at the point (x, y) where $x = 1$, $y = 0$:

(a) -4, (b) 2, (c) 0, (d) -3

10. [5 marks] Consider the function f whose rule is defined by $f(x) = x^{3/5}(4 - x)$. Its critical points (numbers) are:

(a) $x = 0,\ x = 4$ (b) $x = 3/2$ only (c) $x = 0,\ x = 3/2,\ x = 4$ (d) $x = 0,\ x = 3/2$.

11. [5 marks] Consider the function f whose rule is given by $f(x) = 3x^5 - 80x^3 + 100$. This function is DECREASING AND CONCAVE DOWN on the interval(s):

(a) $(-\infty, -\sqrt{8}) \cup (0, \sqrt{8})$ (b) $(-4, -\sqrt{8}) \cup (0, \sqrt{8})$ (c) $(-\sqrt{8}, 0) \cup (\sqrt{8}, 4)$ (d) $(-4, 4)$

12. [5 marks] Determine the asymptotes (vertical and horizontal) of the function f whose rule is given by $f(x) = \dfrac{x - 3}{5x^2 - 11x - 12}$.

(a) Vertical asymptotes at $x = -4/5, x = 3$ only. Horizontal asymptote at $y = 0$. (b) Vertical asymptotes at $x = -4/5$. Horizontal asymptote given by $y = 1/5$. (c) Vertical asymptote at $x = -4/5$ only. Horizontal asymptote given by $y = 0$. (d) Vertical asymptotes at $x = -4/5$. No Horizontal asymptote.

13. [5 marks] An antiderivative of the function $f(x) = \tan(\frac{x}{2} + 1)$ is given by

(a) $\ln|\sec^2(\frac{x}{2} + 1)| - 1$ (b) $\ln|\sec(\frac{x}{2} + 1)|$ (c) $\ln|2\csc(\frac{x}{2} + 1)|$ (d) $2\ln|\csc(\frac{x}{2} + 1)|$.

14. [5 marks] Let $f : \mathbf{R} \to \mathbf{R}$ be a function whose second derivative is such that $f''(x) = (x - 1)(x - 2)^2(x - 3)^3(x - 4)^4$. Find the point(s) of inflection of the function f.

(a) $x = 1,\ x = 2$ only (b) $x = 1,\ x = 3$ only (c) $x = 2,\ x = 3$ only
(d) $x = 1,\ x = 2,\ x = 3$ and $x = 4$

15. [5 marks] Evaluate $\displaystyle\int_{-1}^{0} (x + 2)\ \sqrt[3]{x + 1}\ dx$

(a) $\dfrac{3}{11}$ (b) $\dfrac{33}{25}$ (c) $\dfrac{33}{28}$ (d) $\dfrac{11}{28}$

16. [5 marks] The improper integral $\displaystyle\int_0^{\infty} x^2\, e^{-x}\, dx$ is given by

(a) 1 (b) 3 (c) $\dfrac{1}{3}$ (d) 2

17. [5 marks] The value of $\displaystyle\int t^2 \ln t\ dt$ is given by

(a) $-t^2(\ln t)^2 - \dfrac{t^2}{9} + C$ (b) $\dfrac{1}{3}t^3 \ln t - \dfrac{t^3}{9} + C$ (c) $2(\ln t)^2 + \dfrac{1}{9} + C$ (d) $(\ln t)^2 + C$

18. [5 marks] Evaluate and simplify the definite integral: $\displaystyle\int_0^1 \frac{2x}{(x+1)(x+2)}\ dx.$

(a) $\ln 3$ (b) $\ln(81/64)$ (c) $\ln(3/2)$ (d) $\ln(1/2)$

19. [5 marks] Find an expression for the area between the curves defined by $y = 2$ and $y = 2x^2$ in the first quadrant.

(a) $2\displaystyle\int_0^1 (1 - x^2)\ dx$ (b) $\displaystyle\int_0^1 (x - \sqrt{x})\ dx$ (c) $\displaystyle\int_0^1 (x - 2x^2)\ dx$ (d) $\displaystyle\int_0^1 (x - x^2)\ dx$

20. [5 marks] Find an expression for the volume of the solid of revolution obtained by rotating the region in the first quadrant bounded by the curve defined by $y = \cos x$ between $x = 0$ and $x = \pi/2$ about the y–axis.

(a) $\pi\displaystyle\int_0^{\pi/2} x^2\cos x\, dx$ (b) $\displaystyle\int_0^{\pi/2} \cos x\, dx$ (c) $\displaystyle\int_0^{\pi/2} x\sin x\, dx$ (d) $2\pi\displaystyle\int_0^{\pi/2} x\cos x\, dx$