ENGR361/2 V 2011 Fall Midterm Exam

(Total Problems: 5; Total Pages: 7; Total Time: 75 minutes)

Closed Book and Notes

October 7th 2011

Name: x x x x x x **ID:** x x x x x x

Problem 1 (25 Points) – Concepts and Definitions

(a) Write the equation to define Dynamic Viscosity, explain each term briefly (5 points)

$$\tau = \mu \frac{du}{dy}$$ (2)

μ : dynamic or absolute viscosity, $kg \cdot m^{-1} \cdot s^{-1}$ (1)

τ : shear stress, N (1)

$\frac{du}{dy}$: velocity gradient, s^{-1} (1)

(b) Write the equation to define Kinematic Viscosity, explain each term briefly (5 points)

$$\nu = \frac{\mu}{\rho}$$ (2)

ν : kinematic viscosity, $m^2 \cdot s^{-1}$ (1)

μ : dynamic viscosity, $kg \cdot m^{-1} \cdot s^{-1}$ (1)

ρ : density, $kg \cdot m^{-3}$ (1)

(c) What is the definition of fluids and Newtonian Fluids? (5 points)

fluids : deform continuously under shear stress (2)

Newtonian fluids : shear stress is linearly proportional to the rate of shear strain. $\tau = \mu \frac{du}{dy}$ (3)

(d) What is the difference of Bernoulli equation in the following two forms? What is each of three terms called on the left hand side of both equations? (5 points)

$$\frac{p}{\gamma}+\frac{v^2}{2g}+z=\text{constant} \quad \text{(Eq. 1)} \qquad\qquad p+\frac{\rho v^2}{2}+\gamma z=\text{constant} \quad \text{(Eq. 2)}$$

Eq. 1 : Bernoulli equation in "head form" (2)

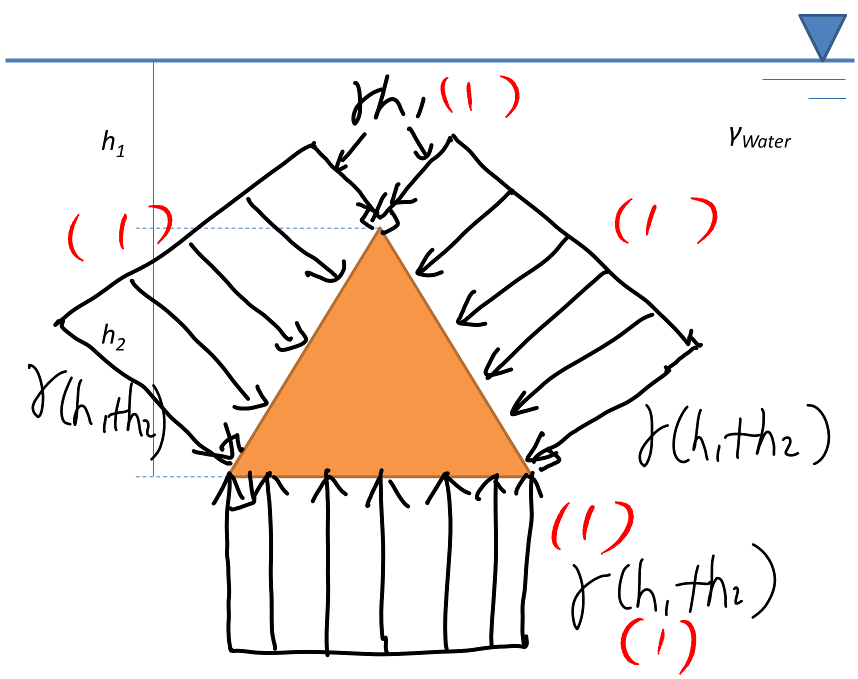
Eq. 2 : Bernoulli equation in "pressure form"

$\frac{p}{\gamma}$: pressure head ; $\frac{v^2}{2g}$: velocity head ; (1)

z : elevation head ; p : static pressure ; (1)

$\frac{\rho v^2}{2}$: dynamic pressure ; γz : hydrostatic pressure. (1)

(e) Draw pressure distribution on EACH edge of the submerged object in the water. (5 points)



Problem 2 (20 Points)

If u is a velocity, x is length, and t is time. What are the dimensions of the following equations:

Dimensions **might** be useful (Problem 1)	
V (velocity)	LT^{-1}
Force or Weight	F
Length	L
Force or Weight	F
p (pressure)	FL^{-2}
ρ (density)	$FL^{-4}T^{2}$
γ (specific weight)	FL^{-3}

(a) $\dfrac{\partial u}{\partial t}$; (b) $\dfrac{\partial^2 u}{\partial x \partial t}$; (c) $\int \dfrac{\partial u}{\partial t} dx$

$$\left[\frac{\partial u}{\partial t}\right] = \frac{[\partial u]}{[\partial t]} = \frac{LT^{-1}}{T} = LT^{-2} \quad (5)$$

$$\left[\frac{\partial^2 u}{\partial x \partial t}\right] = \frac{[\partial u]}{[\partial x][\partial t]} = \frac{LT^{-1}}{LT} = T^{-2} \quad (5)$$

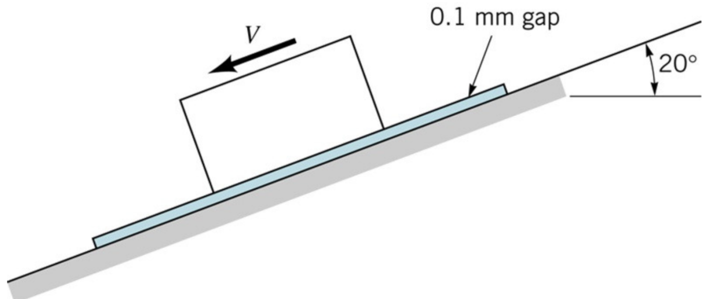
$$\left[\int \frac{\partial u}{\partial t} dx\right] = \frac{[\partial u][dx]}{[\partial t]}$$

$$= \frac{LT^{-1} \cdot L}{T} \quad (10)$$

$$= L^2T^{-2}$$

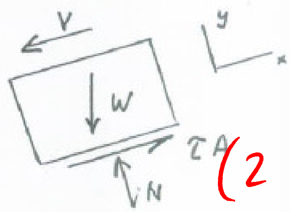
Problem 3 (20 Points)

A 10-kg block slides down a smooth inclined surface. Determine the terminal velocity of the block if the 0.1-mm gap between the block and the surface contains SAE 30 oil at 60 °F. Assume the velocity distribution in the gap is linear, and the area of the block in contact with the oil is 0.1 m^2. (NOTE: SAE 30 oil viscosity: $\mu = 0.38\ \mathrm{N \cdot s/m^2}$; Equations might be used: $\tau = \mu \dfrac{du}{dy}$)



(Problem 3)

$\Sigma F_x = 0$



(2)

Thus,

$W \sin 20° = \tau A$ (5)

Since

$\tau = \mu \dfrac{V}{b}$, where b is film thickness,

$W \sin 20° = \mu \dfrac{V}{b} A$ (5)

Thus, (with $W = mg$)

$$V = \frac{b W \sin 20°}{\mu A} = \frac{(0.0001\,m)(10\,kg)(9.81\,\frac{m}{s^2})(\sin 20°)}{(0.38\,\frac{N \cdot s}{m^2})(0.1\,m^2)}$$

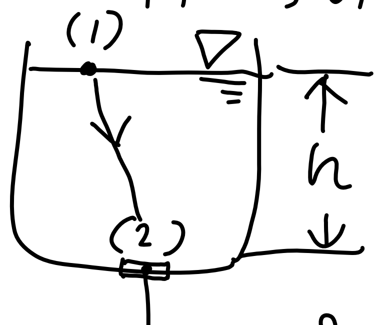
$= \underline{\underline{0.0883\ \frac{m}{s}}}$ (5) (3)

Problem 4 (15 Points)

Water flows through a hole in the bottom of a large, open tank with a speed of 8 m/s. Determine the depth of water in the tank. Viscous effects are negligible. (NOTE: Show steps of finding the equation to solve the problem. Using the final equation from the textbook will not get a full mark. g = 9.81 m/s^2)

$P_1 = 0;\ V_1 = 0;$

$z_1 = h$



$V_2 = 8\,m/s$

$P_2 = 0$

$z_2 = 0$

Assumptions:

(1) neglect viscous effects;

(2) steady state;

(3) incompressible flow;

(4) 1-D case

(5) large tank, free jets at (2)

(5)

Apply Bernoulli equation from (1) → (2)

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 \quad (2)$$

$$P_1 = P_2 = z_2 = V_1 = 0 \quad (3)$$

so: $\gamma z_1 = \frac{1}{2}\rho V_2^2$ where $\gamma = \rho g$

$z_1 = h$

$$\rho g h = \frac{1}{2}\rho V_2^2 \quad (3)$$

$$h = \frac{V_2^2}{2g} = \frac{(8\frac{m}{s})^2}{2(9.81\frac{m}{s^2})} = 3.26m \quad (2)$$

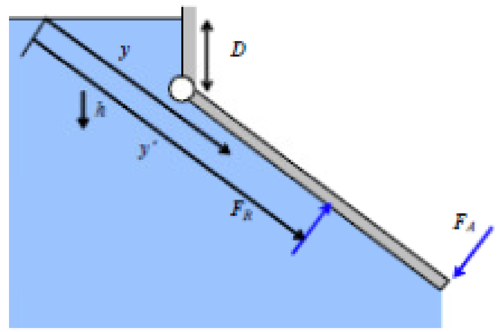
Problem 5 (20 Points)

The gate shown is hinged at *H*. The gate is *3 m* wide normal to the plane of the diagram. Calculate the force required at *A* to hold the gate closed. (Note: neglect the weight of the gate, *F* is perpendicular to the gate)

(**Equations might be useful**: $y_R = \dfrac{I_{xc}}{y_c A} + y_c$, $I_{xc} = \dfrac{1}{12} b a^3$, $\gamma = 9810\ kg \cdot m^{-2} \cdot s^{-2}$)

Given: Geometry of gate

Find: Force at A to hold gate closed



Solution:

Basic equation $\quad \dfrac{dp}{dh} = \rho \cdot g \qquad \Sigma M_z = 0$

Computing equations $\quad F_R = p_c \cdot A \qquad y' = y_c + \dfrac{I_{xx}}{A \cdot y_c} \qquad I_{xx} = \dfrac{w \cdot L^3}{12}$

Assumptions: Static fluid; ρ = constant; p_{atm} on other side; no friction in hinge (5)

For incompressible fluid $\quad p = \rho \cdot g \cdot h \quad$ where p is gage pressure and h is measured downwards

The hydrostatic force on the gate is that on a rectangle of size L and width w.

Hence $\quad F_R = p_c \cdot A = \rho \cdot g \cdot h_c \cdot A = \rho \cdot g \cdot \left(D + \dfrac{L}{2} \cdot \sin(30 \cdot deg)\right) \cdot L \cdot w$ (3)

$$F_R = 1000 \cdot \frac{kg}{m^3} \times 9.81 \cdot \frac{m}{s^2} \times \left(1.5 + \frac{3}{2} \sin(30 \cdot deg)\right) \cdot m \times 3 \cdot m \times 3 \cdot m \times \frac{N \cdot s^2}{kg \cdot m} \qquad F_R = 199 \cdot kN$$ (2)

The location of this force is given by $y' = y_c + \dfrac{I_{xx}}{A \cdot y_c}$ where y' and y_c are measured along the plane of the gate to the free surface

$$y_c = \frac{D}{\sin(30 \cdot deg)} + \frac{L}{2} \qquad y_c = \frac{1.5 \cdot m}{\sin(30 \cdot deg)} + \frac{3 \cdot m}{2} \qquad y_c = 4.5 m$$ (3)

$$y' = y_c + \frac{I_{xx}}{A \cdot y_c} = y_c + \frac{w \cdot L^3}{12} \cdot \frac{1}{w \cdot L} \cdot \frac{1}{y_c} = y_c + \frac{L^2}{12 \cdot y_c} = 4.5 \cdot m + \frac{(3 \cdot m)^2}{12 \cdot 4.5 \cdot m} \qquad y' = 4.67 m$$ (2)

Taking moments about the hinge $\quad \Sigma M_H = 0 = F_R \cdot \left(y' - \dfrac{D}{\sin(30 \cdot deg)}\right) - F_A \cdot L$ (3)

$$F_A = F_R \cdot \frac{\left(y' - \frac{D}{\sin(30 \cdot deg)}\right)}{L} \qquad F_A = 199 \cdot kN \cdot \frac{\left(4.67 - \frac{1.5}{\sin(30 \cdot deg)}\right)}{3} \qquad F_A = 111 \cdot kN$$ (2)

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Mark (Total = 100): X X X X X X